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MATHEMATICAL MODEL OF THE DYNAMICS CHANGE DEPARTURE OF THE JIB SYSTEM MANIPULATOR WITH THE SIMULTANEOUS MOVEMENT OF ITS LINKS

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An equation of motion of the manipulator is obtained taking into account the influence of the inertial component of each link of the boom system and the effect of the oscillatory movement of the cargo on the dynamic loads of the metalware elements and hydraulic drive elements. The influence of the simultaneous movement of the first jib section, the second jib section and the telescopic jib section on cargo oscillation, as well as the effect of cargo oscillation on dynamic loads that occur in the boom system and manipulator hydraulic drive elements, is determined.

Keywords: mathematical model, varying the radius, combination of movements, manipulator, Lagrange equations of the second kind, dynamic loads, load oscillations.

Introduction

During the process of unloading and loading operations in the elements of the boom system and the elements of the drive of the manipulator considerable dynamic loads occur. These loads are the result of the oscillatory movement of the load and the uneven rotation of the boom system with the uniform movement of the rods of the hydraulic cylinders [1-2]. Dynamic loads depend on the kinematic parameters of the manipulator and the nature of the speed of movement of the links of the boom system with the cargo. According to the normative-technical documentation, which regulates the operation of manipulators, it is allowed to combine operations of simultaneous movement of several links of the boom system. Combining the operations of the simultaneous movement of the links of the boom system can significantly reduce the dynamic loads and accordingly increase the performance, reliability of the elements of the boom system and the hydraulic equipment of the manipulator. To determine the actual dynamic loads in the elements of the design of the manipulator when combined movements of the links of the boom system, it is necessary to have adequate mathematical models [3-8].

Analysis of publications

Known [3-11] methods for constructing a mathematical model of the manipulator. In these works, the boom system of the manipulator is presented as a holonomic mechanical system in which the centres of gravity of the links of the metal structure coincides with their geometrical parameters. The mathematical model of the manipulator is considered, the relation between the kinematic dependences of the drive link of the manipulator and the load. The influence of dynamic loads on the elements of metalwork of the boom system of the manipulator is analyzed. In the papers [12-15], an analysis of the solution of optimization problems for reducing load oscillations is considered. The analysis of the influence of dynamic loads on the elements of the boom system and the hydraulic drive is considered in the papers [16-19]. With a large amount of consideration of the problem of dynamic analysis of the combination of simultaneous movement of the links of the boom system, the solution of this problem for manipulators with hydraulic drive is not considered taking into account the load fluctuations at the end of the boom system.

Purpose and research task statement

The purpose of this work is to build a mathematical model of the dynamics of change of departure of the boom system of the manipulator when combining operations of simultaneous movement of the first jib section, movement of the second jib section and movement of the telescopic jib section with cargo oscillation at the end of the boom system.

Research results

When investigating the dynamics of change of departure of the boom system of the manipulator with simultaneous movement of the first jib section, movement of the second jib section, moving the telescopic jib section and the cargo oscillations, we accept the following assumptions:

- we believe that all links in the boom system are perfectly rigid except for the cargo, which oscillates in the plane of change of departure;
- semi-dry friction in moving elements of eye joint and viscous friction of fluid in pipelines is not taken into account;
- the compressibility of the working fluid in the elements of the hydraulic drive is not taken into account.

Based on the above assumptions, the boom system of the manipulator in the process of changing the departure of the cargo with the combination of the three main movements and oscillates of the cargo is presented as a holonomic mechanical system with four degrees of freedom. The angular and linear coordinates of the moving of the first jib section, the second jib section and the telescopic jib section are calculated from the x axis, and the angular coordinate of the cargo deviation from the y axis, (Fig. 1). For the generalized coordinates of the boom system we take the angular coordinates: rotate the first jib section, rotate the second jib section, the linear coordinate of movement of the telescopic jib section, and the vertical deviation of the cargo (Fig. 1).

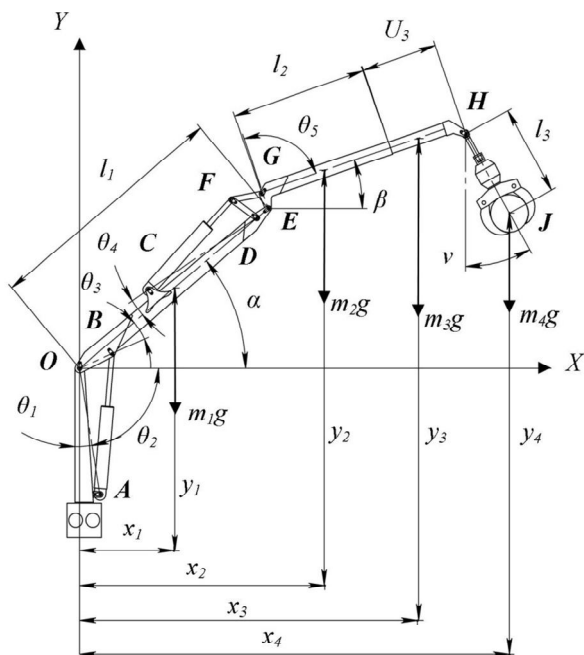


Fig. 1. Dynamic model of boom system of the manipulator

In Fig. 1 accepted the following designations: l_1 - length of the first jib section; l_2 - length of the second jib section; l_3 - the length of the suspension; m_1, m_2, m_3, m_4 - weight respectively of the first boom section, the second boom section, the telescopic section and the cargo; $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ - angles formed by the geometrical parameters of the elements of the boom system and the hydraulic cylinders of the manipulator; x_1, x_2, x_3, x_4 - horizontal coordinates of the centers of mass of the first jib section, the second jib section, the telescopic jib section and the cargo; y_1, y_2, y_3, y_4 - vertical coordinates of the centers of mass of the first jib section, the second jib section, the telescopic jib section and the cargo.

Expressed of the coordinates of the centres of mass for the first jib section, the second jib section, the telescopic jib section and the cargo in the generalized coordinates:

$$\begin{cases} x_1 = \frac{l_1}{2} \cos(\alpha); \\ y_1 = \frac{l_1}{2} \sin(\alpha); \end{cases} \quad (1)$$

$$\begin{cases} x_2 = l_1 \cdot \cos(\alpha) + \frac{l_2}{2} \cdot \cos(\beta); \\ y_2 = l_1 \cdot \sin(\alpha) + \frac{l_2}{2} \cdot \sin(\beta); \end{cases} \quad (2)$$

$$\begin{cases} x_3 = l_1 \cdot \cos(\alpha) + \left(\frac{l_2}{2} + U_3\right) \cdot \cos(\beta) + l_3 \cdot \sin(\nu); \\ y_3 = l_1 \cdot \sin(\alpha) + \left(\frac{l_2}{2} + U_3\right) \cdot \sin(\beta) - l_3 \cdot \cos(\nu). \end{cases} \quad (3)$$

$$\begin{cases} x_4 = l_1 \cdot \cos(\alpha) + (l_2 + U_3) \cdot \cos(\beta) + l_3 \cdot \sin(\nu); \\ y_4 = l_1 \cdot \sin(\alpha) + (l_2 + U_3) \cdot \sin(\beta) - l_3 \cdot \cos(\nu). \end{cases} \quad (4)$$

To compile the equations of motion of the manipulator in the process of changing the departure of the boom system with the cargo, while simultaneously moving the first jib section, the second jib section, the telescopic jib section and the cargo, we use the second-order Lagrange equations, which for the system shown in Fig. 1, have the form:

$$\begin{cases} \frac{d}{dt} \frac{\partial T}{\partial \dot{\alpha}} - \frac{\partial T}{\partial \alpha} = Q_\alpha - \frac{\partial V}{\partial \alpha}; \\ \frac{d}{dt} \frac{\partial T}{\partial \dot{\beta}} - \frac{\partial T}{\partial \beta} = Q_\beta - \frac{\partial V}{\partial \beta}; \\ \frac{d}{dt} \frac{\partial T}{\partial \dot{U}_3} - \frac{\partial T}{\partial U_3} = Q_{U_3} - \frac{\partial V}{\partial U_3}; \\ \frac{d}{dt} \frac{\partial T}{\partial \dot{\nu}} - \frac{\partial T}{\partial \nu} = Q_\nu - \frac{\partial V}{\partial \nu}, \end{cases} \quad (4)$$

where: t - time; T, V - respectively, the kinetic and potential energy of the boom system of the manipulator; $Q_\alpha, Q_\beta, Q_{U_3}, Q_\nu$ - non-conservative components of the generalized forces of the system corresponding to the generalized coordinates α, β, U_3, ν .

Then the kinetic energy of the boom system of the manipulator will take the form:

$$\begin{aligned} T = & \frac{1}{2} \cdot J_1 \cdot \dot{\alpha}^2 + \frac{1}{2} \cdot m_2 \cdot (\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2} \cdot J_2 \cdot \dot{\beta}^2 + \frac{1}{2} \cdot m_3 \cdot (\dot{x}_3^2 + \dot{y}_3^2) + \\ & + \frac{1}{2} \cdot J_3 \cdot \dot{\beta}^2 + \frac{1}{2} \cdot m_4 \cdot (\dot{x}_4^2 + \dot{y}_4^2), \end{aligned} \quad (5)$$

where: m_2, m_3, m_4 - the masses respectively of the second jib section, the telescopic jib section and the cargo; J_1, J_2, J_3 - moments of inertia respectively of the first jib section, the second jib section and the telescopic jib section (Fig. 1).

The potential energy of the boom system of the manipulator in the process of change of departure will be as follows:

$$V = (m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4) \cdot g, \quad (6)$$

where g - free fall acceleration.

Take the derivatives of kinetic energy (5) that are included in the system of equations (4):

$$\frac{\partial T}{\partial \alpha} = m_2 \left(\dot{x}_2 \frac{\partial \dot{x}_2}{\partial \alpha} + \dot{y}_2 \frac{\partial \dot{y}_2}{\partial \alpha} \right) + m_3 \left(\dot{x}_3 \frac{\partial \dot{x}_3}{\partial \alpha} + \dot{y}_3 \frac{\partial \dot{y}_3}{\partial \alpha} \right) + m_4 \left(\dot{x}_4 \frac{\partial \dot{x}_4}{\partial \alpha} + \dot{y}_4 \frac{\partial \dot{y}_4}{\partial \alpha} \right);$$

$$\frac{\partial T}{\partial \beta} = m_2 \left(\dot{x}_2 \frac{\partial \dot{x}_2}{\partial \beta} + \dot{y}_2 \frac{\partial \dot{y}_2}{\partial \beta} \right) + m_3 \left(\dot{x}_3 \frac{\partial \dot{x}_3}{\partial \beta} + \dot{y}_3 \frac{\partial \dot{y}_3}{\partial \beta} \right) + m_4 \left(\dot{x}_4 \frac{\partial \dot{x}_4}{\partial \beta} + \dot{y}_4 \frac{\partial \dot{y}_4}{\partial \beta} \right);$$

$$\frac{\partial T}{\partial U_3} = m_2 \left(\dot{x}_2 \frac{\partial \dot{x}_2}{\partial U_3} + \dot{y}_2 \frac{\partial \dot{y}_2}{\partial U_3} \right) + m_3 \left(\dot{x}_3 \frac{\partial \dot{x}_3}{\partial U_3} + \dot{y}_3 \frac{\partial \dot{y}_3}{\partial U_3} \right) + m_4 \left(\dot{x}_4 \frac{\partial \dot{x}_4}{\partial U_3} + \dot{y}_4 \frac{\partial \dot{y}_4}{\partial U_3} \right);$$

$$\frac{\partial T}{\partial v} = m_2 \left(\dot{x}_2 \frac{\partial \dot{x}_2}{\partial v} + \dot{y}_2 \frac{\partial \dot{y}_2}{\partial v} \right) + m_3 \left(\dot{x}_3 \frac{\partial \dot{x}_3}{\partial v} + \dot{y}_3 \frac{\partial \dot{y}_3}{\partial v} \right) + m_4 \left(\dot{x}_4 \frac{\partial \dot{x}_4}{\partial v} + \dot{y}_4 \frac{\partial \dot{y}_4}{\partial v} \right);$$

$$\frac{\partial T}{\partial \dot{\alpha}} = J_1 \cdot \dot{\alpha} + m_2 \left(\dot{x}_2 \frac{\partial \dot{x}_2}{\partial \alpha} + \dot{y}_2 \frac{\partial \dot{y}_2}{\partial \alpha} \right) + m_3 \left(\dot{x}_3 \frac{\partial \dot{x}_3}{\partial \alpha} + \dot{y}_3 \frac{\partial \dot{y}_3}{\partial \alpha} \right) + m_4 \left(\dot{x}_4 \frac{\partial \dot{x}_4}{\partial \alpha} + \dot{y}_4 \frac{\partial \dot{y}_4}{\partial \alpha} \right);$$

$$\frac{\partial T}{\partial \dot{\beta}} = (J_2 + J_3) \cdot \dot{\beta} + m_2 \left(\dot{x}_2 \frac{\partial \dot{x}_2}{\partial \beta} + \dot{y}_2 \frac{\partial \dot{y}_2}{\partial \beta} \right) + m_3 \left(\dot{x}_3 \frac{\partial \dot{x}_3}{\partial \beta} + \dot{y}_3 \frac{\partial \dot{y}_3}{\partial \beta} \right) + m_4 \left(\dot{x}_4 \frac{\partial \dot{x}_4}{\partial \beta} + \dot{y}_4 \frac{\partial \dot{y}_4}{\partial \beta} \right);$$

$$\frac{\partial T}{\partial U_3} = m_2 \left(\dot{x}_2 \frac{\partial \dot{x}_2}{\partial U_3} + \dot{y}_2 \frac{\partial \dot{y}_2}{\partial U_3} \right) + m_3 \left(\dot{x}_3 \frac{\partial \dot{x}_3}{\partial U_3} + \dot{y}_3 \frac{\partial \dot{y}_3}{\partial U_3} \right) + m_4 \left(\dot{x}_4 \frac{\partial \dot{x}_4}{\partial U_3} + \dot{y}_4 \frac{\partial \dot{y}_4}{\partial U_3} \right);$$

$$\frac{\partial T}{\partial \dot{v}} = m_2 \left(\dot{x}_2 \frac{\partial \dot{x}_2}{\partial v} + \dot{y}_2 \frac{\partial \dot{y}_2}{\partial v} \right) + m_3 \left(\dot{x}_3 \frac{\partial \dot{x}_3}{\partial v} + \dot{y}_3 \frac{\partial \dot{y}_3}{\partial v} \right) + m_4 \left(\dot{x}_4 \frac{\partial \dot{x}_4}{\partial v} + \dot{y}_4 \frac{\partial \dot{y}_4}{\partial v} \right);$$

$$\begin{aligned} \frac{d}{dt} \frac{\partial T}{\partial \dot{\alpha}} &= J_1 \cdot \ddot{\alpha} + m_2 \left(\ddot{x}_2 \frac{\partial \dot{x}_2}{\partial \alpha} + \dot{x}_2 \frac{\partial \ddot{x}_2}{\partial \alpha} + \ddot{y}_2 \frac{\partial \dot{y}_2}{\partial \alpha} + \dot{y}_2 \frac{\partial \ddot{y}_2}{\partial \alpha} \right) + \\ &+ m_3 \left(\ddot{x}_3 \frac{\partial \dot{x}_3}{\partial \alpha} + \dot{x}_3 \frac{\partial \ddot{x}_3}{\partial \alpha} + \ddot{y}_3 \frac{\partial \dot{y}_3}{\partial \alpha} + \dot{y}_3 \frac{\partial \ddot{y}_3}{\partial \alpha} \right) + m_4 \left(\ddot{x}_4 \frac{\partial \dot{x}_4}{\partial \alpha} + \dot{x}_4 \frac{\partial \ddot{x}_4}{\partial \alpha} + \ddot{y}_4 \frac{\partial \dot{y}_4}{\partial \alpha} + \dot{y}_4 \frac{\partial \ddot{y}_4}{\partial \alpha} \right); \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \frac{\partial T}{\partial \dot{\beta}} &= (J_2 + J_3) \cdot \ddot{\beta} + m_2 \left(\ddot{x}_2 \frac{\partial \dot{x}_2}{\partial \beta} + \dot{x}_2 \frac{\partial \ddot{x}_2}{\partial \beta} + \ddot{y}_2 \frac{\partial \dot{y}_2}{\partial \beta} + \dot{y}_2 \frac{\partial \ddot{y}_2}{\partial \beta} \right) + \\ &+ m_3 \left(\ddot{x}_3 \frac{\partial \dot{x}_3}{\partial \beta} + \dot{x}_3 \frac{\partial \ddot{x}_3}{\partial \beta} + \ddot{y}_3 \frac{\partial \dot{y}_3}{\partial \beta} + \dot{y}_3 \frac{\partial \ddot{y}_3}{\partial \beta} \right) + m_4 \left(\ddot{x}_4 \frac{\partial \dot{x}_4}{\partial \beta} + \dot{x}_4 \frac{\partial \ddot{x}_4}{\partial \beta} + \ddot{y}_4 \frac{\partial \dot{y}_4}{\partial \beta} + \dot{y}_4 \frac{\partial \ddot{y}_4}{\partial \beta} \right); \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \frac{\partial T}{\partial \dot{U}_3} &= m_2 \left(\ddot{x}_2 \frac{\partial \dot{x}_2}{\partial U_3} + \dot{x}_2 \frac{\partial \ddot{x}_2}{\partial U_3} + \ddot{y}_2 \frac{\partial \dot{y}_2}{\partial U_3} + \dot{y}_2 \frac{\partial \ddot{y}_2}{\partial U_3} \right) + m_3 \left(\ddot{x}_3 \frac{\partial \dot{x}_3}{\partial U_3} + \dot{x}_3 \frac{\partial \ddot{x}_3}{\partial U_3} + \right. \\ &+ \left. \ddot{y}_3 \frac{\partial \dot{y}_3}{\partial U_3} + \dot{y}_3 \frac{\partial \ddot{y}_3}{\partial U_3} \right) + m_4 \left(\ddot{x}_4 \frac{\partial \dot{x}_4}{\partial U_3} + \dot{x}_4 \frac{\partial \ddot{x}_4}{\partial U_3} + \ddot{y}_4 \frac{\partial \dot{y}_4}{\partial U_3} + \dot{y}_4 \frac{\partial \ddot{y}_4}{\partial U_3} \right); \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \frac{\partial T}{\partial \dot{v}} &= m_2 \left(\ddot{x}_2 \frac{\partial \dot{x}_2}{\partial v} + \dot{x}_2 \frac{\partial \ddot{x}_2}{\partial v} + \ddot{y}_2 \frac{\partial \dot{y}_2}{\partial v} + \dot{y}_2 \frac{\partial \ddot{y}_2}{\partial v} \right) + m_3 \left(\ddot{x}_3 \frac{\partial \dot{x}_3}{\partial v} + \dot{x}_3 \frac{\partial \ddot{x}_3}{\partial v} + \right. \\ &+ \left. \ddot{y}_3 \frac{\partial \dot{y}_3}{\partial v} + \dot{y}_3 \frac{\partial \ddot{y}_3}{\partial v} \right) + m_4 \left(\ddot{x}_4 \frac{\partial \dot{x}_4}{\partial v} + \dot{x}_4 \frac{\partial \ddot{x}_4}{\partial v} + \ddot{y}_4 \frac{\partial \dot{y}_4}{\partial v} + \dot{y}_4 \frac{\partial \ddot{y}_4}{\partial v} \right). \end{aligned}$$

Take partial derivatives of potential energy (6):

$$\begin{aligned}\frac{\partial V}{\partial \beta} &= \left(m_2 \frac{\partial y_2}{\partial \beta} + m_3 \frac{\partial y_3}{\partial \beta} + m_4 \frac{\partial y_4}{\partial \beta} \right) g; \\ \frac{\partial V}{\partial \alpha} &= \left(m_1 \frac{\partial y_1}{\partial \alpha} + m_2 \frac{\partial y_2}{\partial \alpha} + m_3 \frac{\partial y_3}{\partial \alpha} + m_4 \frac{\partial y_4}{\partial \alpha} \right) g; \\ \frac{\partial V}{\partial U_3} &= \left(m_3 \frac{\partial y_3}{\partial U_3} + m_4 \frac{\partial y_4}{\partial U_3} \right) g; \\ \frac{\partial V}{\partial v} &= m_4 \frac{\partial y_4}{\partial v} \cdot g.\end{aligned}\quad (8)$$

The variation of the displacement of the rods of the hydraulic cylinders is expressed by the variation of the generalized coordinates:

$$\begin{aligned}Q_\alpha &= F_1 \frac{\partial U_1}{\partial \alpha} + F_2 \frac{\partial U_2}{\partial \alpha}; \\ Q_\beta &= F_2 \frac{\partial U_2}{\partial \beta}; \\ Q_{U_3} &= F_3.\end{aligned}\quad (8)$$

where: F_1 - efforts in lifting the first jib section; F_2 - efforts in the hydraulic cylinder of the second jib section; F_3 - efforts in the hydraulic cylinder to move the telescopic jib section.

Substituting expressions (7 - 9) into the system of equations (4), we obtain a system of differential equations of motion of the manipulator in the process of changing the departure of the boom system with the cargo when the three main motions of the boom system are combined:

$$\begin{cases} J_1 \ddot{\alpha} + m_2 \left(\ddot{x}_2 \frac{\partial x_2}{\partial \alpha} + \ddot{y}_2 \frac{\partial y_2}{\partial \alpha} \right) + m_3 \left(\ddot{x}_3 \frac{\partial x_3}{\partial \alpha} + \ddot{y}_3 \frac{\partial y_3}{\partial \alpha} \right) + m_4 \left(\ddot{x}_4 \frac{\partial x_4}{\partial \alpha} + \ddot{y}_4 \frac{\partial y_4}{\partial \alpha} \right) = F_1 \frac{\partial U_1}{\partial \alpha} + F_2 \frac{\partial U_2}{\partial \alpha} - \left(m_1 \frac{\partial y_1}{\partial \alpha} + m_2 \frac{\partial y_2}{\partial \alpha} + m_3 \frac{\partial y_3}{\partial \alpha} + m_4 \frac{\partial y_4}{\partial \alpha} \right) g; \\ (J_2 + J_2) \ddot{\beta} + m_2 \left(\ddot{x}_2 \frac{\partial x_2}{\partial \beta} + \ddot{y}_2 \frac{\partial y_2}{\partial \beta} \right) + m_3 \left(\ddot{x}_3 \frac{\partial x_3}{\partial \beta} + \ddot{y}_3 \frac{\partial y_3}{\partial \beta} \right) + m_4 \left(\ddot{x}_4 \frac{\partial x_4}{\partial \beta} + \ddot{y}_4 \frac{\partial y_4}{\partial \beta} \right) = -F_2 \frac{\partial U_2}{\partial \beta} - \left(m_2 \frac{\partial y_2}{\partial \beta} + m_3 \frac{\partial y_3}{\partial \beta} + m_4 \frac{\partial y_4}{\partial \beta} \right) g; \\ m_3 \left(\ddot{x}_3 \frac{\partial x_3}{\partial U_3} + \ddot{y}_3 \frac{\partial y_3}{\partial U_3} \right) + m_4 \left(\ddot{x}_4 \frac{\partial x_4}{\partial U_3} + \ddot{y}_4 \frac{\partial y_4}{\partial U_3} \right) = -F_3 - \left(m_3 \frac{\partial y_3}{\partial U_3} + m_4 \frac{\partial y_4}{\partial U_3} \right) g; \\ m_4 \left(\ddot{x}_4 \frac{\partial x_4}{\partial v} + \ddot{y}_4 \frac{\partial y_4}{\partial v} \right) = -m_4 \frac{\partial y_4}{\partial v} g.\end{cases}\quad (10)$$

Find the coordinates of the driving mechanisms that are part of the system of equations (11).

$$AB = U_1 = \sqrt{AO^2 + OB^2 - 2 \cdot AO \cdot OB \cdot \cos(\theta_2 + \alpha - \theta_3)}.\quad (11)$$

$$CF = U_2 = \sqrt{CD^2 + DF^2 - 2 \cdot CD \cdot DF \cdot \cos(\angle CDF)}. \quad (12)$$

To determine, $\angle CDF$ first consider the four link mechanism $EDFG$ (Fig. 2), and define the diagonal DG :

$$DG = \sqrt{EG^2 + DE^2 - 2 \cdot EG \cdot DE \cdot \cos(\angle DEG)}. \quad (13)$$

$$\angle DEG = \pi - (\theta_5 + \beta - \alpha). \quad (14)$$

After substitution of expression (13) in dependence (14) we obtain:

$$DG = \sqrt{EG^2 + DE^2 - 2 \cdot EG \cdot DE \cdot \cos(\theta_5 + \beta - \alpha)}. \quad (15)$$

Using the sine theorem, we write:

$$\frac{\sin \angle DEG}{DG} = \frac{\sin \angle EDG}{EG}. \quad (16)$$

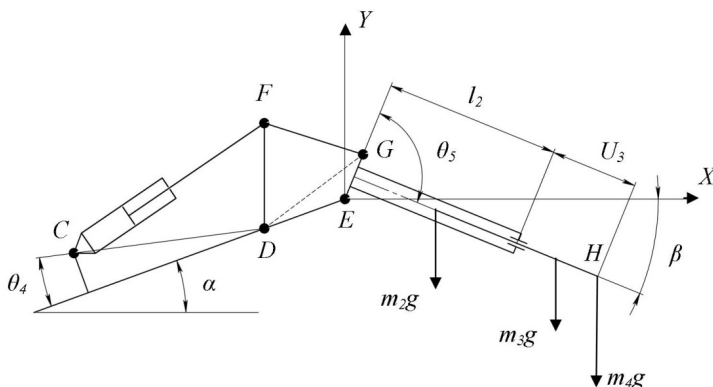


Fig. 2. The kinematic scheme of the drive of the second jib section

From equation (16) we find:

$$\angle ADE = \text{ArcSin} \left(\frac{EG \cdot \sin(\theta_5 + \beta - \alpha)}{DG} \right). \quad (17)$$

We find the angle $\angle FDG$ from the expression:

$$FG^2 = DF^2 + DG^2 - 2 \cdot DF \cdot DG \cdot \cos \angle FDG. \quad (18)$$

Then:

$$\angle FDG = \arccos \left(\frac{DG^2 + DF^2 - FG^2}{2 \cdot DF \cdot DG} \right). \quad (19)$$

Adding expressions (18) and (20), we find the angle $\angle EDF$:

$$\angle EDF = \arcsin \left(\frac{EG \cdot \sin(\theta_5 + \beta - \alpha)}{DG} \right) + \arccos \left(\frac{DG^2 + DF^2 - FG^2}{2 \cdot DF \cdot DG} \right). \quad (20)$$

Now can find the angle $\angle CDF$:

$$\angle CDF = \pi - \angle EDF - \theta_4. \quad (21)$$

After substitution of expression (21) in dependence (12) we obtain:

$$U_2 = \sqrt{CD^2 + DF^2 + 2 \cdot CD \cdot DF \cdot \cos(\angle EDF - \theta_4)}. \quad (22)$$

To move the links of the boom system, hydraulic cylinders develop driving forces, which are determined by mechanical characteristics.

The characteristics are presented in the form of quadratic relationships between the acting forces and the displacement rates of the rods of the hydraulic cylinders:

$$\begin{aligned} F_1 &= P_n \cdot A_1 \cdot \sqrt{1 - \frac{A_1 \cdot \dot{U}_1}{Q_1}}, \\ F_2 &= P_n \cdot A_2 \cdot \sqrt{1 - \frac{A_2 \cdot \dot{U}_2}{Q_2}}, \\ F_3 &= P_n \cdot A_3 \cdot \sqrt{1 - \frac{A_3 \cdot \dot{U}_3}{Q_3}}, \end{aligned} \quad (23)$$

where: P_n - fluid pressure in the hydraulic system; A_1, A_2, A_3 - respectively, the piston area of the hydraulic cylinders of the first jib section, the second jib section and the telescopic jib section; $\dot{U}_1, \dot{U}_2, \dot{U}_3$ according the speed of the cylinder rods of the first jib section, the second jib section and the telescopic section. The flow of the working fluid flowing through the hydraulic distributor to provide the hydraulic cylinders with the required start mode and speed of movement of the boom system is determined by the following dependencies, respectively, for the hydraulic cylinder of the first jib section, the second jib section and the telescopic jib section:

$$\begin{aligned} Q_1 &= \mu \cdot f_1 \cdot \sqrt{\frac{2 \cdot \Delta P_1}{\rho}}, \\ Q_2 &= \mu \cdot f_2 \cdot \sqrt{\frac{2 \cdot \Delta P_2}{\rho}}, \\ Q_3 &= \mu \cdot f_3 \cdot \sqrt{\frac{2 \cdot \Delta P_3}{\rho}}, \end{aligned} \quad (24)$$

where: $\Delta P_1, \Delta P_2, \Delta P_3$ - respectively the pressure drop in the cylinders; μ - coefficient of consumption of working fluid; f_1, f_2, f_3 - according of the area cross-sectional of the hydraulic distributor; ρ - the specific gravity of the liquid.

To calculate the dynamics of change of departure of the boom system of the manipulator we use the following output parameters: $m_1 = 350$ kg, $m_2 = 155$ kg, $m_3 = 65$ kg, $m_4 = 500$ kg, $l_1 = 4$ m, $l_2 = 2$ m, $l_3 = 0,8$ m, $AO = 1,6$ m, $OB = 0,5$ m, $CD = 1,6$ m, $DF = 0,425$ m, $FG = 0,425$ m, $EG = 0,425$ m, $DE = 0,255$ m, $P_n = 20 \cdot 10^6$ Pa, $A_1 = 0,012265$ m²,

$A_2 = 0,00915 \text{ m}^2$, $A_3 = 0.00185 \text{ m}^2$, $\theta_1 = 0,192 \text{ rad}$, $\theta_2 = 1,378 \text{ rad}$,
 $\theta_3 = 0,384 \text{ rad}$, $\theta_4 = 0,157 \text{ rad}$, $\theta_5 = 1,57 \text{ rad}$, $\rho = 850 \text{ kg/m}^3$. Initial
 conditions of manipulator movement: $\alpha[0] = -0.3$, $\dot{\alpha}[0] = 0$, $\beta[0] = -1.2$,
 $\dot{\beta}[0] = 0$, $U_3[0] = 1$, $\dot{U}_3[0] = 0$, $v[0] = 0$, $\dot{v}[0] = 0$.

Substituting the initial parameters and initial conditions into the system of equations (10) and solving it, the graphical dependences of the kinematic and force characteristics of the manipulator with the cargo are determined and constructed (Fig. 3 - Fig. 7). The following assumptions were made when solving the equation system:

- switching time of the hydraulic distributor is 0,1 s;
- the cross-sectional area of the hydraulic distributor varies according to linear law.

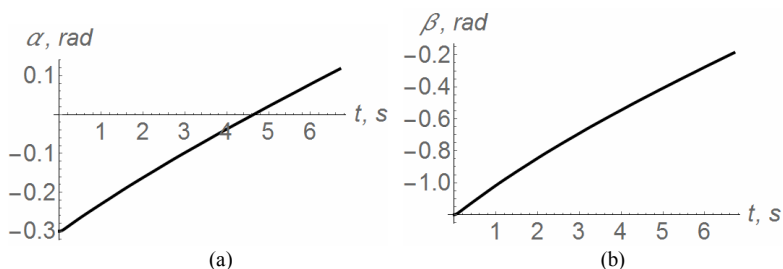


Fig. 3. Graphical dependencies of angular displacement:
 (a) the first jib section; (b) the second jib section

From the system of equations (10) and expressions (11 - 22), the displacement of the rods of the hydraulic cylinders, respectively, of the first jib section, the second jib section, and the telescopic jib section were determined. Depending on the movement of the rod of hydraulic cylinders, the angular movement of the first jib section and the second jib section is determined. Analyzing the graphical dependences of movement of the units of the boom system (Fig. 3), it is possible to determine, in accordance with the geometric and kinematic characteristics, the functional dependence of the angular movement of the boom system units in accordance with the linear movement of the rods of hydraulic cylinders.

Having solved the system of equations with initial parameters and initial conditions, graphical dependences of speeds of movement of rods of drive hydraulic cylinders (Fig. 4) and respectively elements of the boom system (Fig. 5) were constructed with the condition of simultaneous movement of the first jib section, the second jib section, the telescopic jib section and cargo.

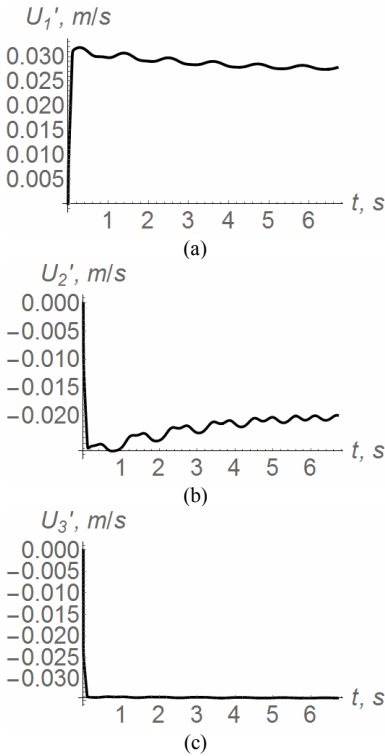


Fig. 4. Graphical dependences of speeds of movement of rods of hydraulic cylinders: (a) the first jib section; (b) the second jib section; (c) telescopic jib section

first jib section and second jib section with steady motion, caused by the kinematic parameters of the manipulator and, accordingly, the oscillatory movement of the cargo (Fig. 7).

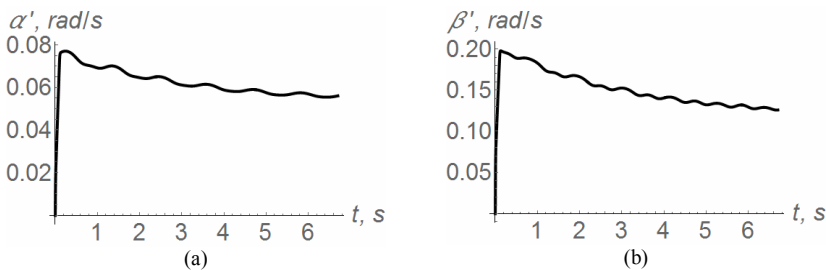


Fig. 5. Graphical dependences of the angular speed of movement of the links of the boom system: (a) the first jib section; (b) the second jib section

Acceleration of rod of the hydraulic cylinder of the telescopic jib section (Fig. 4(c)) occurs within 0.1 s, which corresponds to the time of movement of the hydraulic control valve spool. The speed of movement of the rod of the hydraulic cylinder is 0.058 m/s, and with further movement is accompanied by dynamic loads.

Exit to on the steady motion of the first boom section and the second boom section (Fig. 5), occurs in accordance with of the rod steady motion of hydraulic cylinder. The angular speed of movement of the first section of the jib is 0.077 rad/s, at the beginning of steady motion, with further movement has a slight decrease.

The decrease in the angular speed of movement coincides with the decrease in the linear velocity of the rod of the hydraulic cylinder (Fig. 4(a)). The angular speed of movement of the second jib section at the beginning of steady motion is 0.2 rad / s, with further movement has a slight decrease. A gradual decrease in the angular speed of the

As can be seen from the graphical dependencies, the pressure at the beginning of the motion is equal $2 \cdot 10^7$ Pa, which corresponds to the pressure of the working fluid in the hydraulic system. Upon further movement of the boom system and their exit in a steady motion, the pressure in the hydraulic cylinder of the first jib section and the second jib section is equal to $5 \cdot 10^6$ Pa, and is accompanied by dynamic loads. This is caused by the inertial component of the units of the boom system and, accordingly, the occurrence in it and the elements of the hydraulic drive of dynamic loads caused by the oscillatory movement of the cargo (Fig. 7).

Taking into account the inertial components of the units of the boom system and the fluctuations of the working fluid pressure in the hydraulic cylinders, the dependence of the cargo oscillation at the end of the jib was constructed (Fig. 7). From the graphical dependence you can see the characteristic correspondence of the deviation of the cargo from the vertical (Fig. 7), which coincides in time with the dynamic loads in the elements of the hydraulic drive (Fig. 4), links of the boom system (Fig. 5) and fluctuations in the working fluid pressure (Fig. 6).

Based on the initial conditions, there is no deviation of the cargo at the beginning of the movement. At the beginning of the steady movement of elements of the boom system (Fig. 4 - Fig. 5) the cargo deviation is maximized and equals $0,34$ rad.

Such high load

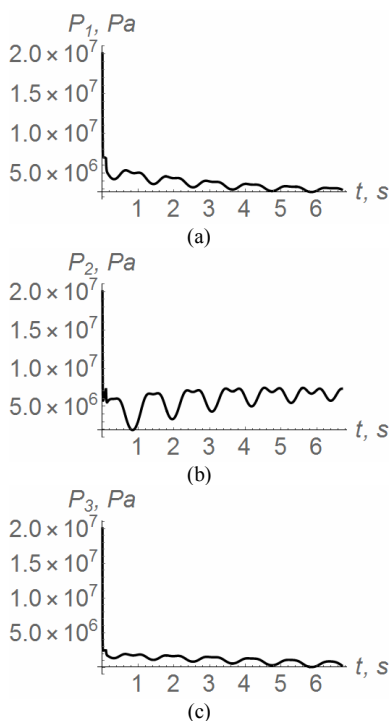


Fig. 6. Graphic dependencies of pressure change in the hydraulic cylinder: (a) the first jib section; (b) the second jib section; (c) the telescopic jib section

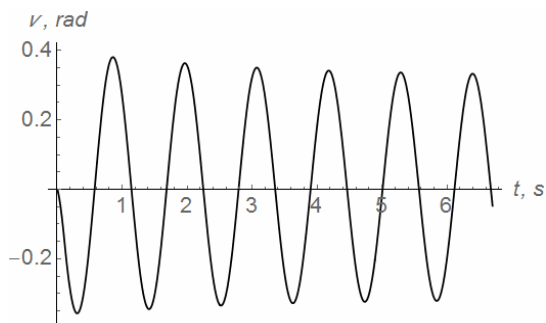


Fig. 7. Graphical dependence of the cargo oscillation

variability results in considerable dynamic loads in the elements of the boom system and the elements of the hydraulic drive of the manipulator. Thus reducing its reliability and speeding up the failure of the mechanical system as a whole.

Conclusions. As a result of the research, a mathematical model of the dynamics of change of departure in the plane of movement of the boom system of the manipulator was constructed, provided that the motions of the elements of the boom system were combined with the cargo. Dynamic analysis of the mechanism of simultaneous movement of the first jib section, the second jib section, the telescopic jib section, and the cargo oscillations is performed. Graphical dependences of dynamic loads in the boom system and elements of the hydraulic drive of the manipulator were obtained. The proposed mathematical model makes it possible to determine the actual dynamic loads in the elements of the manipulator design and drive mechanisms. The results obtained can be used in further practical design of hydraulic manipulators.

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Ловеїкін В.С., Ромасевич Ю.О., Сподоба О.О., Ловеїкін А.В., Почка К.І.

МАТЕМАТИЧНА МОДЕЛЬ ДИНАМІКИ ЗМІНИ ВІЛЬОТУ СТІРЛОВОЇ СИСТЕМИ МАНІПУЛЯТОРА ЗА ОДНОЧАСНОГО ПЕРЕМІЩЕННЯ ЇЇ ЛАНОК

З метою підвищення продуктивності та надійності маніпулятора згідно з нормативно-технічною документацією, яка регламентує безпечну експлуатацію маніпуляторів допускається суміщення рухів з одночасним переміщенням декількох ланок стрілової системи. В результаті в роботі розглянута методика побудови математичної моделі в площині зміни вильоту стрілової системи маніпулятора із вантажем. Математична модель побудована із врахуванням трьох одночасних рухів, а саме, одночасного переміщення першої секції стріли, другої секції стріли, телескопічної секції стріли та коливання вантажу. Розраховано функції зміни кінематичних та динамічних характеристик стрілової системи за одночасного переміщення її ланок. Побудова математичної моделі виконана із застосуванням рівнянь Лагранжа другого роду. При цьому за узагальнені координати моделі маніпулятора прийнято, кутові координати положення ланок стрілової системи та кутове відхилення від вертикалі вантажу. А механічні характеристики гідравлічного приводу, представлені у вигляді квадратичних залежностей між діючими зусиллями та швидкостями переміщень штоків гідроциліндрів. Керування елементами приводу представлено у вигляді рівнянь витрати робочої рідини зі зміною площі прохідного перерізу гідравлічного розподільника за лінійним законом. В результаті отримано рівняння руху маніпулятора з врахуванням впливу інерційної складової кожної ланки стрілової системи та впливу коливань вантажу на динамічні навантаження елементів металоконструкції та елементів гідравлічного приводу. Розроблена математична модель дозволяє теоретично визначити вплив одночасного переміщення першої секції стріли, другої секції стріли та телескопічної секції стріли на коливання вантажу, а також вплив коливання вантажу на динамічні навантаження, які при цьому виникають в стріловій системі та елементах гідравлічного приводу маніпулятора.

Ключові слова: математична модель, зміна вильоту, суміщення рухів, маніпулятор, рівняння Лагранжа другого роду, динамічні навантаження, коливання вантажу.

Loveikin V.S., Romasevich Yu.O., Spodoba O.O. Loveikin A.V., Pochka K.I.,

MATHEMATICAL MODEL OF THE DYNAMICS CHANGE DEPARTURE OF THE JIB SYSTEM MANIPULATOR WITH THE SIMULTANEOUS MOVEMENT OF ITS LINKS

In order to increase the productivity and reliability of the manipulator according to the normative and technical documentation, which regulates the safe operation of the manipulators, it is allowed to combine movements with the simultaneous movement of several elements of the boom system. As a result, the paper methodology reviewed for constructing a mathematical model in the plane of the departure change of the boom system of a manipulator with a load. The mathematical model is constructed from the calculation of three simultaneous movements, namely, the simultaneous movement of the first jib section, the second jib section, the telescopic jib section and the oscillation of the cargo. The functions of changing the kinematic and dynamic characteristics of the boom system with the simultaneous movement of its links are calculated. The construction of a mathematical model is carried out using Lagrange equations of the second kind. Moreover, the generalized coordinates of the manipulator model are taken as the angular coordinates of the position of the links of the boom system and the angular deviation from the vertical of the cargo. And the mechanical characteristics of the hydraulic drive are presented in the form of square dependencies between the acting forces and the speeds of movement of the hydraulic cylinder rods. The control of the drive elements is presented in the form of equations of the flow rate of the working fluid with a change in the area of the flow cross-section of the hydraulic distributor according to a linear law. As a result, the equation of motion of the manipulator is obtained, taking into account the influence of the inertial component of each link of the boom system and the influence of cargo oscillations on the dynamic loads of metalware elements and hydraulic drive elements. The developed mathematical model allows one to theoretically determine the effect of simultaneous movement of the first jib section, the second jib section and the telescopic jib section on cargo oscillation, as well as the effect of cargo oscillation on dynamic loads that occur in the boom system and manipulator hydraulic drive elements.

Keywords: mathematical model, varying the radius, combination of movements, manipulator, Lagrange equations of the second kind, dynamic loads, load oscillations.

Ловейкин В.С., Ромасевич Ю.А., Сподоба А.А., Ловейкин А.В., Почка К.И.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ДИНАМИКИ ИЗМЕНЕНИЯ ВЫЛЕТА СТРЕЛОВОЙ СИСТЕМЫ МАНИПУЛЯТОРА ПРИ ОДНОВРЕМЕННОМ ПЕРЕМЕЩЕНИИ ЕЕ ЗВЕНЬЕВ

С целью повышения производительности и надежности манипулятора согласно нормативно-технической документации, которая регламентирует безопасную эксплуатацию манипуляторов, допускается совмещение движений с одновременным перемещением нескольких элементов стреловой системы. В результате в работе рассмотрена методика построения математической модели в плоскости изменения вылета стреловой системы манипулятора с грузом. Математическая модель построена из расчета трех одновременных движений, а именно, одновременного перемещения первой секции стрелы, второй секции стрелы, телескопической секции стрелы и колебания груза. Рассчитаны функции изменения кинематических и динамических характеристик стреловой системы при одновременном перемещении ее звеньев. Построение математической модели проводится с использованием уравнений Лагранжа второго рода. При этом за обобщенные координаты модели манипулятора принято, угловые координаты положения звеньев стреловой системы и углового отклонения от вертикали груза. А механические характеристики гидравлического привода, представлены в виде квадратных зависимостей между действующими усилиями и скоростями перемещения штоков гидроцилиндров. Управление элементами привода представлено в виде уравнений расхода рабочей жидкости со сменой площадью проходного сечения гидравлического распределителя за линейным законом. В результате получено уравнение движения манипулятора с учетом влияния инерционной составляющей каждого звена стреловой системы и влияния колебаний груза на динамические нагрузки элементов металлоконструкции и элементов гидравлического привода. Разработанная математическая модель позволяет теоретически определить влияние одновременного перемещения первой секции стрелы, второй секции стрелы и телескопической секции стрелы на колебания груза, а также влияние колебаний груза на динамические нагрузки, которые при этом возникают в стреловой системе и элементах гидравлического привода манипулятора.

Ключевые слова: математическая модель, изменение вылета, совмещение движений, манипулятор, уравнение Лагранжа второго рода, динамические нагрузки, колебания груза.

УДК 621.87

Ловейкін В.С., Ромасевич Ю.О., Сподоба О.О., Ловейкін А.В., Почка К.І. **Математична модель динаміки зміни вильоту стрілової системи маніпулятора за одночасного переміщення її ланок** // Опір матеріалів та теорія споруд: Наук.-техн. збірник. - К.: КНУБА, 2020. – Вип. 104. - С. 175-190.

Отримано рівняння руху маніпулятора з врахуванням впливу інерційної складової кожної ланки стрілової системи та впливу коливань вантажу на динамічні навантаження елементів металоконструкції та елементів гідравлічного приводу. Встановлено вплив одночасного переміщення першої секції стріли, другої секції стріли та телескопічної секції стріли на коливання вантажу, а також вплив коливання вантажу на динамічні навантаження, які при цьому виникають в стріловій системі та елементах гідравлічного приводу маніпулятора.

Іл. 7. Бібліогр. 19 назв.

UDC 621.87

Loveikin V.S., Romasevich Yu.O., Spodoba O.O. Loveikin A.V., Pochka K.I. **Mathematical model of the dynamics change departure of the jib system manipulator with the simultaneous movement of its links** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 104. – P. 175-190.

An equation of motion of the manipulator is obtained taking into account the influence of the inertial component of each link of the boom system and the effect of the oscillatory movement of the cargo on the dynamic loads of the metalware elements and hydraulic drive elements. The influence of the simultaneous movement of the first jib section, the second jib section and the telescopic jib section on cargo oscillation, as well as the effect of cargo oscillation on dynamic loads that occur in the boom system and manipulator hydraulic drive elements, is determined.

Fig. 7. Ref. 19.

УДК 621.87

Ловейкін В.С., Ромасевич Ю.А., Сподоба А.А., Ловейкін А.В., Почка К.І. **Математическая модель динамики изменения вылета стреловой системы манипулятора при одновременном перемещении ее звеньев** // Сопротивление материалов и теория сооружений. – 2020. – Вип. 104. – С. 175-190.

Получено уравнение движения манипулятора с учетом воздействия инерционной составляющей каждого звена стреловой системы и воздействия колебательного движения груза на динамические нагрузки элементов металлоконструкции и элементов гидравлического привода. Определено влияние одновременного перемещения первой секции стрелы, второй секции стрелы и телескопической секции стрелы на колебания груза, а также воздействие колебаний груза на динамические нагрузки, которые при этом возникают в стреловой системе и элементах гидравлического привода манипулятора.

Ил. 7. Библиогр. 19 назв.

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