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PECULIARITIES OF WAVE PROPAGATION PROCESSES IN POROELASTIC MEDIA

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In this paper are presented peculiarities of wave propagation processes in porous media; parameters that determine properties of fluid-saturated materials; basic methods for solution of poroelastic problems, one of which is Boundary Integral Equation Method; boundary integral equations and graphs of fundamental solutions functions versus frequency parameter.

Key words: poroelasticity, porous media, boundary integral equations, fundamental solution.

Introduction. Many natural and unnatural materials have pores structure especially fluid- or gas-saturated soils, rocks and also porous building materials: timbers, sandstones, bricks, fillers for light concretes. That's why investigation of wave propagation processes in porous bodies and media has practical interest. Presence of filler changes the behavior of such materials therefore laws of the theory of elasticity can't be used for studying of wave propagations in saturated materials.

1. Basic methods. In the end of XVIII century seriously problems of dams and dikes building and necessity of understanding of cooperation and common work of water and the solid ware a reason for first description of the porous media. Now in civil construction problems of the soil-water processes are described on the basis of the theory of the porous media that consists of the theory of mixes and the conception of volume factions. The theory of mixes was based on the mechanics of continuous media that is consists of multi-component materials with different physical properties.

Mathematical modeling of the multi-component fluid- or gas-saturated porous media began in thirties of last century. Works by Y.I. Frenkel [2] and M.A. Biot [3, 4] were first works in this direction. In their works was given great attention to models of the porous media dissipation and methods for considering it in the equilibrium equations. Works by M.A. Biot are the linear theory of the effective two phase media and are supposed as the basic and classic theory for solving similar problems. In this works for the porous fluid-saturated media the two phase model that is consists from the porous solid and the fluid that fills up pores was proposed. Also additional parameters for considering cooperation of these phases was introduced such as: the porosity, the fluid viscosity, the permeability, the Biot coefficient of effective stress, the mass densities, the shear modulus and the bulk modulus of the porous material.

Procedures for determining these parameters are presented in works [4, 5]. Also analysis of porous materials properties are elucidated in works [6, 7, 8].

During analyzing of porous structures stress-strain stain is assumed that the pores are disseminate uniformly in the body. The fluid- or gas-saturated porous region when it's considered from the point of view by the mechanics of the continuous media is essentially the two-phase continuous media. The porous solid elements are belonging to the first phase and the elements of pores fluid filler are belong to the second phase. It should be taking into account during studying of the peculiarities of the porous media behavior that are foredoomed by differences of both phase mechanical properties. Breaking of all elements to two classes is also needful because the difference of the one phase elements behavior is less significant than of the different phase elements behavior. The assumption remains that the elementary volume space is full of two continuous media that can interact with each other. Also the fundamental characteristic of the porous media is propagation of three different compression waves: the longitudinal fast wave, the second longitudinal slow wave, and the third transversal slow wave.

In nineties of twentieth century began to appear the science works that are dedicated to studying of poroelastic problems and application of the Boundary Elements Method and the Boundary Integral Equation Method for solving these problems. The two-dimensional poroelastic equations ware represented almost at the same time in works [9] i [10]. The equations [9] were written in terms of the solid displacements and stresses and the fluid pressure when the boundary integral equations in [10] were consists of the dynamic and kinematic parameters.

One of the methods that now are used for solving systems of the differential equations is the integral and numerical Laplace transformation. This method was used to obtain the fundamental solutions for poroelastic systems in [11, 12] where for solving the problems three phase model was used in which porous skeleton is partially saturated by fluid and partially saturated by gas. In [13] were presented the methods for numerical modeling of the three-dimensional poroelastic bodies dynamic and for solving the model problems about wave propagation in such bodies with different boundary conditions. Solving the problem about elastic wave propagation in the porous region that is not full of the fluid is adducing in [14] with presenting of the differential equations for the not saturated space in three-dimensional transform Laplace region. The work [15] presents the fundamental solutions for the singular boundary integral equations of poroelasticity. Some aspects of linear dynamic poroelasticity in the fluid-saturated bodies are in [16-20]. Despite the fact that now are presented a significant number of the singular boundary integral equations variants are only unitary BE-solutions of the poroelastic problems. That's why questions are actual in this direction.

2. Basic Relations. Whereas the components of the different phases in the porous elastic saturated media have the different densities the total density (the total mass of the fluid-solid aggregate per unit volume) should be considering in calculation. It can be determined by the following expression [3]:

$$\rho=\rho_s+\beta(\rho_f-\rho_s)\,, \tag{1}$$

where β is the porosity of the porous solid, the parameters ρ_s and ρ_f are the mass densities of the solid and fluid, respectively. Should be taking into account the assumption that the relative motion between the solid and the fluid is not exists.

Another peculiarity of the porous fluid-saturated media are, proposed by M.A. Biot [3, 4] and analyzed in [6], the coefficients of the poroelastic material: Q, R, B i M that are expressed from the porosity β, the Biot coefficient of effective stress α and the drained and undrained bulk modules of elasticity K i K_u:

$$Q=\frac{\beta(\alpha-\beta)(K_u-K)}{\alpha^2}\,, \tag{2}$$

$$R=\frac{\beta^2(K_u-K)}{\alpha^2}\,, \tag{3}$$

$$B=\frac{\alpha M}{K+\alpha^2 M}\,, \tag{4}$$

$$M=\frac{R}{\beta^2}\,, \tag{5}$$

where the coefficient α is determined:

$$\alpha=\frac{\beta(Q+R)}{R}\,. \tag{6}$$

The bulk modules of elasticity are determined after three types of the laboratory tests (the drained test, the unjacketed test and the undrained test) [6]:

$$K=\frac{V\cdot\Delta P}{\Delta V}\,, \tag{7}$$

$$K_u=\frac{V_u\cdot\Delta P}{\Delta V_u}\,, \tag{8}$$

where V and V_u are the primary volumes of the drained and undrained rock samples; ΔP is the incremental load in time that is applied on the rock as the pressure; ΔV i ΔV_u are the volume changes of the drained and undrained samples.

The algorithmic bases of the BEM are the boundary analogues of Somiliani's formulas for the solid displacements and the fluid pressure that under zero body conditions can be written [9]:

$$c_{ij}u_i+\int_\Gamma t^*_{ij}u_i d\Gamma+\int_\Gamma \tau^*_j U_i d\Gamma=\int_\Gamma u^*_{ij}t_i d\Gamma+\int_\Gamma \tau U^*_{nj} d\Gamma\,, \tag{9}$$

$$\int_\Gamma t^*_{i3}u_i d\Gamma+\int_\Gamma \tau^*_3 U_n d\Gamma=\int_\Gamma u^*_{i3}t_i d\Gamma+\int_\Gamma \tau\left(U^*_{n3}-JX_i'^{*}n_i\right)d\Gamma+Jc_{33}\tau\,, \tag{10}$$

where c is the coefficient that is equal 0.5 for points where the boundary is smooth, u_i, U_i, t_i, τ are the displacements and stresses in the solid and fluid pressure; n is the normal to the boundary; $J=1/(i\omega b-\omega^2\rho_{22})$; u_{ij}^*, t_{ij}^*, τ_j^*, U_{nj}^* are the weighting displacement fields or the fundamental solutions. The first components of the fundamental solution tensors may be written as:

$$u_{11}^*(r,\omega)=\sum_{m=1}^{3}\left[\frac{\delta(m,3)}{2\pi\mu}K_0(i\lambda_m r)+\frac{i\lambda_m a_m}{r}K_1(i\lambda_m r)+\lambda_m^2 a_m (r_{,1})^2 K_2(i\lambda_m r)\right], \quad (11)$$

$$u_{12}^*(r,\omega)=u_{21}^*(r,\omega)=\sum_{m=1}^{3}\lambda_m^2 a_m r_{,1} r_{,2} K_2(i\lambda_m r), \quad (12)$$

$$u_{22}^*(r,\omega)=\sum_{m=1}^{3}\left[\frac{\delta(m,3)}{2\pi\mu}K_0(i\lambda_m r)+\frac{i\lambda_m a_m}{r}K_1(i\lambda_m r)+\lambda_m^2 a_m (r_{,2})^2 K_2(i\lambda_m r)\right], \quad (13)$$

where λ_m are the wave numbers that can be obtained as the roots of the characteristic equation; $K_\alpha(i\lambda_m r)$ is the modified Bessel functions;

$$K(\omega)=\frac{R}{b+i\omega\rho_{22}},$$

$$\alpha_1(\omega)=\frac{-(1-i\omega/K\lambda_1^2)}{2\pi(\lambda_2^2-\lambda_1^2)(\lambda+2\mu)},$$

$$\alpha_2(\omega)=\frac{1-i\omega/K\lambda_2^2}{2\pi(\lambda_2^2-\lambda_1^2)(\lambda+2\mu)},$$

$$\alpha_3(\omega)=\frac{-1}{2\pi\rho\omega^2}.$$

When similar fundamental solutions for the elastic region are:

$$U_{11}^*(r,\omega)=\frac{i}{4\mu}\left[H_0(k_2r)-\frac{1}{k_2r}\left(H_1(k_2r)-\frac{C_2^2}{C_1^2}H_1(k_1r)\right)+\right.$$
$$\left.+r_{,1}^2\left(H_2(k_2r)-\frac{C_2^2}{C_1^2}H_2(k_1r)\right)\right], \quad (14)$$

$$U_{12}^*(r,\omega)=U_{21}^*(r,\omega)=\frac{i}{4\mu}r_{,1}r_{,2}\left(H_2(k_2r)-\frac{C_2^2}{C_1^2}H_2(k_1r)\right), \quad (15)$$

$$U_{22}^*(r,\omega)=\frac{i}{4\mu}\left[H_0(k_2r)-\frac{1}{k_2r}\left(H_1(k_2r)-\frac{C_2^2}{C_1^2}H_1(k_1r)\right)+\right.$$
$$\left.+r_{,2}^2\left(H_2(k_2r)-\frac{C_2^2}{C_1^2}H_2(k_1r)\right)\right], \quad (16)$$

where ρ is the density of the elastic material; $k_\alpha=\dfrac{\omega}{C_\alpha}$; $C_1=\sqrt{(\lambda+2\mu)/\rho}$, $C_2=\sqrt{\mu/\rho}$ are the velocities of elastic wave propagation.

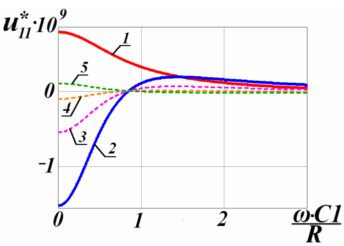

Fig. 1

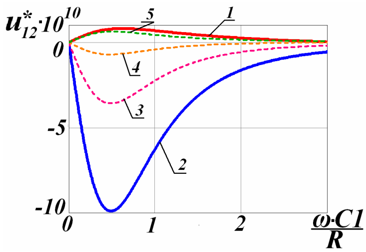

Fig. 2

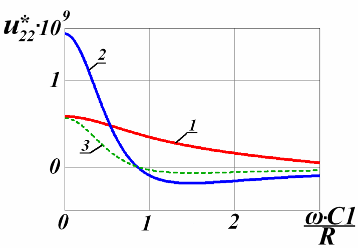

Fig. 3

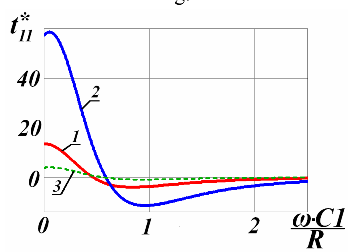

Fig. 4

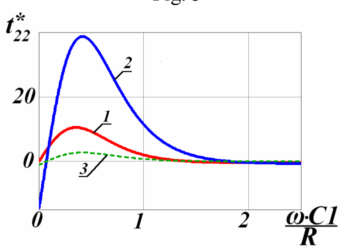

Fig. 5

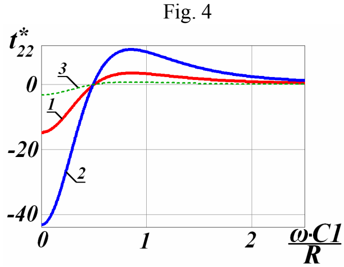

Fig. 6

Figures present the graphs of the fundamental solutions functions: the displacements u_{11}, u_{12}, u_{22} (fig. 1, 2, 3) and the stresses t_{11}, t_{12}, t_{22} (fig. 4, 5, 6) versus frequency parameter $\omega r/C_1$. The curves with designation 1 correspond to the graphs of functions for elastic media and the curves with designation 2 correspond to the graphs of functions for poroelastic fluid-saturated media, respectively.

Conclusion. The graphs of the weighting displacements and stresses fields functions for the elastic and poroelastic regions have different characters and different values depending on the frequency parameter because the body with gas- or fluid-saturated pores is differ from the continuous homogeneous elastic media and it should be modeling with applying of the two phase or the three phase model and the poroelastic equations with additional poroelastic parameters. Figures show that the graphs for the poroelastic region may be gradual approximated to the elastic analogues during changing some parameters. On the graphs in the figures 1-3 was changing of the parameter *R* namely gradual increase of it for the some order (curves 3, 4, 5). When the graphs of generalized derivatives functions on the figures 4-6 had changing of the parameter *Q* – one gradual increase for one order was enough (curves 3).

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Кара І.Д.

ОСОБЛИВОСТІ ПРОЦЕСІВ ПОШИРЕННЯ ХВИЛЬ В ПОРОПРУЖНОМУ СЕРЕДОВИЩІ

При дослідженні процесів розповсюдження хвиль в насичених пористих середовищах, на відміну від теорії пружності, має прийматись запропонована М.А.Біо двофазна модель середовища, в якій до першої фази належать тверді часточки пористого каркасу, а до другої відносяться елементи рідини, яка заповнює пори. Іноді для задач використовується трифазна модель середовища, в якій пористий пружний каркас частково заповнений рідиною, частково газом. Для пружного пористого середовища вводяться параметри, зокрема: пористість, в'язкість рідини, проникність, коефіцієнт ефективних напружень Біо, модулі зсуву та об'ємного стиснення, ефективні щільності та узагальнена густина пористого матеріалу. Також фундаментальною властивістю пружно-пористого насиченого середовища є те, що в ньому можуть розповсюджуватись три типи хвиль, а саме: дві поздовжні хвилі: швидка і повільна, а також поперечна повільна хвиля. Одним із методів розв'язання проблем поропружності є метод граничних інтегральних рівнянь. Алгоритмічною основою методу є граничні аналоги формули Соміліани для переміщень в пружному скелеті і тиску в рідині. Граничні інтегральні рівняння та фундаментальні розв'язки, які входять до складу рівнянь поропружності, суттєво відрізняються від аналогічних в теорії пружності, оскільки тіло, в якому містяться заповнені рідиною пори, відрізняється від суцільного однорідного пружного середовища. З рисунків видно, що змінюючи певні параметри, графіки для поропружної області можна поступово наближати до аналогічних для пружної. Найбільший вплив на функції переміщень дає зміна параметра R а саме поступове збільшення його на декілька порядків. В той час як для зміни графіків функцій узагальнених похідних фундаментального розв'язку достатньо одного збільшення на порядок значення параметра модуля Q.

Ключові слова: поропружність, пористе середовище, граничні інтегральні рівняння, фундаментальний розв'язок.

Kara I.D.

PECULIARITIES OF WAVE PROPAGATION PROCESSES IN POROELASTIC MEDIA

During analyzing of wave propagation processes in the fluid-saturated porous media unlike the theory of elasticity should be applied proposed by Biot the two phase model of media in which porous the solid elements are belonging to the first phase and the elements of pores fluid filler are belong to the second phase. Sometimes, for solving problems three phase model are used in which porous skeleton is partially saturated by fluid and partially saturated by gas. For the elastic porous media are introduced parameters such as: the porosity, the fluid viscosity, the permeability, the Biot coefficient of effective stress, the shear modulus and the bulk modulus, the mass densities and the total density of the porous material. Also the fundamental characteristic of the porous media is propagation of three different compression waves: the longitudinal fast wave, the second longitudinal slow wave, and the third transversal slow wave. One of the methods that are used for solving problems of poroelasticity is the Boundary Integral Equation Method. The algorithmic bases of it are the boundary analogues of Somiliani's formulas for the solid displacements and the fluid pressure. The boundary integral equations and the fundamental solutions that are comprised in the poroelastic equations are different from the theory of elasticity analogues because the body with fluid-saturated pores is differ from the continuous homogeneous elastic media. Figures show that the graphs for the poroelastic region may be gradual approximated to the elastic analogues during changing some parameters. The biggest influence for displacements functions has change of the parameter R especially gradual increase of it for the some order. When for changing the functions graphs of the generalized derivatives one gradual increase of the parameter Q for one order is enough.

Key words: poroelasticity, porous media, boundary integral equations, fundamental solution.

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Кара І.Д. **Особливості процесів поширення хвиль в поропружному середовищі** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип. 105. – С. 247 – 254.
Наведені особливості хвильових процесів в пористих середовищах; величини, які визначають властивості насичених пористих матеріалів; основні підходи до розв'язання проблем поропружності, одним із яких є метод граничних інтегральних рівнянь; граничні інтегральні рівняння та графіки залежності фундаментальних розв'язків поропружності від частотного параметра
Табл. 0. Іл. 6. Бібліогр. 19 назв.

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Kara I.D. **Peculiarities of wave propagation processes in poroelastic media** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2020. – Issue 105. – P. 247 – 254.
In this paper are presented peculiarities of wave propagation processes in porous media; parameters that determine properties of fluid-saturated materials; basic methods for solution of poroelastic problems, one of which is Boundary Integral Equation Method; boundary integral equations and graphs of fundamental solutions functions versus frequency parameter.
Tabl. 0. Fig. 6. Ref. 20.

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Кара И.Д. **Особенности процессов распространения волн в пороупругих средах** // Сопротивление материалов и теория сооружений. – К.: КНУБА, 2020. – №. 105. – С. 247 – 254.
Приведены особенности волновых процессов в пористых средах; параметры, которые определяют свойства насыщенных пористых материалов; основные подходы к решению проблем пороупруугости, одним из которых есть метод граничных интегральных уравнений; граничные интегральные уравнения и графики зависимости фундаментальных решений от частотного параметра.
Табл. 0. Ил. 6. Библиогр. 20 назв.

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