

UDC 539.375

CONVERGENCE OF THE FINITE ELEMENT METHOD AND THE SEMI-ANALYTICAL FINITE ELEMENT METHOD FOR PRISMATIC BODIES WITH VARIABLE PHYSICAL AND GEOMETRIC PARAMETERS

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DOI: 10.32347/2410-2547.2021.106.92-104

In this paper, a numerical study of the convergence of solutions obtained on the basis of the developed approach [1, 3, 4, 5] is carried out. A wide range of test problems for bodies with smoothly and abruptly varying physical and geometric characteristics in elastic and elastic-plastic formulation are considered. In all cases, the semi-analytic finite element method is not inferior in approximation accuracy, and in some problems it is 1.5-2 times superior to the traditional method of scheduling elements. finite element method.

Keywords: finite element method (FEM), semi-analytical finite element method (SAFEM), finite element (FE), stress-strain state, physical and geometric nonlinearity, elastic and elastic-plastic deformation, curvilinear prismatic bodies.

Introduction. The approach developed within the framework of the semi-analytical method to study the stress-strain state of inhomogeneous curvilinear prismatic bodies, taking into account physical and geometric nonlinearity, requires substantiation of its effectiveness in relation to the traditional FEM and confirmation of the reliability of the results obtained on its basis.

The main indicators that allow comparing the SAFEM and FEM include the rate of convergence of solutions with an increase in the number of unknowns and the amount of charges associated with solving linear and nonlinear equations. For the considered class of problems, the convergence is determined by such factors as the nature of the change along Z^3 of the geometric and mechanical parameters of the object. The uneven distribution of mechanical characteristics is associated with the presence of the initial heterogeneity of the material, the development of plastic deformations, and the

dependence of material properties on temperature. The same factors also affect the convergence of the iterative process, since the conditionality of the SAFEM matrix depends on them. In order to determine the area of effective application of SAFEM, a wide range of test cases are considered.

Convergence of SAFEM and FEM for bodies with a smooth change along the coordinate of expansion of geometric and physicomachanical characteristics. The comparison of the convergence of solutions with an increase in the number of unknowns was carried out by assessing the accuracy of the obtained solutions in relation to the reference ones, which were taken as the results of other authors or those obtained by the FEM.

A comparison is made of the convergence of SAFEM and FEM for bodies with a smooth change along the decomposition coordinate in the geometric and physicomachanical characteristics, which does not lead to significant local features in the distribution of the parameters of the stress-strain state.

Consider the elastic equilibrium of a bar (Fig. 1), the lower surface of which is described by a parabola, and, therefore, its height is a smooth continuously function of the coordinate $Z^{3'}$:

$$h(Z^{3'}) = h_0 - 0,4(Z^{3'})^2. \quad (1)$$

The boundary conditions at the ends $Z^{3'} = -1$ and $Z^{3'} = 1$ correspond to the support on diaphragms that are absolutely rigid in the plane and flexible from it:

$$U^{1'}/Z^{3'} = \pm 1 = \sigma^{3'3'}/Z^{3'} = \pm 1 = 0. \quad (2)$$

To simulate the conditions of plane deformation in this and other test examples, a layer of finite thickness was distinguished in the $Z^{2'}$ direction, which was approximated by one FE, fixed from displacements along $U^{2'}$.

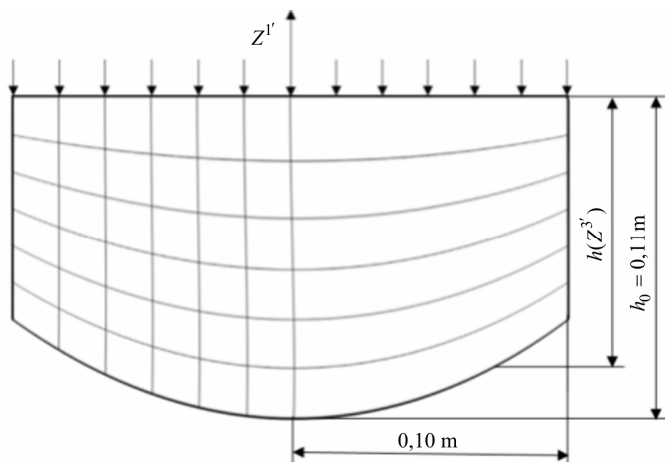


Fig. 1

Loading is carried out on the upper surface of the body with a uniformly distributed load. In the calculations, a unit load intensity $q=1$ and a unit modulus

of elasticity were taken. To obtain a reference solution, the finite element discretization of the object was used, one of the variants of the finite element mesh is shown in Fig. 1 to the left of the axis of symmetry. Studies on convergence (Table 1) showed that it is enough to use 144 FE, since a further increase in their number leads to a slight change in the result. Studies of the influence of the number of retained members of the series m_3 were carried out with a fixed number of elements along Z^1 equal to 8.

Table 1

m	$V_{\max}^{1'}$	%	$V_{\max}^{2'}$	%
25	$1.535 \cdot 10^2$	3.3	$9.109 \cdot 10^3$	5.2
81	$1.579 \cdot 10^2$	0.6	$9.503 \cdot 10^3$	1.1
169	$1.588 \cdot 10^2$	-	$9.608 \cdot 10^3$	-

The character of the SAFEM convergence is shown in Fig. 2 in the form of a graph (solid line), reflecting the dependence of the error in calculating the maximum relative displacement ($V_{\max}^{1'} = U_{\max}'/h_0$), $h_0 = 0.11$ m on the number of retained expansion terms in the third direction. To compare the efficiency of the polynomial and piecewise linear approximation, the dotted line in the same figure shows the FEM convergence graph. As we can see only the three terms of the expansion ensures that the result is only 3,6% different from the reference, while the use of three nodes of the FEM mesh region leads to more than 2 times the percentage of error.

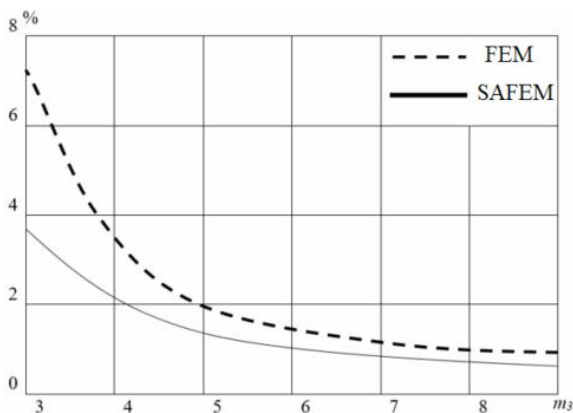


Fig. 2

Approbation of the methodology for calculating prismatic bodies with smoothly varying physical and mechanical parameters and the study in this case of the rate of convergence of solutions are given on the problem of elastic

equilibrium of a prismatic bar, the modulus of elasticity of which changes according to the law:

$$E(Z^3) = E_0 - (E_0 - E_1) \left| Z^3 \right|, \quad (3)$$

$$E_0 = 2.1 \cdot 10^5 \text{ MPa}, \quad E_1 = 1.4 \cdot 10^5 \text{ MPa}.$$

In Fig. 3 shows the geometric dimensions of the beam, as well as the division into finite elements for FEM and SAFEM. The boundary conditions at the ends of the body are determined in accordance with (2). The object is loaded on the upper surface with a uniformly distributed load of unit intensity.

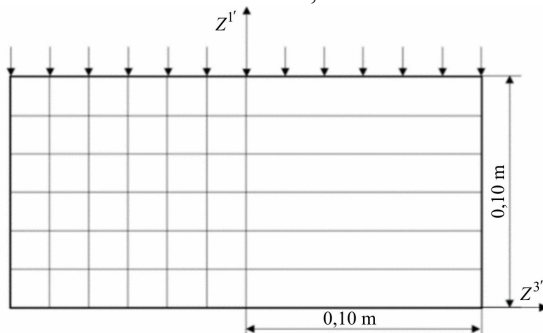


Fig. 3

The reference solution was obtained

by the finite element method using 169 nodes of the grid region (Table 2). When comparing the convergence of the SAFEM [3, 5] and the FEM in the Z^1 direction, the approximation was carried out by 8 finite elements.

Table 2

m	V_{\max}^1	%	V_{\max}^2	%
25	$1.517 \cdot 10^{-2}$	3.3	$8.762 \cdot 10^{-3}$	5.2
81	$1.557 \cdot 10^{-2}$	0.6	$9.239 \cdot 10^{-3}$	1.1
169	$1.565 \cdot 10^{-2}$	-	$9.373 \cdot 10^{-3}$	-

Graphs of changes in the error in determining the maximum relative displacements U^1 depending on the number of unknown m_3 are shown in Fig. 4.

The character of convergence is observed, similar to that obtained earlier for bodies with variable geometry.

Convergence of SAFEM and FEM for inhomogeneous curvilinear prismatic bodies with various types of inserts, notches and holes. At this stage, we will consider inhomogeneous curvilinear prismatic

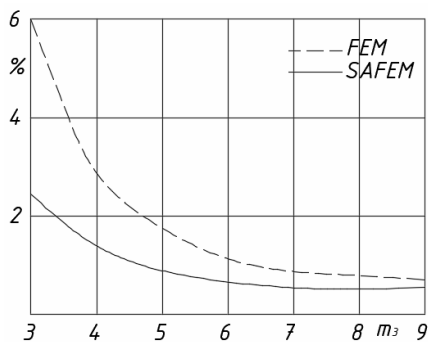


Fig. 4

bodies with various types of inserts, cuts and holes. The geometric and

physical characteristics of such structures are described by piecewise continuous functions. Since the presence of concentrators leads to a local redistribution of stresses, then to approximate objects of this type, it is required to increase the number of retained expansion terms.

To assess the effect of the notch depth on the convergence of the FEM and SAFEM, the problem of stretching a strip weakened by notches, the shape of

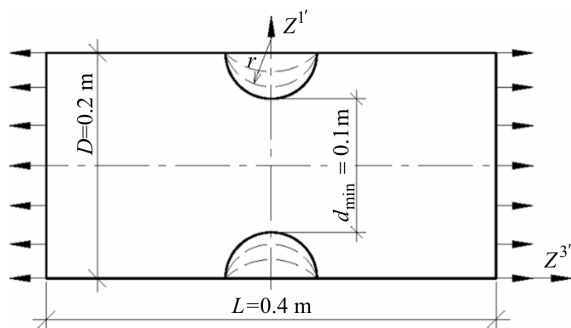


Fig. 5

which is described by an arc of radius z with a gradual increase in the notch depth, was solved. Calculation scheme shown in Fig. 5. The width of the strip is $D = 0,2$ m, its length is $0,4$ m, and the length of the concentrator l is taken equal to $0,1$ m.

D is the width of the neck.

The concentration factor is defined as the ratio of the maximum stresses acting in the weakened section of the sample to the nominal stresses

$$K_t = \frac{\sigma_{\max}}{\sigma_{\text{nom}}}. \quad (4)$$

The nominal stresses σ_{nom} are found by the formula:

$$\sigma_{\text{nom}} = \frac{P}{b \cdot d}, \quad (5)$$

where $P = q \cdot D$.

Concentration coefficients, calculated on the basis of calculated SAFEM data for D/d varying in the range from 1 to 2, are compared with those obtained in [6]. Based on the results shown in Table 3, we can conclude about the reliability of the solutions obtained, since the percentage of the error in calculating K_t by the semi-analytical finite element method in relation to the reference value for cutouts of various depths does not exceed 2,0%.

Table 3

D/d		L	1.1	1.54	2
K_t	SAFEM	1,01	1,215	1,44	1,485
	FEM	1,0	1,23	1,46	1,51
	%	1	1,2	1,4	1,65

For the limiting case of a strip with semicircular notches, the convergence of the FEM and SAFEM solutions to the reference one is compared [6]. Table

4 shows the change in the error in calculating σ_{\max} with an increase in the accuracy of approximation by Z^3 .

Table 4

m_3	SAFEM		FEM	
	σ_{\max}	%	σ_{\max}	%
7	2.81	7.1	2.73	9.8
9	3.01	0.3	2.9	4
13	2.97	1.6	2.98	1.3

In this test case, both methods, for an equal number of unknowns, yield solutions with the same accuracy.

The comparison of the rate of convergence of solutions for the traditional and semi-analytical FEM variants with a stepwise change in the elastic modulus was carried out using the example of elastic deformation of a rectangular plate with a square insert (Fig. 6). Solutions to the problem are obtained with a gradual decrease in the elastic modulus of the insert material E from the initial value E_0 (the

modulus of elasticity of the plate material), up to the formation of a hole.

A uniformly distributed load is applied to two opposite sides of the plate, intensity $q = 1$. The reference solution was obtained by the finite element method with a grid uniformly in the Z^1 and Z^3 directions and a total number of nodes equal to 247. The solution of the SAFEM problem was carried out using 12 elements in the Z^1 direction.

A study of the convergence of the resulting stresses on the notch contour (point A) with a decrease in the elastic modulus of the insert from E_0 to 0 and keeping the value of the elastic modulus of the plate material to the reference solution obtained by the FEM for a plate with a square hole was carried out. When the ratio of the elastic moduli of the material and the insert is equal to 4 orders of magnitude, the values of the considered parameters of the stress-strain state are already quite close to the reference ones (Fig. 6).

When modeling cuts bounded by coordinate planes, a region with a zero elastic modulus, the question of satisfying the natural boundary conditions on the free surface of the inner contour becomes important for substantiating the reliability of the results.

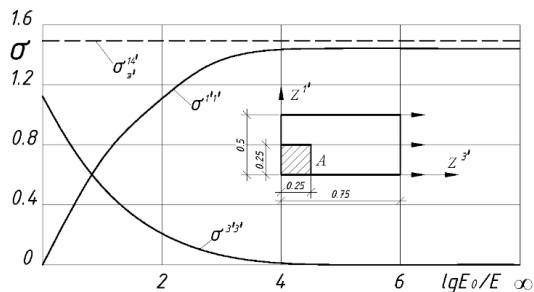


Fig. 6

Consider the stress distribution in finite elements passing through the cutout. The $\sigma^{3'3'}$ graphs are shown in Fig. 7. indicate that the boundary conditions on the free surface are satisfied, since when approaching the contour of the hole, the value of stresses tends to zero. To confirm the results obtained by semi-analytical methods when calculating a plate with a square hole, let us compare the stress diagrams in different sections with the stresses determined in [2].

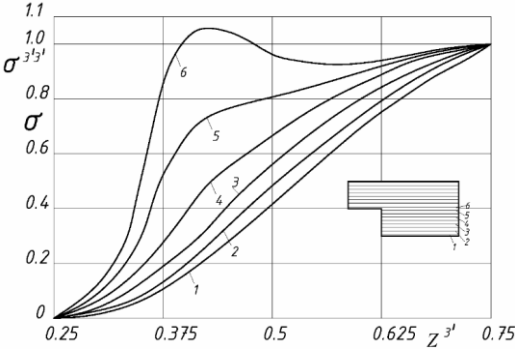


Fig. 7

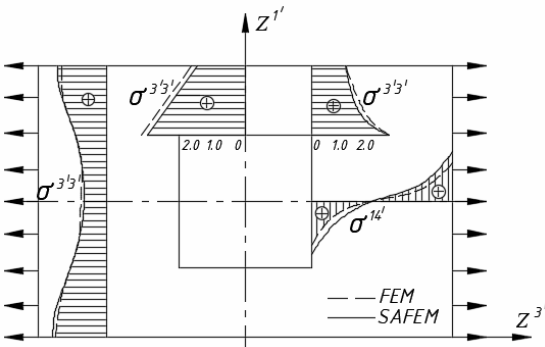


Fig. 8

The data presented indicate that SAFEM allows obtaining results with a higher accuracy than the traditional finite element method with the same number of unknowns.

Table 5

m_3	SAFEM		FEM	
	$V_{\max}^{1'}$	%	$V_{\max}^{2'}$	%
4	$7.645 \cdot 10^{-6}$	2.6	$7.452 \cdot 10^{-6}$	5.2
7	$7.913 \cdot 10^{-6}$	-0.8	$7.737 \cdot 10^{-6}$	1.44
10	$7.920 \cdot 10^{-6}$	-0.9	$7.806 \cdot 10^{-6}$	0.6

The nature of the stress-strain state of a plate with a cut depends on its linear dimensions. In Fig. 9 shows the stress diagrams $\sigma^{1'1'}$ in the section passing through the $Z^{3'}$ axis when the length of the cut l is changed.

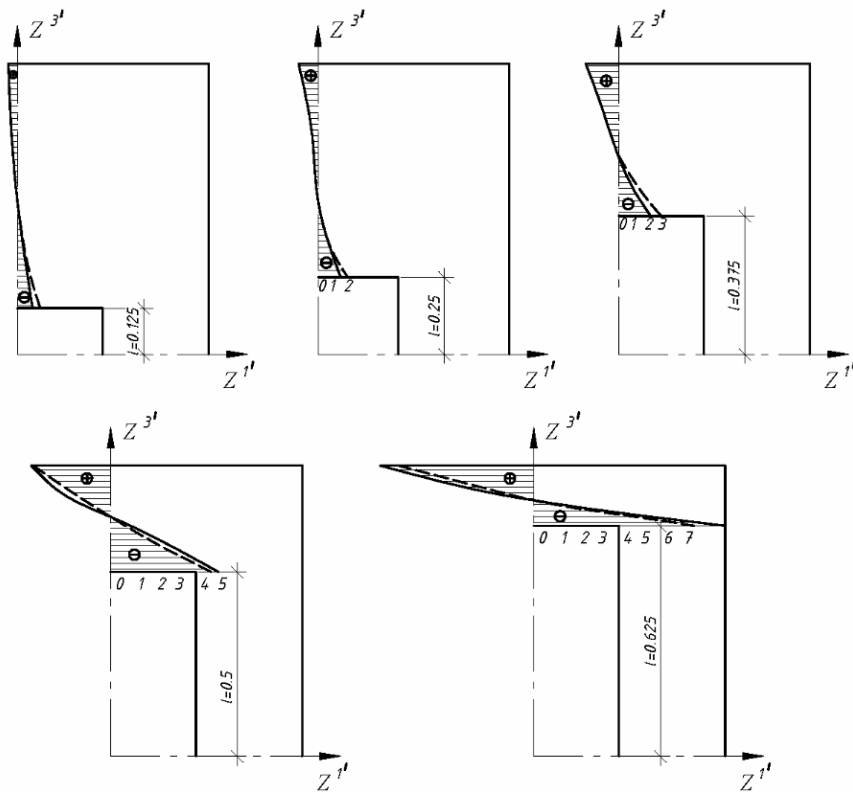


Fig. 9

Solid lines correspond to the solution obtained by the SAFEM, dashed lines - to the FEM. Both methods give similar results. With the ratio $l/L = 5/6$ the diagram $\sigma^{1'1'}$ is linear and the flange can be calculated as a restrained beam. In this case, in order to achieve a given accuracy with both FEM variants, it is required to keep the same number of approximating functions in the $Z^{3'}$ direction Fig. 10.

As you can see, when considering prismatic bodies with piecewise-continuous variation of geometric and physical-mathematical

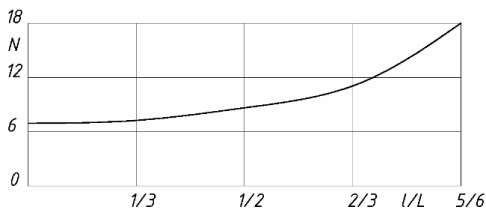


Fig. 10

parameters, the accuracy of approximating displacements by polynomial expansion is not inferior, and in some cases even exceeds piecewise-linear approximation.

The presence of plastic deformations further complicates the picture of the stress-strain state, further complicates the picture of the stress-strain state of the object.

A comparison of the convergence of the SAFEM and the FEM in this case

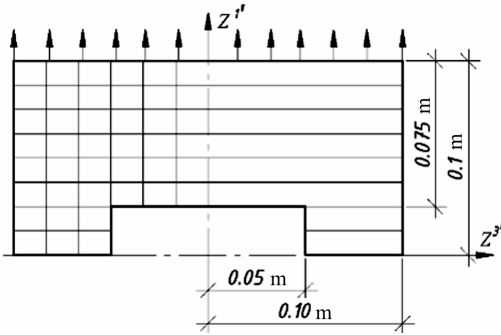


Fig. 11

was carried out using the example of elastic-plastic deformation of an infinite strip of rectangular cross-section with a notch (Fig. 11). The strip is under the influence of a load evenly distributed over the upper surface $q = 0.5\tau_s$. The boundary conditions at the ends are taken in the form (2).

The data in Table 6, the values of the maximum relative displacements obtained by the finite element method allow us to conclude that the solution using 289 nodes of the grid region can be taken as a reference.

Table 6

m	V_{\max}^1	%	V_{\max}^3	%
25	$1.460 \cdot 10^{-2}$	3.3	$-1.075 \cdot 10^{-2}$	5.2
81	$1.508 \cdot 10^{-2}$	0.6	$-1.129 \cdot 10^{-2}$	1.1
169	$1.519 \cdot 10^{-2}$	0.6	$-1.144 \cdot 10^{-2}$	1.1
289	$1.523 \cdot 10^{-2}$	-	$-1.149 \cdot 10^{-2}$	-

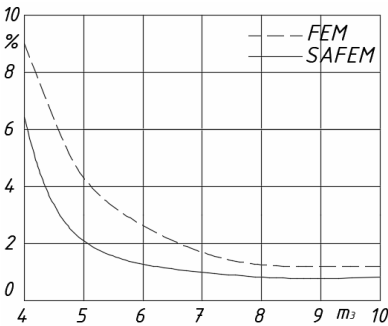


Fig. 12

Convergence studies were carried out for a fixed number of elements along Z^1 , equal to 13. Graphs of changes in the error in determining the maximum displacements of the SAFEM and FEM for the elastic-plastic solution are shown in Fig. 12. The character of convergence is observed, similar to that obtained for the elastic solution.

Consequently, the accuracy of approximation of displacements in solving problems of elastic and elastic-plastic formulation for the finite element method and its semi-analytical version has the same order of magnitude.

When considering the convergence of semi-analytical [3, 5] and traditional finite element methods for a wide class of curvilinear prismatic objects with variable physical and mechanical characteristics and a rather complicated law of geometry change in problems of elastic and elastic-plastic equilibrium, the accuracy of the polynomial approximation of displacements in the Z^3 'direction is not inferior, and in some cases even exceeds the piecewise linear approximation.

Conclusion. The use of SAFEM makes it possible to obtain solutions for objects with smoothly varying geometric and physical parameters of a given accuracy with a smaller number of unknowns than FEM. As shown earlier, the rate of convergence of the iterative process of solving systems of linear and nonlinear equations of the SAFEM [1, 4] by the method of block iterations is an order of magnitude higher than the rate of convergence of solutions of FEM systems. Thus, the efficiency of SAFEM - based solution of elastic and elastic-plastic problems for a new wide class of structures - curvilinear prismatic bodies of complex configuration with variable physical and mechanical parameters in the presence of cuts and holes exceeds the efficiency of using traditional FEM.

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ЗБІЖНІСТЬ МЕТОДА СКІНЧЕНИХ ЕЛЕМЕНТІВ І НАПІВНАЛІТИЧНОГО МЕТОДУ СКІНЧЕНИХ ЕЛЕМЕНТІВ ДЛЯ ПРИЗМАТИЧНИХ ТІЛ З ПЕРЕМІННИМИ ФІЗИЧНИМИ І ГЕОМЕТРИЧНИМИ ПАРАМЕТРАМИ

В роботі виконано чисельне дослідження збіжності розв'язання, одержуваних на базі розробленого підходу [1, 3, 4, 5]. Розглянуто широке коло тестових завдань для тіл з плавно і стрибкоподібно змінюваними фізичними та геометричними характеристиками в пружній і пружно-пластичній поведінці. Розроблений в рамках напіваналітичного методу підхід до дослідження напружено-деформованого стану неоднорідних криволінійних призматичних тіл з урахуванням фізичної і геометричної нелінійності вимагає обґрунтування його ефективності по відношенню до традиційного МСЕ і підтвердження достовірності одержуваних на його основі результатів.

До числа основних показників, що дозволяють провести зіставлення НМСЕ і МСЕ, відносяться швидкість збіжності розв'язання при збільшенні числа невідомих і обсяг обчислення, пов'язаних з розв'язання лінійних і нелінійних рівнянь. Для розглянутого класу задач збіжності визначається такими факторами, як характер зміни уздовж Z^3 геометричних і механічних параметрів об'єкта. Нерівномірний розподіл механічних характеристик пов'язано з наявністю початкової неоднорідності матеріалу, розвитком пластичних деформацій і залежністю властивостей матеріалу від температури. Ці ж фактори впливають і на збіжність ітераційного процесу, оскільки від них залежить обумовленість матриці НМСЕ. З метою визначення області ефективного застосування НМСЕ розглянуто широке коло контрольних прикладів.

У всіх випадках напіваналітичний метод скінчених елементів по точності апроксимації не поступається, а в деяких задачах в 1.5-2 рази перевершує традиційний метод скінчених елементів.

Ключові слова: метод скінчених елементів, напіваналітичний метод скінчених елементів, скінчений елемент, напружено-деформований стан, фізична і геометрична нелінійність, пружне і пружно-пластичне деформування, криволінійні призматичні тіла.

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The main indicators that allow comparing the SAFEM and FEM include the rate of convergence of solutions with an increase in the number of unknowns and the amount of charges associated with solving linear and nonlinear equations. For the considered class of problems, the convergence is determined by such factors as the nature of the change along Z^3 of the geometric and mechanical parameters of the object. The uneven distribution of mechanical characteristics is associated with the presence of the initial heterogeneity of the material, the development of plastic deformations, and the dependence of material properties on temperature. The same factors also affect the convergence of the iterative process, since the conditionality of the SAFEM matrix depends on them. In order to determine the area of effective application of the SAFEM, a wide range of test cases are considered.

In all cases, the semi-analytic finite element method is not inferior in approximation accuracy, and in some problems it is 1.5-2 times superior to the traditional method of scheduling elements. finite element method.

Keywords: finite element method, semi-analytical finite element method, finite element, stress-strain state, physical and geometric nonlinearity, elastic and elastic-plastic deformation, curvilinear prismatic bodies.

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СХОДИМОСТЬ МЕТОДА КОНЕЧНЫХ ЭЛЕМЕНТОВ И ПОЛУАНАЛИТИЧЕСКОГО МЕТОДА КОНЕЧНЫХ ЭЛЕМЕНТОВ ДЛЯ ПРИЗМАТИЧЕСКИХ ТЕЛ С ПЕРЕМЕННЫМИ ФИЗИЧЕСКИМИ И ГЕОМЕТРИЧЕСКИМИ ПАРАМЕТРАМИ

В работе выполнено численное исследование сходимости решения, получаемых на базе разработанного подхода [1, 3, 4, 5]. Рассмотрен широкий круг тестовых задач для тел с плавно и скачкообразно меняющимися физическими и геометрическими характеристиками в упругой и упруго-пластической постановке. Во всех случаях полуаналитический метод конечных элементов по точности аппроксимации не уступает, а в некоторых задачах в 1.5-2 раза превосходит традиционный метод конечных элементов.

Ключевые слова: метод конечных элементов, полуаналитический метод конечных элементов, конечный элемент, напряженно-деформированное состояние, физическая и геометрическая нелинейность, упругое и упруго-пластическое деформирование, криволинейные призматические тела.

УДК 539.375

Баженів В.А., Горбач М.В., Мартинюк І.Ю., Максим'юк О.В. Збіжність метода скінчених елементів і напіваналітичного методу скінчених елементів для призматичних тіл з перемінними фізичними і геометричними параметрами // Опір матеріалів і теорія споруд: наук.-тех. збірн. – Київ: КНУБА, 2021. – Вип. 106. – С. 92-104.

В роботі виконано чисельне дослідження збіжності результатів, одержуваних на базі розробленого підходу [1, 3, 4, 5]. Розглянуто широке коло тестових завдань для тіл з плавно і стрибкоподібно змінюваними фізичними та геометричними характеристиками в пружно і пружно-пластичній постановці. У всіх випадках напіваналітичний метод скінчених елементів по точності апроксимації не поступається, а в деяких задачах в 1.5-2 рази перевершує традиційний метод скінчених елементів.

Табл. 6. Іл. 12. Бібліогр. 6 назв.

UDC 539.375

Bazhenov V.A., Horbach M.V., Martyniuk I.Yu., Maksymyuk O.V. Convergence of the finite element method and the semi-analytical finite element method for prismatic bodies with variable physical and geometric parameters // Strength of Materials and Theory of Structures: Scientific-&Technical collected articles – Kyiv: KNUBA, 2021. – Issue 106. – P. 92-104.

The paper performs a numerical study of the convergence of the results obtained on the basis of the developed approach [1, 3, 4, 5]. A wide range of test tasks for bodies with smoothly and abruptly changing physical and geometric characteristics in elastic and elastic-plastic formulation is considered. In all cases, the semi-analytical method of finite elements is not inferior in accuracy of approximation, and in some problems is 1.5-2 times higher than the traditional method of finite elements.

Табл. 6. Fig. 12. Ref. 6.

УДК 539.375

Баженів В.А., Горбач М.В., Мартинюк І.Ю., Максим'юк О.В. Полуаналітичний метод скінчених елементів в упругій і упругопластичній постановці для криволинійних призматических об'єктів // Спротивлення матеріалів і теорія споруд: науч.-тех. сборн. – К.: КНУСА, 2020. – Вип. 106. – С. 92-104.

В роботі виконано чисельне дослідження сходимости решения, получаемых на базе разработанного подхода [1, 3, 4, 5]. Рассмотрен широкий круг тестовых задач для тел с плавно и скачкообразно меняющимися физическими и геометрическими характеристиками в упругой и упруго-пластической постановке. Во всех случаях полуаналитический метод конечных элементов по точности аппроксимации не уступает, а в некоторых задачах в 1.5-2 раза превосходит традиционный метод конечных элементов.

Табл. 6. Ил. 4. Библиогр. 6 назв.

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