ISSN 2410-2547
Опір матеріалів і теорія споруд/Strength of Materials and Theory of Structures. 2018. № 100

UDK 624.012.3

DESIGN SOLUTIONS OF OPTIMAL SYSTEMS UNDER ACTION OF DEAD AND LIVE MOBILE LOAD

Yu.P. Kitov,
PhD in Technical Sciences, Associate professor

M.A. Verevicheva,
PhD in Technical Sciences, Associate professor

G.L. Vatulia,
Dr. Sci. Eng., Professor of Structural Mechanics and Hydraulics Department

S.V. Deryzemlia,
Postgraduate student of Structural Mechanics and Hydraulics Department

Ukrainian State University of Railway Transport, Kharkiv

Abstract. The authors have considered the design of a three-span beam bridge of a given length under live load. The purpose of this study is obtaining an equally strong system, in which the maximum stresses in all elements are equal to the estimated ones. A few statically determinate and statically indeterminate systems have been considered to determine the optimal one. It has been proved that optimal solutions exist both in the set of statically determinate and indeterminate systems.
Keywords: reinforced steel concrete beam, live load, dead load, beam bridge, equally strong system, design optimization.

1. General

In his monograph [1], A.I. Vinogradov considered the problem of design optimizing and showed that there is no optimal solution in the set of statically indeterminate systems if the geometrical characteristics of the cross-sections of the elements are taken as optimization arguments. The optimal system can be obtained by neglecting under-stressed constraints and is in the set of statically determinate systems.

In this article, the example of optimization of a three-span continuous beam was used to show that an optimal solution can be an element of both statically determinate and statically indeterminate sets if a displacement of support fastenings is allowed.

As before, the optimal internal stress distribution in the system shall be understood as such a set of internal stresses with which the system becomes optimal, for example, a system of minimum volume or value [3, 8-17]. The system is optimal if the maximum stresses are equal to the estimated ones in all its elements, that is, if the system is equally strong. If the stress-strain state and the cross-sections of all the elements are also the same, then the system of equal strength is a system with equal stresses in the design sections.

In the set of statically determinate systems, the stresses are determined from the static equilibrium equations ; their distribution under the given load depends only on the linear dimensions of the elements and on their mutual arrangement and does not depend on the displacement of the support fastenings.

In the set of statically indeterminate systems, the stresses are determined from the equilibrium equations, as well as from the strain compatibility equations [4-7]. Their distribution depends on the dimensions of the structural elements, as well as on the displacement of the support fastenings. Therefore, not only the linear dimensions of the elements should be changed, but also the support fastenings should be displaced to obtain an optimal system in a set of statically indeterminate systems.

2. Problem statements

In this article, the optimization algorithm of the multi-span statically determinate and indeterminate uniform section beams is generalized as compared to [2] for the case of action of dead and live mobile loads and is also applicable to the beams with displaced supports.

A continuous beam of the constant cross-section on the fixed supports is accepted as the initial system (Fig. 1, a). The beam length l and the number of support fastenings n are considered to be given.

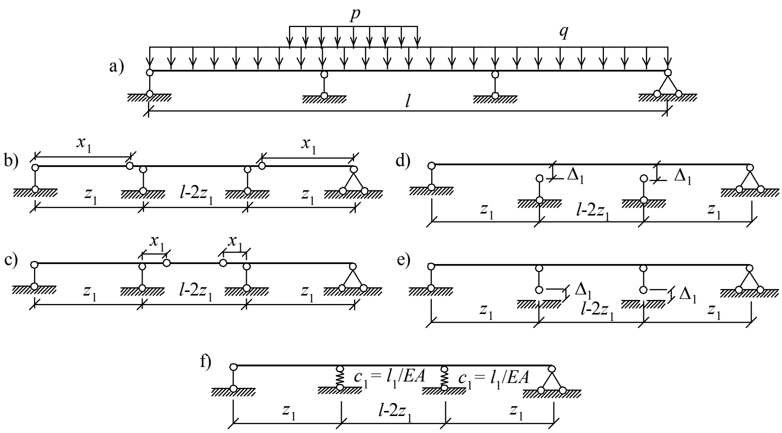

Fig. 1. Design solutions of optimal systems:
a) given continuous three-span beam; b) multi-span beam with hinge joints on the end spans; c) multi-span beam with hinge joints in the middle span; d) beam with unilateral support fastenings; e) beam with displaced supports; f) beam with resilient connections

If the optimal solution is searched for in the set of statically determinate systems, $n-2$ connections should be dropped by mounting the same number of hinge joints (Fig. 1, b, c). In such case, distances from the hinge joints to the support fastenings x_i and span lengths z_i are accepted as optimization parameters. If the number of hinge joints $(n-2)$ is even, then since the system is symmetric, the number of parameters x_i will be two times less, i.e. $(n-2)/2$. If the number of hinge joints is uneven, the number of unknowns is $(n-3)/2+1$. The number of optimization parameters z_i is accepted being ($n-2$) minus the number of parameters x_i.

If the optimal design is searched for in the set of statically indeterminate systems, it is possible with one of three design solutions:

– a beam with unilateral support connections (Fig. 1, d);

– a pre-tensioned system obtained by vertical displacement of the supports (Fig. 1, e);
– a beam with resilient connections (Fig. 1, f).

In all above cases, the optimization parameters which effect the stress distributions are span lengths z_i. Moreover, depending in the choice of the design solution as optimization parameters k_i (coefficient k_i determines the part of distributed load $k_i q$, at which the beam touches the supports, i.e. becomes statically indeterminate); $\Delta_i EJ$ (Δ_i – support displacement) or $c_i EJ$ (c_i – support yield) are chosen, respectively. The number of optimized parameters is determined the same way as with the statically determinate constructions, while the number k_i, $\Delta_i EJ$ or $c_i EJ$ is equal to number x_i.

3. Design optimization of the three-span beam bridge

3.1 Problem statements. An optimal design of a three-span beam bridge of the specified length l under dead load $q = \text{const}$ and live load p (Fig. 1, a) shall be obtained. The beam cross-section is uniform and equal in all spans, the arrangement is symmetrical.

3.2 Optimization conditions. Since the load-bearing elements are beams with the uniform cross-section, they experience the same stress-strain states under the load. Therefore, the optimal bridge design is equal-strength beams - with equal maximum bending moments in the typical cross-sections: in the first span, above the support and in the second span (Fig. 1, a).

There are three typical cross-sections with maximum moments, therefore two optimality conditions should be provided, namely:

$$M_{1\max} = \left| M_{\sup} \right|, \tag{1}$$

$$\left| M_{\sup} \right| = M_{2\max}. \tag{2}$$

Optimality equations are compiled based on these conditions, there are two of them for this problem. Solving these equations together, we determine the optimal design parameters for different types of beams.

Since the number of the unknowns $c_i EJ$, $\Delta_i EJ$ and k_i in this case is 1, we omit index i: cEJ, ΔEJ, k.

The formulas for defining moments in the typical cross-sections under the combined action of dead and live loads are given below.

4. Optimization of a multi-span statically determinate beam

4.1 Multi-span beam with hinge joints in the end spans (Fig. 1, b)

After making the storey plan of the multi-span beam, we obtain the values of the moments in the typical sections from dead load q [2].

Based on the analysis of the influence line of the bending moment, we conclude on the location of the unfavorable load case with the live load (Fig. 2).

Thus, having accepted that the live load intensity p is equal in all parts, we obtain the expressions as follows for the moments in the typical cross-sections:

$$M_{1\max} = \frac{(q+p)x^2}{8}, \tag{3}$$

$$M_{\sup} = \frac{(q+p)z_1(x-z_1)}{2}, \tag{4}$$

$$M_{2\max} = -q\frac{z_1(z_1-x)}{2} + (q+p)\frac{(l-2z_1)^2}{8}. \tag{5}$$

Form (1), we obtain the expression for x:

$$x = 2z_1(\sqrt{2}-1). \tag{6}$$

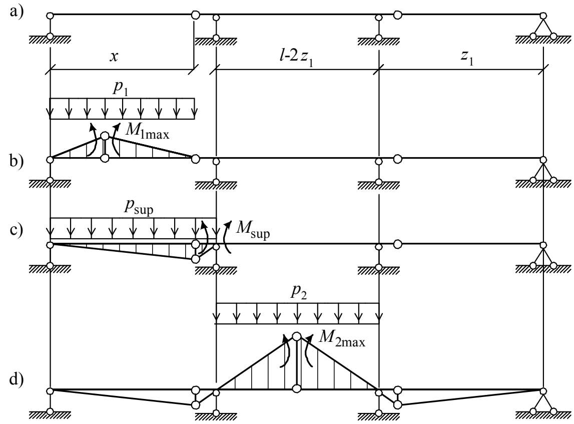

Fig. 2. Multi-span beam with the hinge joints on the end spans (a) and unfavorable load cases for M_1 (b), $M_{\sup}$ (c) and M_2 (d), where $M_{\sup}$ support moment

Parameter z_1 is numerically defined by iterate over the values with increment of 0.001, until the condition is fulfilled (2).

4.2 Multi-span beam with hinge joints in the middle span (Fig. 1, c)

After making the storey plan of the multi-span beam, we obtain the values of the moments in the typical sections from dead load q [2].

Based on the analysis of the influence line of the bending moment, we conclude on the location of the unfavorable load case of the mobile load (Fig. 3).

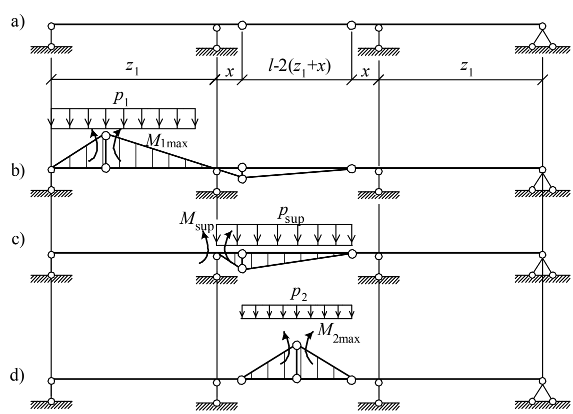

Fig. 3. Multi-span beam with the hinge joints on the end spans (a) and unfavorable load cases for M_1 (b), $M_{\sup}$ (c) и M_2 (d), where $p_{\sup}$ support load

Thus, having accepted that the live load intensity p is equal in all parts, we obtain:

$$M_{1\max} = R_1 x_{\max} - (p+q)\frac{x_{\max}^2}{2}, \tag{7}$$

$$R_1 = \frac{q}{2z_1}\left(z_1^2 - x^2 - z_2 x\right) + \frac{p z_1}{2}, \quad x_{\max} = \frac{R_1}{p+q}, \quad z_2 = l - 2(x + z_1),$$

$$M_{\sup} = -\frac{(q+p)x(z_2 - x)}{2}, \tag{8}$$

$$M_{2\max} = (q+p)\frac{z_2^2}{8}. \tag{9}$$

From (2), we obtain the expression for x:

$$x = \frac{\sqrt{2}-1}{2\sqrt{2}}(l - 2z_1). \tag{10}$$

Parameter z_1 is numerically defined by iterate over the values with increment of 0.001, until the condition is fulfilled (1).

5. Optimization of the continuous beam

We will show the algorithm for determining stresses using the continuous three-span bean (Fig. 4 (a)). Since the beam will be loaded with dead and live mobile loads, stresses will be determined using the influence lines. We will choose the main system by inserting the hinge joints in the middle sections above the supports (Fig. 4 (b)). We accept moments at support as redundancies. Isolated and load stress diagrams of the bending moments in the main system which are required to determine the coefficients of the system of fundamental equations, are shown in Fig.. 4,(c–g)..

The unknown values X_1, X_2 under the live load are defined from the system of fundamental equations

$$\begin{cases} X_1\delta_{11} + X_2\delta_{12} + \delta_{1F} = 0 \\ X_1\delta_{12} + X_2\delta_{22} + \delta_{2F} = 0 \end{cases} \tag{11}$$

and are

$$X_1 = \frac{\delta_{11}\delta_{1F} - \delta_{12}\delta_{2F}}{\delta_{12}^2 - \delta_{11}^2}, \quad X_2 = \frac{\delta_{11}\delta_{2F} - \delta_{12}\delta_{1F}}{\delta_{12}^2 - \delta_{11}^2}. \tag{12}$$

Coefficients of the system of fundamental equations at the unknowns for all spans are equal and determined by the formulas

$$\delta_{11} = \int \frac{\bar{M}_1^2}{EJ} du = \frac{2(l - z_1)}{6EJ}, \tag{13}$$

$$\delta_{12} = \delta_{21} = \int \frac{\bar{M}_1 \bar{M}_2}{EJ} du = \frac{l - 2z_1}{6EJ}. \tag{14}$$

In case of a symmetrical beam $\delta_{22} = \delta_{11}$.

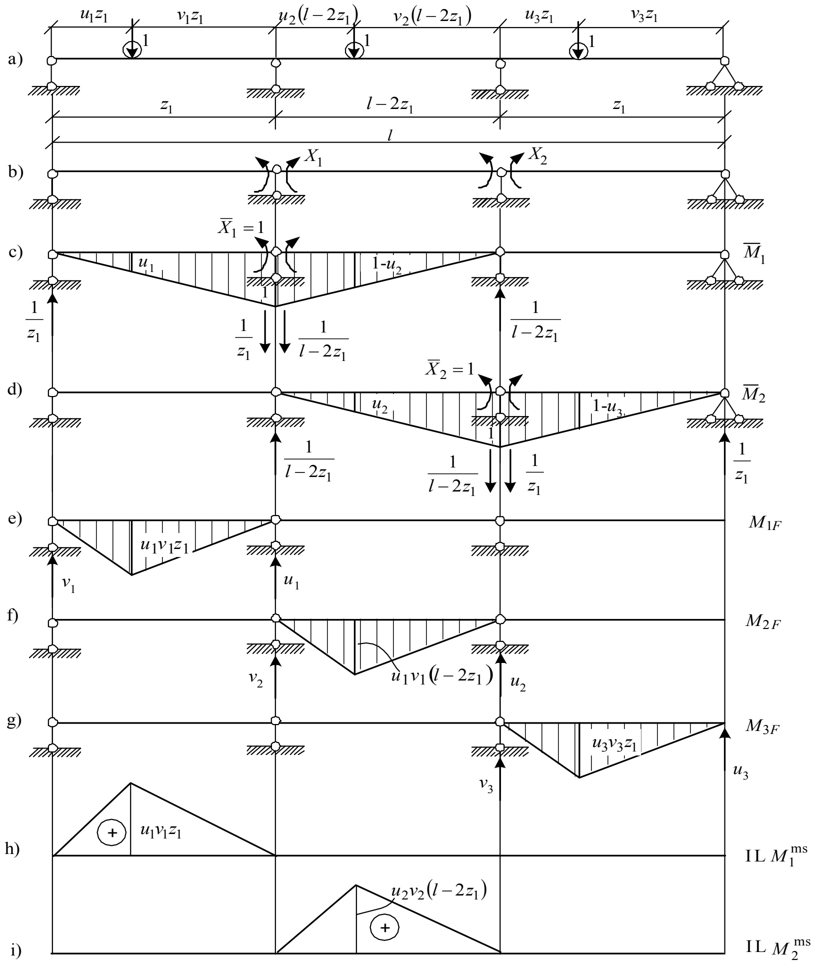

Fig. 4. Continuous beam (a); main system (b); isolated (c, d) and load (e, f, g) stress diagrams in the main systems; influence line M_1^{ms}, M_2^{ms} in cross-sections u_1, u_2 (h, i)

Free terms of the system are written for each span:

– on the first span

$$\delta_{1F(1)} = \int \frac{M_{1F}\bar{M}_1}{EJ} du = \frac{u_1 z_1^2 (1-u_1^2)}{6EJ}, \tag{15}$$

$$\delta_{2F(1)} = \int \frac{M_{1F}\bar{M}_2}{EJ} du = 0; \tag{16}$$

–on the second span

$$\delta_{1F(2)} = \int \frac{M_{2F}\bar{M}_1}{EJ} du = \frac{u_1(1-u_1)(2-u_1)(l-2z_1)^2}{6EJ}, \tag{17}$$

$$\delta_{2F(2)} = \int \frac{M_{2F}\bar{M}_2}{EJ} du = \frac{u_1(1-u_1^2)(l-2z_1)^2}{6EJ}; \tag{18}$$

– on the third span

$$\delta_{1F(3)} = \int \frac{M_{3F}\bar{M}_1}{EJ} du = 0 , \quad (19)$$

$$\delta_{2F(3)} = \int \frac{M_{3F}\bar{M}_2}{EJ} du = \frac{u_1(1-u_1)(2-u_1)z_1^2}{6EJ} . \quad (20)$$

When we solve the system of fundamental equations (11) in each cross-section u, in accordance to (12) we plot the influence lines X_1, X_2. For this purpose, during the numerical solution we divide each span into the given number of equal intervals, and accept points u on the limits of ranges.

Ordinates of the influence lines of the moments in the cross-section u are determined using the formulas

$$\text{IL } M_{1,u} = M_1^{\text{ms}} + \bar{M}_1(u)\,\text{IL } X_1 + \bar{M}_2(u)\,\text{IL } X_2 , \quad (21)$$

$$\text{IL} M_{2,u} = M_2^{\text{ms}} + \bar{M}_1(u)\,\text{IL } X_1 + \bar{M}_2(u)\,\text{IL } X_2 , \quad (22)$$

where M_1^{ms}, M_2^{ms} – influence lines of the bending moment in the main system on the 1st and 2nd spans, accordingly (Fig. 4 (h),(i)).

5.1 Continuous beam with unilateral support connections (Fig. 1, d)

Expressions for the moments in the typical cross-sections loaded with dead load, according to [2], are:

$$M_{1,x} = \left[\frac{xk(l^3 - 2lz_1^2 - z_1^3)}{4z_1(3l - 4z_1)} + \frac{x(-l^3 - 6lz_1^2 + 6l^2 z_1 - z_1^3)}{4z_1(3l - 4z_1)} - \frac{x^2}{2} \right] q , \; 0 \le x \le z_1 , \quad (23)$$

$$M_{\text{sup}} = \frac{-(l - 2z_1)^3 - z_1^3 + (l^3 - 2lz_1^2 + z_1^3)k}{4(3l - 4z_1)} q , \quad (24)$$

$$M_{2\max} = \left[\frac{2(l^3 - 2lz_1^2 + z_1^3)k + l^3 - 4l^2 z_1 + 4lz_1^2 - 2z_1^3}{8(3l - 4z_1)} \right] q . \quad (25)$$

Based on the analysis of the influence line of the bending moment, we conclude on the location of the unfavorable load case with the live load (Fig. 5).

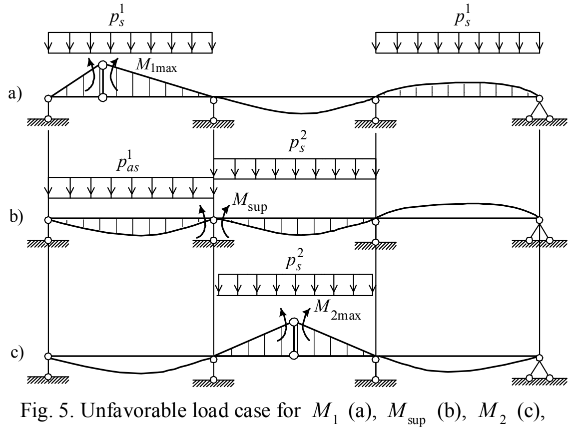

Fig. 5. Unfavorable load case for M_1 (a), M_{sup} (b), M_2 (c), where p_s - symmetrical load; p_{as} - asymmetrical load

We suppose that the live load p is equal in all parts, thus, p: $p_s^1 = p_s^2 = p_{as}^1 = p$.

We obtain the unknown moments X_1, X_2 (Fig. 4 (b)) at the live load from the fundamental equations (11), (12). The coefficients at the unknowns are determined in accordance with (13), (14).

We take as free terms:

– to determine $M_{1\max}$ $\delta_{1F} = \delta_{2F} = \delta_{1F}^{1\max} = \delta_{2F}^{1\max} = \dfrac{pz_1^3}{24EJ}$.

Using the obtained values X_1 and expressions for M_1 from dead load, we obtain [2]

$$M_{1\max} = \frac{\left[(l^3 - 2lz_1^2 + z_1^3)k - l^3 + 6l^2 z_1 - 6lz_1^2 - z_1^3\right]q + 3z_1^2(2l - 3z_1)p}{4z_1(3l - 4z_1)} x_{\max} - (q+p)\frac{x_{\max}^2}{2}, \quad (26)$$

$$x_{\max} = \frac{\left[(l^3 - 2lz_1^2 + z_1^3)k - l^3 + 6l^2 z_1 - 6lz_1^2 - z_1^3\right]q + 3z_1^2(2l - 3z_1)p}{4z_1(3l - 4z_1)(q+p)};$$

– to determine $M_{\sup}$ $\delta_{1F} = \delta_{1F}^{\sup} = \dfrac{p}{24EJ}\left[(l - 2z_1)^3 + z_1^3\right]$;

$$\delta_{2F} = \Delta_{2F}^{\sup} = \frac{p(l - 2z_1)^3}{24EJ}.$$

Using the obtained values X_1 and expressions for $M_{\sup}$ from dead load, we obtain [2]

$$M_{\sup} = -\frac{l\left[(l - 2z_1)^3 + z_1^3 - (l^3 - 2lz_1^2 + z_1^3)k\right]q + \left[l(l - 2z_1)^3 - 2z_1^3(z_1 - l)p\right]}{4l(3l - 4z_1)}; \quad (27)$$

– to determine $M_{2\max}$ $\delta_{1F} = \delta_{2F} = \delta_{1F}^{2\max} = \delta_{2F}^{2\max} = \dfrac{p(l - 2z_1)^3}{24EJ}$.

Using the obtained values X_1 and expressions for $M_{2\max}$ from dead load, we obtain [2]

$$M_{2\max} = \frac{\left[2(l^3 - 2lz_1^2 + z_1^3)k + l^3 - 4l^2 z_1 + 4lz_1^2 - 2z_1^3\right]q + l(l^2 - 4lz_1 + 4z_1^2)p}{8(3l - 4z_1)}. \quad (28)$$

Parameters z_1, k are numerically defined by iterate over the values with increment of 0.001 and 0.0001, respectively, until the conditions are fulfilled (1), (2).

5.2 Continuous beam with the displaced supports (Fig. 1, e)

First, let us calculate support displacement of the beam under only dead load (Fig. 6 (a)), for instance, using the area-moment method. Fig. 6 (c), (d) shows isolated and load stress diagrams in the main system (Fig. 6 (b)), which are required for calculation.

Thus,

ISSN 2410-2547
Опір матеріалів і теорія споруд/Strength of Materials and Theory of Structures. 2018. № 100

$$X_1 = \frac{-l^3 - 2lz_1^2 + z_1^3}{4z_1(3l - 4z_1)} q - 6\frac{\Delta EJ}{z_1^2(3l - 4z_1)},$$

$$M_{\sup}(q) = \frac{-l^3 + 6l^2 z_1 - 12lz_1^2 + 7z_1^3}{4()3l - 4z_1} q + 6\frac{\Delta EJ}{z_1(3l - 4z_1)}, \qquad (29)$$

$$M_{1,u}(q) = \frac{quz_1(l - uz_1)}{2} - X_1 u, \qquad (30)$$

$$M_{2\max}(q) = \frac{l^3 - 4l^2 z_1 + 4lz_1^2 - 2z_1^3}{8(3l - 4z_1)} q + 6\frac{\Delta EJ}{z_1(3l - 4z_1)}. \qquad (31)$$

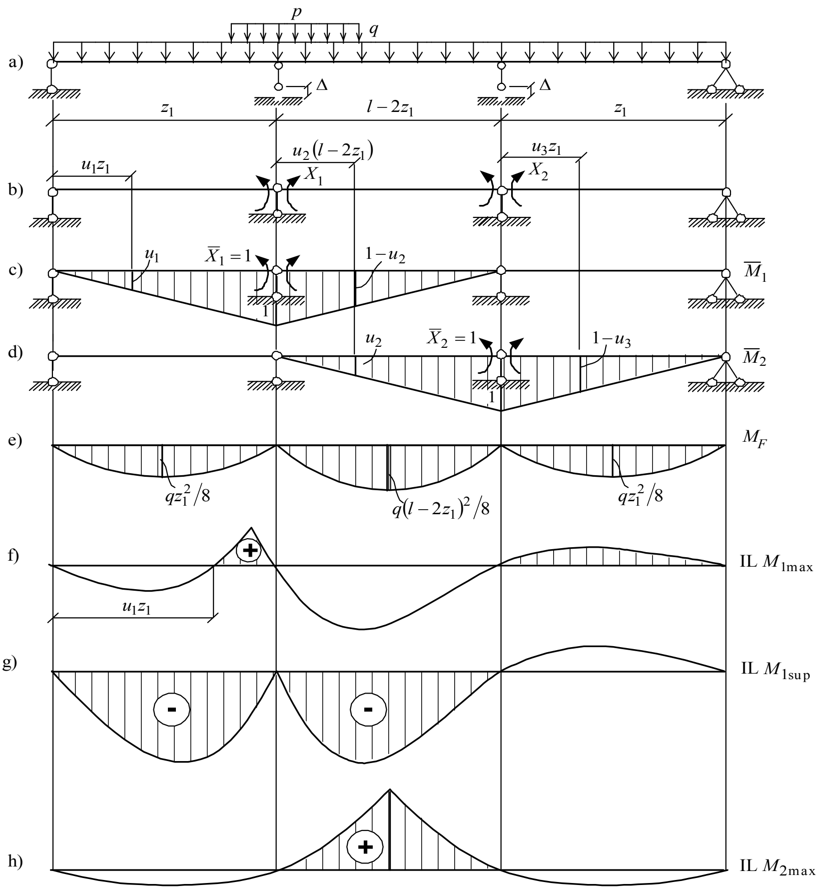

Fig. 6. Beam with the displaced supports (a); main system (b); isolated (c, d) and stress diagrams (e) in the main system; influence lines of the moments in the typical points (f – h)

Let us plot in accordance with (21), (22) using Fig. 4 (e) – and influence lines X_1, X_2 without taking into account the support displacement. After we load them according to Fig. 7 (f – h), we can find the moments in the typical points from the live load without taking into account the support displacement (we accept that the live load intensity p is equal in all parts):

$$M_{1,u}(p)=\int_{u_1 z_1}^{z_1} p\left(\text{IL } M_{1,u}\right)\mathrm{d}u+\int_{l-z_1}^{l} p\left(\text{IL } M_{1,u}\right)\mathrm{d}u\ , \tag{32}$$

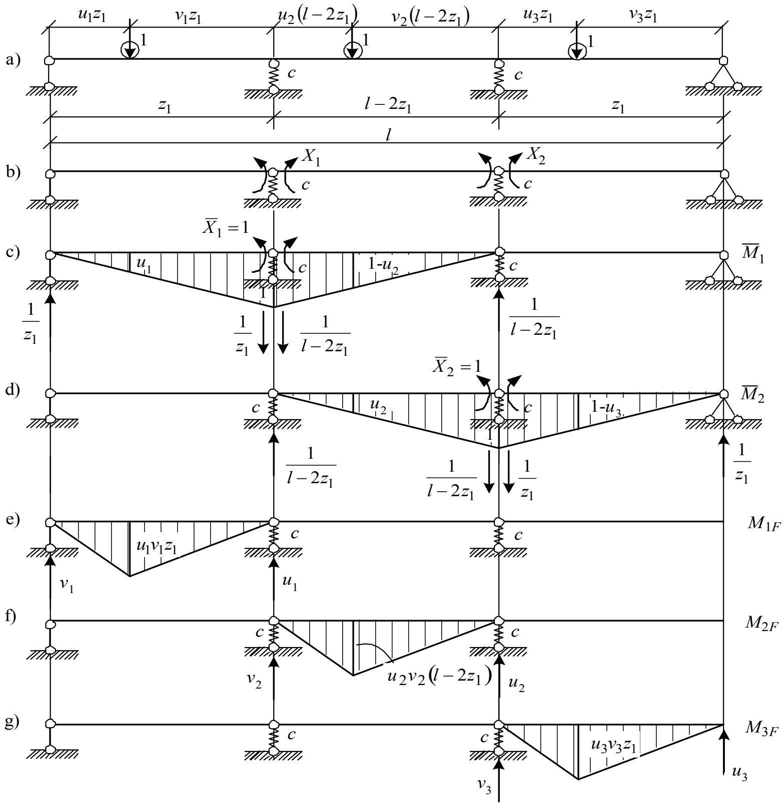

Fig. 7. Beam on the resilient supports (a); main system (b); isolated (c, d) and load (e, f, g) stress diagrams in the main system

$$M_{\sup}(p)=\int_{0}^{l-z_1} p\left(\text{IL } X_1\right)\mathrm{d}u\,, \tag{33}$$

$$M_{2\max}(p)=2\int_{z_1}^{l-z_1} p\left(\text{IL } M_{2,u}\right)\mathrm{d}u\,. \tag{34}$$

If we add the values of the moments which occur only from the support displacement, we obtain

$$M_{1,u}(p)=\int_{u_1 z_1}^{z_1} p\left(IL\ M_{1,u}\right)\mathrm{d}u+\int_{l-z_1}^{l} p\left(\text{IL } M_{1,u}\right)\mathrm{d}u+\frac{6\Delta EJu}{z_1\left(3l-4z_1\right)}\,, \tag{35}$$

$$M_{\sup}(p) = \int\limits_{0}^{l-z_1} p\left(\text{IL } X_1\right)\mathrm{d}u + \frac{6\Delta EJ}{z_1\left(3l-4z_1\right)}, \tag{36}$$

$$M_{2\max}(p) = 2\int\limits_{z_1}^{l-z_1} p\left(\text{IL } M_{2,u}\right)\mathrm{d}u + \frac{6\Delta EJ}{z_1\left(3l-4z_1\right)}. \tag{37}$$

Using the formulas (29) – (31), (35) – (37), we can find the maximum values of moments $M_{\sup}$, $M_{2\max}$ under the combined action of dead and live loads:

$$M_{2\max} = M_{2\max}(q) + M_{2\max}(p), \tag{38}$$

$$M_{\sup} = M_{\sup}(q) + M_{\sup}(p), \tag{39}$$

$$M_{1\max} = \max_{0<u<z_1}\left(\frac{quz_1(l-uz_1)}{2} - X_1 uz_1 + M_{1,u}(p) + \frac{6\Delta EJu}{z_1(3l-4z_1)}\right). \tag{40}$$

Parameters z_1, dEJ are numerically defined by iterate over the values with increment of 0.001 and 0.0001, respectively, until the conditions are fulfilled (1), (2).

5.3 Continuous beam on the resilient supports (Fig. 1, f)

Analytical determination of the values of the maximum moments under the combined action of dead and live loads is difficult, so the loads are considered separately.

Under the action of dead load according to [2]

$$M_{1x}(q) = \frac{xX_1}{z_1} + \frac{qx(z_1 - x)}{2}, \; 0 \le x \le z_1, \tag{41}$$

where

$$X_1 = -\frac{(l-2z_1)^3 + z_1^3 - 12cEJ(l-z_1)/z_1}{3l - 4z_1 + 6cEJ/z_1^2/(l-2z_1)^2}\,\frac{q}{4},$$

$$M_{\sup}(q) = X_1, \tag{42}$$

$$M_{2\max}(q) = X_1 + \frac{q(l-2z_1)^2}{8}. \tag{43}$$

Under action of the live load, the values of moments in the typical points are numerically determined; for this purpose the respective influence lines are plotted and loaded (Fig. 7). We accept that the live load intensity p is equal in all parts.

The unknown values X_1, X_2 under the live load are determined from the system of fundamental equations (11) using the formulas (12), where coefficients at the unknowns for all spans are equal and are determined in accordance with (13), (14) taking into account the support yield (Fig. 7):

$$\delta_{11}^{c} = \delta_{22}^{c} = \delta_{11} + \sum R_i c_i = \frac{1}{6EJ}\left[2(l-z_1) + 6cEJ\frac{(l-z_1)^2 + z_1^2}{z_1^2(l-2z_1)^2}\right], \tag{44}$$

$$\delta_{12}^{c} = \delta_{21}^{c} = \delta_{12} + \sum R_i c_i = \frac{1}{6EJ}\left[(l-2z_1) - 6cEJ\frac{2(l-z_1)}{z_1(l-2z_1)^2}\right]. \tag{45}$$

Free terms of the system are written for each span. Taking into account the support yield according to (15) – (20), we obtain (Fig. 7):

– on the first span

$$\delta_{1c(1)} = \delta_{1F(1)} + \sum R_i c_i = \frac{u_1}{6EJ}\left[z_1^2(1-u_1^2) + 6cEJ\frac{z_1 - l}{z_1(l-2z_1)} \right], \qquad (46)$$

$$\delta_{2c(1)} = \delta_{2F(1)} + \sum R_i c_i = \frac{1}{EJ}\frac{cEJu_1}{l-2z_1}; \qquad (47)$$

– on the second span

$$\delta_{1c(2)} = \delta_{1F(2)} + \sum R_i c_i = \frac{1}{6EJ}\left[u_2(1-u_2)(2-u_2)\left(l-2z_1\right)^2 + 6cEJ\frac{z_1 + l(u_2-1)}{z_1(l-2z_1)} \right], \qquad (48)$$

$$\delta_{2c(2)} = \delta_{2F(2)} + \sum R_i c_i = \frac{1}{6EJ}\left[u_2(1-u_2^2)(l-2z_1)^2 + 6cEJ\frac{z_1 - u_2 l}{z_1(l-2z_1)} \right]; \qquad (49)$$

– on the third span

$$\delta_{1c(3)} = \delta_{1F(3)} + \sum R_i c_i = \frac{1}{EJ}\frac{cEJ(1-u_3)}{l-2z_1}, \qquad (50)$$

$$\delta_{2c(3)} = \delta_{2F(3)} + \sum R_i c_i = \frac{1-u_3}{6EJ}\left[u_3\left(2-u_3\right)z_1^2 - 6cEJ\frac{l-z_1}{z_1(l-2z_1)} \right]. \qquad (51)$$

When we solve the system of the fundamental equations in each cross-section u, we plot according to (12) taking into account the support yield of the influence lines X_1, X_2 (Fig. 8 (b),(c)):

$$X_1 = \frac{\delta_{11}^c \delta_{1c} - \delta_{12}^c \delta_{2c}}{(\delta_{12}^c)^2 - (\delta_{11}^c)^2}, \quad X_2 = \frac{\delta_{11}^c \delta_{2c} - \delta_{12}^c \delta_{1c}}{(\delta_{12}^c)^2 - (\delta_{11}^c)^2}. \qquad (52)$$

Ordinates of the influence lines of the moments in the cross-section u of the beam on the resilient supports are defined in accordance with (21), (22) and Fig. 8 (d), (e).

The values of the moment $M_{2\max}$ and the support bending moment from the live load are determined by the unfavorable load cases of the influence lines X_1 (on the negative parts of the influence line X_1) and $M_{2,u}$ (on the positive parts of the influence line $M_{2,u}$), while the value $M_{2\max}$ is reached in the middle of the beam due to the symmetry (Fig. 8 (b), (e)):

$$M_{\sup}(p) = \int_0^{u_1^x z_1} p\left(\text{IL } X_1\right)\mathrm{d}u + \int_{z_1 + u_2^x z_2}^{l} p\left(\text{IL } X_1\right)\mathrm{d}u, \qquad (53)$$

$$M_{2\max}(p) = 2\int_{u_1^2 z_1}^{l/2} p\left(\text{IL } M_{2,u}\right)\mathrm{d}u. \qquad (54)$$

The points of intersection of the influence lines with the axis $u_1^x z_1$, $u_2^x z_2$, $u_1^2 z_1$ and further $u_1^1 z_2$ (Fig. 8) are numerically defined.

Using the formulas (42), (43), (53), (54), we can find the maximum values of moments $M_{\sup}$, $M_{2\max}$ under the combined action of dead and live loads:

$$M_{2\max} = M_{2\max}(q) + M_{2\max}(p), \qquad (56)$$

$$M_{\text{sup}} = M_{\text{sup}}(q) + M_{\text{sup}}(p) \,. \tag{56}$$

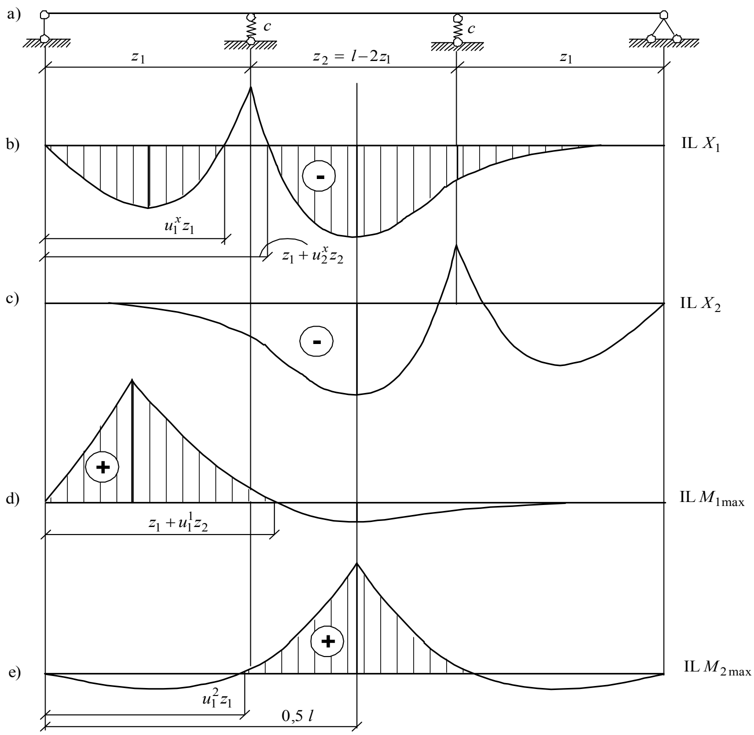

Fig. 8. Diagram of the beam (a); influence lines X_1, X_2 (b, c) and $M_{1\max}$, $M_{2\max}$ (d, e)

$M_{1\max}$ is the maximum value of the total moment under the combined action of dead and live loads. Moment $M_{1,u}$ under action of dead load is determined using the formula (41) and is

$$M_{1,u}(q) = u\,X_1 + \frac{qz_1^2 u(1-u)}{2} \,. \tag{57}$$

Under action of the live load (Fig. 8, d)

$$M_{1,u}(p) = \int_0^{z_1+u_1^1 z_2} p\left(ILM_{1,u}\right) \mathrm{d}u \,. \tag{58}$$

Thus,

$$M_{1\max} = \max_{0<u<z_1} \left(u\,X_1 + \frac{qz_1^2 u(1-u)}{2} + M_{1,u}(p) \right) \cdot \tag{59}$$

Parameters z_1, cEJ are numerically defined by iterate over the values with increment of 0.001 and 0.0001, respectively, until the conditions are fulfilled (1), (2).

The results of optimization of all beam types are summarized in Table 1 (in case of calculations without limitation of communality, single load density was accepted as q=p=1kN/m).

Table 1

Load	System type		z_1, *m*	*x*, *m*	*cEJ*, m³; Δ*EJ*, kNm³ or *k*	$M_{1\max}$ kNm	$\|M_{\sup}\|$, kNm	$M_{2\max}$ kNm
q	Statically determinate three-span beam	Hinge joints in the end spans	11.351	9.404		11.054	11.050	11.054
		Hinge joints in the middle span	11.352	1.947		11.058	11.049	11.049
	Continuous beam	With unilateral support connections	11.369		*k* 0.0271	10.994	10.996	10.989
		With displaced supports	11.351		Δ*EJ* 495	11.053	11.053	11.052
		With resilient connections	11.351		*cEJ* 40.151	11.053	11.052	11.052
q,p	Statically determinate three-span beam	Hinge joints in the end spans	11.942	9.896		24.483	24.433	24.483
		Hinge joints in the middle span	11.005	2.049		24.470	24.465	24.465
	Continuous beam	With unilateral support connections	11.410		*k* 0.0431	25.750	25.749	25.814
		With displaced supports	11.444		Δ*EJ* 403	25.732	25.726	25.727
		With resilient connections	11.351		*cEJ* 29.50	24.835	24.776	24.709

6. Conclusions

1. It has been shown that an optimal solution exists in the set of statically indeterminate systems under the action of dead load.

2. An optimal design can be part of the set of both statistically determinate, and indeterminate systems.

3. The optimal solution can be designed in the form of various options. The technical issues of making structures require further investigation to select the final version.

4. Under the action of dead and live mobile loads, the estimated stresses in different variants differ insignificantly.

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Стаття надійшла 7.02.2018

Ю.П. Кітов, М.А. Веревічева, Г.Л. Ватуля, С.В. Дериземля

КОНСТРУКТИВНІ РІШЕННЯ ОПТИМАЛЬНИХ СИСТЕМ ПІД ДІЄЮ ПОСТІЙНОГО І ТИМЧАСОВОГО РУХОМОГО НАВАНТАЖЕННЯ

Авторами статті була розглянута конструкція трипрогонового балочного моста заданої довжини під дією тимчасового навантаження. Метою даного дослідження є отримання рівноміцної системи, в усіх елементах якої максимальні напруження дорівнюють розрахунковим. Було доведено, що оптимальні рішення існують як у множині статично визначених систем, так і статично невизначених.

Ключові слова: сталезалізобетонна балка, тимчасове навантаження, постійне навантаження, балочний міст, рівноміцна система, оптимізація конструкції.

Ю.П. Китов, М.А. Веревичева, Г.Л. Ватуля, С.В. Дериземля

КОНСТРУКТИВНЫЕ РЕШЕНИЯ ОПТИМАЛЬНЫХ СИСТЕМ ПРИ ДЕЙСТВИИ ПОСТОЯННОЙ И ВРЕМЕННОЙ ПОДВИЖНОЙ НАГРУЗОК

Авторами статьи была рассмотрена конструкция трехпролетного балочного моста заданной длины под воздействием временной нагрузки. Целью данного исследования является получение равнопрочной системы, во всех элементах которой максимальные напряжения равны расчетным. Для выбора оптимальной системы был рассмотрен ряд статически определимых и статически неопределимых систем. Было доказано, что оптимальные решения существуют как в множестве статически определимых систем, так и неопределимых.

Ключевые слова: сталежелезобетонная балка, временная нагрузка, постоянная нагрузка, балочный мост, равнопрочная система, оптимизация конструкции.

УДК 624.012.3
Кітов Ю.П., Веревічева М.А., Ватуля Г.Л., Дериземля С.В. **Конструктивні рішення оптимальних систем під дією постійного і тимчасового рухомого навантаження** // Опір матеріалів і теорія споруд: наук.-техн. збірник. – К.: КНУБА, 2018. – Вип. 100. – С. 124-139.

Розглядаються конструктивні можливі рішення оптимальних систем під дією зовнішнього навантаження.

Табл. 1. Іл. 8. Бібліогр. 16 назв.

UDC 624.012.3
Kitov Yu., Verevicheva M., Vatulia G., Deryzemlia S. **Design solutions of optimal systems under action of dead and live mobile load //** Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2018. – Issue 100. – P. 124-139.

Authors describe some design solutions of optimal systems under external load.

Table 1. Fig. 8. Ref. 16.

УДК 624.012.3
Китов Ю.П., Веревичева М.А., Ватуля Г.Л., Дериземля С.В. **Конструктивные решения оптимальных систем при действии постоянной и временной подвижной нагрузок** // Сопротивление материалов и теория сооружений: науч.-техн. сборник. – К.: КНУСА, 2018. - Вып. 100. - С. 124-139.

Рассматриваются конструктивные возможные решения оптимальных систем под воздействием внешней нагрузки.

Табл. 1. Рис. 8. Библиогр. 16 назв.

Автор (вчена ступень, вчене звання, посада): *кандидат технічних наук, КІТОВ Юрій Петрович.*
Адреса робоча: *61050 Україна, м. Харків, площа Фейєрбаха 7, Український державний університет залізничного транспорту, КІТОВУ Юрію Петровичу.*
Робочий тел.: *+38(057) 730-10-70;*
E-mail: budmekh@ukr.net

Автор (вчена ступень, вчене звання, посада): *кандидат технічних наук, ВЕРЕВІЧЕВА Марина Анатоліївна.*
Адреса робоча: *61050 Україна, м. Харків, площа Фейєрбаха 7, Український державний університет залізничного транспорту, ВЕРЕВІЧЕВІЙ Марині Анатоліївні*
Робочий тел.: *+38(057) 730-10-70*
E-mail: *budmekh@ukr.net*

Автор (вчена ступень, вчене звання, посада): *доктор технічних наук, професор кафедри будівельної механіки і гідравліки, ВАТУЛЯ Гліб Леонідович.*
Адреса робоча: *61050 Україна, м. Харків, площа Фейєрбаха 7, Український державний університет залізничного транспорту, ВАТУЛІ Глібу Леонідовичу.*
Робочий тел.: *+38(057) 730-10-05;*
Мобільний тел.: *+38(050) 300-77-70;*
E-mail: *glebvatulya@gmail.com*
ORCID ID: https://orcid.org/0000-0002-3823-7201

Автор (вчена ступень, вчене звання, посада): *аспірант кафедри будівельної механіки і гідравліки, ДЕРИЗЕМЛЯ Світлана Володимирівна.*
Адреса робоча: *61050 Україна, м. Харків, площа Фейєрбаха 7, Український державний університет залізничного транспорту, ДЕРИЗЕМЛІ Світлані Володимирівні.*
Робочий тел.: *+38(057) 730-10-70*
Мобільний тел.: *+38(099) 482-97-99*
E-mail: *svetlana.deryzemlia@gmail.com*
ORCID ID: https://orcid.org/0000-0001-6556-4454