

UDC 539.3

**OPTIMAL DESIGN OF SHELL CONSTRUCTIONS TAKING INTO ACCOUNT THE EVOLUTION OF CORROSION DAMAGE****A.P. Dzyuba,**

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DOI: 10.32347/2410-2547.2022.108.17-34

An algorithm for computer modeling of the evolution of corrosion wear of the surface of shell elements under conditions of joint action of mechanical external loads and the impact of aggressive environments has been developed. The algorithm of optimal design of shell structures taking into account corrosion degradation of the material surface is constructed, the essence of which is to optimize the design parameters at the end of its durability, further reverse build-up of the sacrificial layer of material and rational refinement of the project according to technological requirements. The results of weight optimization of a cylindrical tank for storage of petroleum products under conditions of asymmetric bilateral corrosion damage to the material surface are presented.

**Keywords:** modeling of corrosion damage evolution, shells, strength, aggressive environment, optimal design.

**Introduction**

The problem of predicting corrosion degradation and optimal design of structures operating under the combined action of extreme power loads and the effects of corrosion is quite relevant, whereas the operating conditions of structures in aggressive environments are typical of many branches of mechanical engineering, mining, chemical and oil refining industries, metallurgy, construction, etc. [1-10].

The peculiarity of the study of the evolution of surface corrosion damage under the combined action of force loads and aggressive environment is that in the loaded structural element in corrosion conditions there are two interrelated kinetic processes: deformation and corrosion degradation. The increase in stress on the surface of the material causes the acceleration of corrosion degradation, which leads to a decrease in the geometric dimensions of the sections, the wall thickness of the shell, etc., and this, in turn, - to the redistribution (in the direction of increasing) stresses and strains in structural elements [5].

The development of effective algorithms for selecting the optimal material distribution of shell structures taking into account the impact of aggressive environments is complicated by the need to repeatedly test the durability requirements associated with compliance with strength constraints throughout the design life of the structure. The peculiarity of verifying the implementation of this restriction is the need to integrate the differential equations of the selected

mathematical model of corrosion damage, (the right part of which includes the parameters of the stress state of the structure interrelated with the values of corrosion damage) from the initial to the final time of loss of bearing capacity as a result of corrosion degradation of material structural [4, 8, 11-14].

The complexity of the application of classical models for solving the problem of finite-dimensional optimization in the initial time of existence of the structure is due to the presence of a significant number of variables, since the entire surface of the structure undergoes uneven corrosion, and with subject to restrictions, the parameters of which change over time, which leads not only to significant computational costs and accumulation of errors, but also to the need to solve a number of fundamental problems.

Known attempts to introduce simplifying assumptions to solve this problem in many cases lead to the inadequacy of the constructed design scheme of the real object, therefore, the results obtained in them are mainly of scientific and methodological importance, the possibility of practical use of which seems questionable.

It should be noted that experimental studies (which would be a reliable criterion for the reliability of the results) of the behavior of structures in aggressive environments are long in time and associated with significant material costs [2, 3].

As a consequence of this state of affairs, the long-term reliability of structures operated in aggressive environments often has to be increased by introducing a so-called "sacrificial" layer of material, i.e. part of material conventionally intended for corrosion damage. As a rule, such a layer is assigned to be uniform for the entire structure in terms of durability of the most loaded and (or) most corrosion-damaged element, as a certain margin of safety. Since stress and corrosion damage are distributed unevenly on the surface of the material, most of this sacrificial layer of material after the designated period of operation of the structure for corrosion wear remains "unused", which leads to overuse of the material as a whole. Thus, the problem of developing effective approaches to solving the problem of computer modeling of the evolution of corrosion damage and the optimization of structures that are operated under aggressive environments is still far from complete.

### **1. Algorithm of computer modeling of corrosion damage evolution**

In the general case, the mathematical model of corrosion damage, which takes into account the influence of the stress-strain state on the process of corrosion destruction, is taken in the form [5, 9, 15-17]:

$$\frac{d\delta}{dt} = f(\delta, \sigma, \bar{V}, \bar{X}, t), \quad \delta(\bar{X}, t_0) = 0, \quad (1)$$

where  $\sigma(\bar{X}, t)$  is stress and  $\delta(\bar{X}, t)$  is the depth of the corrosion damage at the point  $\bar{X}(x, y)$  on the surface of studied shell element;  $x, y$  are coordinates in the meridional and circumferential directions on the surface of the rotation shell;  $\bar{V}$  is vector of parameters that characterize the degree of corrosion resistance of the material and the level of aggressiveness of the environment;  $t_0 \leq t \leq t_{cr}$  is time.

The critical state of the structure at  $t_{cr}$  is determined by the achievement of a certain limit, destructive for this material stress value  $\sigma_{cr}$  as a result of reducing its stiffness characteristics due to corrosion damage.

The most well-known and tested models that take into account the effect of stresses on the corrosion rate in the calculations of strength are the models of V.M. Dolinsky [15], E.M. Gutman [9] and I.G. Ovchinnikov [5], which are, respectively, in the form of:

$$\begin{aligned} \text{(a)} \quad & \frac{d\delta}{dt} = V(t)(1 + K\sigma); \\ \text{(b)} \quad & \frac{d\delta}{dt} = \varphi(t)\exp(\gamma\sigma); \\ \text{(c)} \quad & \frac{d\delta}{dt} = \psi(t)(1 + \varepsilon E_n), \end{aligned} \quad (2)$$

where  $V(t)$ ,  $\varphi(t)$ ,  $\psi(t)$  are the functions, in the general case, which due to the complexity of their definition, in most cases [5] are accepted as reference experimentally established constants  $V_0$ , that characterizing the corrosion damage of a particular unstressed material in a certain aggressive environment;  $K$ ,  $\gamma$ ,  $\varepsilon$  are coefficients expressing the degree of influence of stresses on the corrosion rate;  $E_n = \sigma_x^2 + \sigma_y^2 - 2\mu\sigma_x\sigma_y + 2(1+\mu)\sigma_{xy}^2 / (2E)$  is specific deformation energy;  $\sigma_x, \sigma_y, \sigma_{xy}$  are relevant components of the stress state;  $E, \mu$  are modulus of elasticity and Poisson's ratio, respectively.

Thus, the task of determining the magnitude and direction of corrosion  $\delta(\bar{X}, t)$  and change the position of each point  $\bar{X}(x, y)$  of the surface deep into the material during  $\Delta t$  corrosion during the initial  $t = t_0$  to some final critical time  $t = t_{cr}$  of the structure is to integrate selected for this aggressive environment equation mathematical model in the form (2, a), (2, b) or (2, c).

Solution of the nonlinear initial problem (1) using one of the known methods, in particular, Kutt–Merson, Adams, Euler or others in the  $(k+1)$ -th step of integration over time can be represented as

$$\delta^{k+1} = \delta^k + f^k(\delta^k, \sigma^k, \bar{V}, \bar{X}^k, t^k) \cdot \Delta t, \quad k=1, 2, 3, \dots \quad (3)$$

When constructing an algorithm for computer simulation of the evolution of corrosion, it is assumed that its magnitude and direction are perpendicular to the surface deep into the material for each  $i$ -th node of the shell and does not depend on the magnitude of corrosion at neighboring points on the current  $k$ -th the steps of integrating the equation of the selected corrosion model (2).

Given that the surface of the shell in the process of corrosion becomes irregular, the meridian line on the surface of the shell material is approximated by a broken line drawn through evenly or unevenly defined nodal points, and the amount of corrosion  $\Delta\delta_i^k$  at these points is calculated by integrating a mathematical model of corrosion (3)  $k$ -th step in the form

$$\Delta\delta_i^{k+1}(x_i, y_i) = \delta_i^{k+1}(x_i, y_i) - \delta_i^k(x_i, y_i), \quad i = \overline{1, n}. \quad (4)$$

Given that each of the angular (nodal) points will belong simultaneously to two lines of the approximating line, changes in the coordinates of these nodal points can be calculated for geometric reasons (Fig. 1).

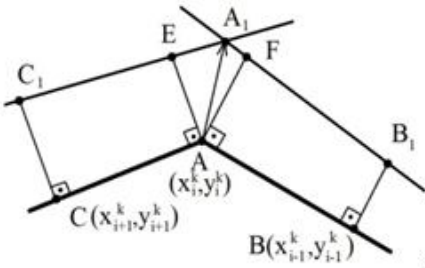


Fig. 1. Calculation scheme to calculate the value of and direction of corrosion damage

Segments  $BB_1 = \Delta\delta_{i-1}^k$ ,  $AE = AF = \Delta\delta_i^k$ ,  $CC_1 = \Delta\delta_{i+1}^k$  in Fig. 1 are correspond to the values of corrosion damage at the nodal points  $i-1, i, i+1$  at the  $k$ -th step of integrating equation (3). The final

value and direction of corrosion at the point  $(x_i^k, y_i^k)$  is determined by the vector  $\overline{AA_1}$ . The location of the point  $A_1$  is calculated as the coordinates of the point of intersection of the lines  $C_1E$  and  $B_1F$ , which are then taken as the angular point approximating the surface of the shell line at  $k+1$  integration step (2).

To calculate the values of the coordinates of the angular points of the approximating line of the surface meridian  $(k+1)$ -th step of integration of the selected corrosion model (2), determine the ort  $\overline{l_{BA}}$  of the vector  $\overline{BA}$  through the coordinates of these points in the  $k$ -th step (Fig. 1), as

$$\overline{l_{BA}} = \frac{\overline{BA}}{|BA|}; \quad \overline{l_{BA}} = \left\{ \frac{x_i^k - x_{i-1}^k}{|BA|}, \frac{y_i^k - y_{i-1}^k}{|BA|} \right\};$$

$$|BA| = \sqrt{(x_i^k - x_{i-1}^k)^2 + (y_i^k - y_{i-1}^k)^2} \quad (5)$$

and further, given that the vectors  $\overline{BB_1}$  and  $\overline{AF}$  are perpendicular to the vector  $\overline{BA}$ , ors  $\overline{l_{BB_1}}$ ,  $\overline{l_{AF}}$  of vectors  $\overline{BB_1}$ ,  $\overline{AF}$ , which are parallel, can be expressed as

$$\overline{l_{AF}} = \overline{l_{BB_1}} = \left\{ \frac{y_i^k - y_{i-1}^k}{|BA|}; -\frac{x_i^k - x_{i-1}^k}{|BA|} \right\}, \quad (6)$$

Vectors  $\overline{BB_1}$  and  $\overline{AF}$ , as directions of corrosion damage  $\Delta\delta_{i-1}^k$ ,  $\Delta\delta_i^k$  at the surface of the material in points  $B$  and  $A$ , can be represented as

$$\overline{AF} = \Delta\delta_i^k \cdot \overline{l_{AF}}; \quad \overline{BB_1} = \Delta\delta_{i-1}^k \cdot \overline{l_{BB_1}}. \quad (7)$$

After that the coordinates of points  $B_1$  and  $F$  can be obtained as follows:

$$x_{B_1} = x_{i-1}^k + \Delta\delta_{i-1}^k \cdot \frac{y_i^k - y_{i-1}^k}{|BA|}; \quad y_{B_1} = y_{i-1}^k - \Delta\delta_{i-1}^k \cdot \frac{x_i^k - x_{i-1}^k}{|BA|};$$

$$x_F = x_i^k + \Delta\delta_i^k \cdot \frac{y_i^k - y_{i-1}^k}{|BA|}; \quad y_F = y_i^k - \Delta\delta_i^k \cdot \frac{x_i^k - x_{i-1}^k}{|BA|}. \quad (8)$$

The ort  $\overline{l_{AC}}$  and length of the vector  $\overline{AC}$  are similarly determined

$$\overline{l_{AC}} = \left\{ \frac{x_{i+1}^k - x_i^k}{|AC|}, \frac{y_{i+1}^k - y_i^k}{|AC|} \right\}; \quad |AC| = \sqrt{(x_{i+1}^k - x_i^k)^2 + (y_{i+1}^k - y_i^k)^2}.$$

Then orts of vectors  $\overline{l_{AE}}, \overline{l_{CC_1}}$  and coordinates of points  $E, C_1$  are calculated:

$$\overline{l_{AE}} = \overline{l_{CC_1}} = \left\{ \frac{y_{i+1}^k - y_i^k}{|AC|}; -\frac{x_{i+1}^k - x_i^k}{|AC|} \right\}, \quad (9)$$

$$x_E = x_i^k + \Delta\delta_i^k \cdot \frac{y_{i+1}^k - y_i^k}{|AC|}; \quad y_E = y_i^k - \Delta\delta_i^k \cdot \frac{x_{i+1}^k - x_i^k}{|AC|};$$

$$x_{C_1} = x_{i+1}^k + \Delta\delta_{i+1}^k \cdot \frac{y_{i+1}^k - y_i^k}{|AC|}; \quad y_{C_1} = y_{i+1}^k - \Delta\delta_{i+1}^k \cdot \frac{x_{i+1}^k - x_i^k}{|AC|}. \quad (10)$$

The angular point  $A_1(x_{A_1}, y_{A_1})$  of the approximating meridian line of the next step of integrating the equation of the mathematical model of corrosion (1) is the point of intersection of the lines  $B_1F, EC_1$  and its coordinates satisfy the system of equations in the form

$$\begin{cases} x_{A_1}(y_F - y_{B_1}) - y_{A_1}(x_F - x_{B_1}) = x_{B_1}y_F - y_{B_1}x_F; \\ x_{A_1}(y_{C_1} - y_E) - y_{A_1}(x_{C_1} - x_E) = x_Ey_{C_1} - y_Ex_{C_1}. \end{cases} \quad (11)$$

Thus, the coordinates  $x_{A_1(i)}, y_{A_1(i)}$  of the  $i$ -th angular point following in the process of corrosion damage during  $\Delta t$  the configuration of the shell surface are calculated from (10), and the final value of corrosion damage to the surface of the material at angular point  $A$  is determined by the length of the vector  $\overline{AA_1}$

$$\Delta\delta_i^{*k} = |\overline{AA_1}| = \sqrt{(x_{A_1(i)}^2 - x_{A(i)}^2)^2 + (y_{A_1(i)}^2 - y_{A(i)}^2)^2}. \quad (12)$$

The author's software implementation of this problem provides various possible options for the mutual location of points  $C, A, B$ , different contour angles, lengths of segments  $CA$  and  $VA$ , parallel offset contours, reducing or increasing the number of nodes, respectively, for corrosion of convex and concave contours corresponding to a decrease or increase in the length of the approximating lines, etc.

## 2. Calculated models of shells of rotation with variables of external and internal components of wall thickness

Assuming that the corrosion rate depends on the intensity of stresses on the surface of the material, and the impact of aggressive environments on the shell structure is usually heterogeneous, the depth of corrosion damage to the inner and outer surfaces of the shell may be different and uneven. In this case, the middle surface of the shell over time will be increasingly different from the original. This leads to the need to develop a mathematical model for calculating the stress-strain state of the shells of rotation, which have different

laws of change of the outer and inner (relative to the middle surface) components of the wall thickness of the shells.

For the case of shell rotation under axisymmetric loading with variable along the meridian wall thickness  $d(s)$  under the Kirchhoff – Love hypotheses, it is assumed that the outer  $H(s)$  and inner  $h(s)$  components of the shell wall thickness as distances in the normal direction to some initial the reduction surface, which remains unchanged despite the corrosive degradation of the shell surfaces, are independent functions of the meridional coordinate  $s$ ,  $h^*(s)$  is the distance from the middle to such the reduce surface (Fig. 2(b)), so that

$$H(s) - h^*(s) = d(s) / 2; \quad h(s) + h^*(s) = d(s) / 2; \quad h^*(s) = (H(s) - h(s)) / 2. \quad (13)$$

It is also assumed that the middle surface and the reduction surface at the initial time of corrosion damage coincide, or are chosen parallel. In this case, the error associated with the subsequent, in the process of corrosion, the difference between the normals to these surfaces, as well as the parameters of the curvature  $\chi_1, \chi_2$  of these surfaces for thin-walled shells can be neglected.

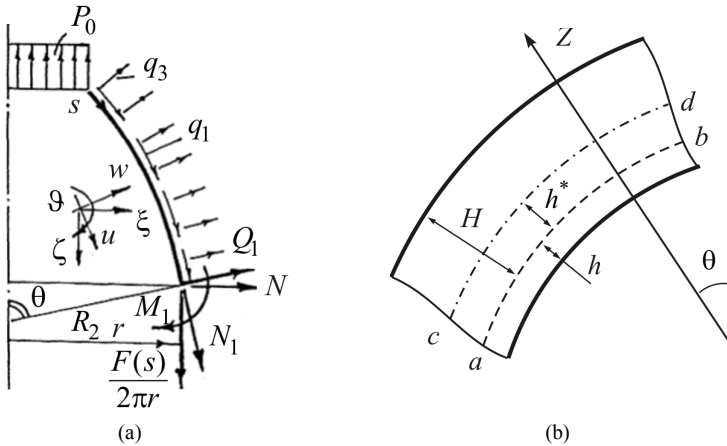


Fig. 2. Effort, displacement and position reduced (ab) and middle (cd) surfaces of the shell

The scheme of forces and movements in the shell is given in Fig. 2(a). Where do you come from

$$N_1 = \frac{F(s)}{2\pi r} \sin \theta + N \cos \theta; \quad Q_1 = -\frac{F(s)}{2\pi r} \cos \theta + N \sin \theta. \quad (14)$$

Here  $\theta(s)$  is the angle between the normal to the reduced surface and the axis of the shell;  $N_1, Q_1$  are longitudinal and transverse forces;  $N$  is expanding

force;  $P_0, q_n, q_1$  are specified axial, distributed normal and meridional loads,

$$F(s) = P_0 + \int_{s_0}^{s_p} (q_n \cos \theta - q_1 \sin \theta) 2\pi r ds \text{ is total axial load.}$$

Radial displacement  $\xi$ , angle of rotation of the normal  $\vartheta$ , axial displacement  $\zeta$ , as well as multiplied by the radius  $r(s)$  of the parallel circle, expanding force  $Nr$  and moment  $M_1r$  are the main variables.

The deformations  $\varepsilon_{1\text{mdl}}, \varepsilon_{2\text{mdl}}$  of the middle surface are related to the deformations  $\varepsilon_1, \varepsilon_2$  of the reduction surface and  $\chi_1, \chi_2$  as follows:

$\varepsilon_{1\text{mdl}} = \varepsilon_1 + h^* \chi_1$ ,  $\varepsilon_{2\text{mdl}} = \varepsilon_2 + h^* \chi_2$ . The relationship between stresses and deformations on the adduced surface in accordance with Hooke's law has the form

$$\begin{aligned} \sigma_1 &= \frac{E}{1-\mu^2} ((\varepsilon_1 + \mu\varepsilon_2) + z(\chi_1 + \mu\chi_2)); \\ \sigma_2 &= \frac{E}{1-\mu^2} ((\varepsilon_2 + \mu\varepsilon_1) + z(\chi_2 + \mu\chi_1)), \end{aligned} \quad (15)$$

The internal forces and moments relative to the adduced surface are determined by known dependencies

$$N_1 = \int_{-h}^H \sigma_1 dz; \quad N_2 = \int_{-h}^H \sigma_2 dz; \quad M_1 = \int_{-h}^H \sigma_1 z dz; \quad M_2 = \int_{-h}^H \sigma_2 z dz \quad (16)$$

and after substitution (15) will be as follows:

$$\begin{aligned} \text{(a)} \quad N_1 &= K_1 (\varepsilon_1 + \mu\varepsilon_2) + K_2 (\chi_1 + \mu\chi_2); \\ \text{(b)} \quad N_2 &= K_1 (\varepsilon_2 + \mu\varepsilon_1) + K_2 (\chi_2 + \mu\chi_1); \end{aligned} \quad (17)$$

$$\begin{aligned} \text{(a)} \quad M_1 &= K_2 (\varepsilon_1 + \mu\varepsilon_2) + D (\chi_1 + \mu\chi_2); \\ \text{(b)} \quad M_2 &= K_2 (\varepsilon_2 + \mu\varepsilon_1) + D (\chi_2 + \mu\chi_1). \end{aligned} \quad (18)$$

Here is the introduction of the notation:

$$K_1 = \frac{E(H+h)}{(1-\mu^2)}; \quad K_2 = \frac{E(H^2-h^2)}{2(1-\mu^2)}; \quad D = \frac{E}{1-\mu^2} \frac{H^3+h^3}{3}. \quad (19)$$

Excluding  $\varepsilon_1$  from (17) and (18) taking into account

$$\varepsilon_2 = \frac{\xi}{r}, \quad \chi_1 = \frac{d\vartheta}{ds}, \quad \chi_2 = \frac{\cos \theta}{r} \vartheta, \quad \frac{1}{R_1} = \frac{d\theta}{ds}, \quad \frac{1}{R_2} = \frac{\sin \theta}{r}, \quad (20)$$

we receive

$$N_2 = \mu N_1 + K_1 (1-\mu^2) \frac{\xi}{r} + K_2 (1-\mu^2) \frac{\cos \theta}{r} \vartheta, \quad (21)$$

$$M_2 = \mu M_1 + K_2 (1-\mu^2) \frac{\xi}{r} + D (1-\mu^2) \frac{\cos \theta}{r} \vartheta. \quad (22)$$

From equation (17, a) taking into account (14), (20) it follows

$$\varepsilon_1 = \frac{1}{K_1} \left( \frac{F(s)}{2\pi r} \sin \theta + N \cos \theta \right) - \mu \frac{\xi}{r} - \frac{K_2}{K_1} \left( \frac{d\vartheta}{ds} + \mu \frac{\cos \theta}{r} \vartheta \right). \quad (23)$$

Substituting further (14), (20), (23) in (16, a), we obtain

$$M_1 = \frac{K_2}{K_1} \left( \frac{\cos \theta}{r} N r + \frac{\sin \theta}{r} \frac{F(s)}{2\pi} \right) + \left( D - \frac{K_2^2}{K_1} \right) \left( \frac{d\vartheta}{ds} + \mu \frac{\cos \theta}{r} \vartheta \right),$$

from which follows one of the equations of the state system

$$\begin{aligned} \frac{d\vartheta}{ds} = -\mu \frac{\cos \theta}{r} \vartheta - \left( \frac{K_2}{DK_1 - K_2^2} \right) \frac{\cos \theta}{r} N r + \left( \frac{K_1}{DK_1 - K_2^2} \right) \frac{M_1 r}{r} - \\ - \left( \frac{K_2}{DK_1 - K_2^2} \right) \frac{\sin \theta}{r} \frac{F(s)}{2\pi}. \end{aligned} \quad (24)$$

Equilibrium equations of axisymmetric deformation of shells of rotation have the form [18]:

$$\begin{aligned} \text{(a)} \quad & \frac{1}{r} \frac{d}{ds} (Q_1 r) - \frac{N_1}{R_1} - \frac{N_2}{R_2} + q_n = 0; \\ \text{(b)} \quad & \frac{1}{r} \frac{d}{ds} (N_1 r) - N_2 \frac{\cos \theta}{r} + \frac{Q_1}{R_1} + q_1 = 0; \\ \text{(c)} \quad & \frac{1}{r} \frac{d}{ds} (M_1 r) - M_2 \frac{\cos \theta}{r} - Q_1 = 0. \end{aligned} \quad (25)$$

After substitution (17), (14), (20), (21) in equation (25, a), and substitution (14), (20), (21) in (25, b) we obtain the following system equations

$$\begin{aligned} \frac{d\xi}{ds} = -\mu \frac{\cos \theta}{r} \xi - \vartheta \sin \theta + \frac{D}{DK_1 - K_2^2} \frac{\cos^2 \theta}{r} N r + \frac{K_2}{DK_1 - K_2^2} \frac{\sin \theta}{r} M_1 r + \\ + \frac{D}{DK_1 - K_2^2} \frac{\sin \theta \cos \theta}{r} \frac{F(s)}{2\pi}, \end{aligned} \quad (26)$$

$$\begin{aligned} \frac{d(Nr)}{ds} = K_1(1-\mu^2) \frac{\xi}{r} + K_2(1-\mu^2) \frac{\cos \theta}{r} \vartheta + \mu \frac{\cos \theta}{r} (Nr) + \\ + \mu \frac{\sin \theta}{r} \frac{F(s)}{2\pi} - q_r r, \end{aligned} \quad (27)$$

where  $\frac{d}{ds} \left( \frac{F(s)}{2\pi} \right) = (q_n \cos \theta - q_1 \sin \theta) 2\pi r$ ;  $q_r = q_1 \cos \theta + q_n \sin \theta$ .

Using the expressions for  $Q_1$  (14) and  $M_2$  (22) in (25, c), we obtain

$$\begin{aligned} \frac{d(M_1 r)}{dr} = K_2(1-\mu^2) \frac{\cos \theta}{r} \xi + D(1-\mu^2) \frac{\cos^2 \theta}{r} \vartheta + \\ + \sin \theta (Nr) + \mu \frac{\cos \theta}{r} (M_1 r) - \cos \theta \frac{F(s)}{2\pi}. \end{aligned} \quad (28)$$



To determine the axial displacement  $\zeta$  of expression (23) for  $\varepsilon_1$  substitute in the equation of continuity of deformation  $\frac{d\zeta}{ds} = \varepsilon_1 \sin \theta + \vartheta \cos \theta$ , which takes the form

$$\frac{d\zeta}{ds} = -\mu \frac{\sin \theta}{r} \xi + \vartheta \cos \theta + \frac{D}{DK_1 - K_2^2} \frac{\sin \theta \cos \theta}{r} (Nr) - \frac{K_2}{DK_1 - K_2^2} \frac{\sin \theta}{r} (M_1 r) + \frac{D}{DK_1 - K_2^2} \frac{\sin^2 \theta}{r} \frac{F(s)}{2\pi}. \quad (29)$$

Thus, the obtained equations (26), (24), (27) - (29) form a system of differential equations with variable coefficients, which describe the stress-strain state of the shell of rotation with bilateral, relative to the reduction surface, variable along the meridian.

For a special case when the adduced surface coincides with the middle one, i.e.  $h^*(s) = 0$ , it follows that  $H(s) = h(s)$ . Then (19) will have the form  $K_1 = Ed / (1 - \mu^2)$ ,  $K_2 = 0$ ,  $D = Ed^3 / (12(1 - \mu^2))$ , where  $d(s) = H(s) + h(s)$  is the thickness of the shell. In this case, the system of obtained equations coincides with the known system given in [18].

It should be noted that equations (24), (26)-(29), which describe the stress-strain state of the axisymmetric shell of rotation with variables along the meridian components of wall thickness  $H(s)$ ,  $h(s)$  together with the boundary conditions of the contour, are linear over the main variables (phase vector components)  $\bar{u} = (\xi, \vartheta, Nr, M_1 r, \zeta)$  and nonlinear in terms of external  $H(s)$  and internal  $h(s)$  components of wall thickness and depth of corrosion damage to the material surface  $\delta(t, s)$ .

In this case, the right-hand sides of these equations have the form

$$\varphi_i = \sum_{j=1}^n a_{ij} (d(s, t), \delta(t, s)) u_j + b_i (d(s, t), s), \quad i = \overline{1, n}, \quad n=5, \quad (30)$$

and boundary conditions describing the conditions of fixing the contour or interaction with other substructures can be presented in the form of linear relations

$$\psi_{ej} = \sum_{i=1}^n a_{ji} u_i (s_e) + b_{je} = 0, \quad (31)$$

where  $s_e = s_0$  or  $s_e = s_L$ ;  $j = \overline{1, p_e}$ ;  $p_e$  - determines the number of boundary conditions ( $e = 0 \vee L$ ,  $p_0 + p_L = n$ ); coefficients (30) are known functions, and coefficients (31) are given constants.

To numerically solving the obtained boundary value problem for a system of ordinary differential equations with variable (due to changes in shell wall thickness) coefficients under given boundary conditions, a fairly effective and repeatedly tested in the problems of mechanics of thin-walled structures run method according to S.K. Godunov [19].

### 3. Algorithm of weight optimization of shells taking into account evolution of corrosion damage

The task of weight optimization of rotating shells taking into account the joint action of axisymmetric external load and corrosion degradation of the surface is to find the optimal variable along the meridian of wall thickness  $d^{\text{opt}}(s)$  from the condition of minimum material volume

$$V_{\text{opt}} = \min 2\pi \int_{s_0}^{s_L} r(s)d(s,t) ds, \quad (32)$$

when performing shell equations given in the form of a boundary value problem for a system of ordinary differential equations with variable coefficients

$$\frac{d\bar{u}}{dx} = \bar{A}(s, t)\bar{u} + \bar{B}(s), \quad s_0 \leq s \leq s_L \quad (33)$$

with boundary conditions for fixing the circuit (31) at the beginning  $s = s_0$  and end  $s = s_L$  of the integration interval:

$$(a) \bar{A}_0\bar{u} = \bar{B}_0; \quad (b) \bar{A}_N\bar{u} = \bar{B}_N \quad (34)$$

and compliance with strength limitations and design requirements for  $t_0 \leq t \leq t_{\text{cr}}$ :

$$(a) \sigma_{\text{equ}}(h(s,t), H(s,t), \bar{u}(s,t), \delta(t,s), s, t) \leq [\sigma];$$

$$(b) d_{\text{min}} \leq d(s) \leq d_{\text{max}}. \quad (35)$$

The dependence  $\sigma_{\text{equ}} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$  is accepted to calculate the equivalent stress on the shell surfaces, where  $\sigma_i = N_i / d \pm 12M_{iz} / d^3$  ( $i = 1, 2$ ),  $z = \pm d(s) / 2$  ( $i = 1, 2$ ).

Internal force factors  $N_1(s), N_2(s), M_2(s)$  are determined from the dependences (14), (21), (22);  $M_1(s), N(s)$  are the solution of the equations of the state of the shell (33), (34), and  $\delta(t, s)$  - is the solution from equation (1).

To solve the problem of optimizing the parameters of the rotation shells, taking into account the corrosive degradation of the surface, it is proposed to use an original approach. The essence of the proposed approach is to find some rational by a certain criterion (for example, the minimum weight of the material) project at the final moment  $t_{\text{cr}}$  depletion (as a result of corrosion of the material surface) bearing capacity of the structure and further increasing the "sacrificial" layer of material on the outer and inner surfaces of the shell in the opposite direction, starting from this final time  $t_{\text{cr}}$ , which corresponds to the durability time of the structure, to the initial  $t_0$ , using the equation of the accepted mathematical model of corrosion (2).

In this case, in contrast to the traditional approach, which consists in finding the optimal distribution of material at the initial time  $t_0$  and, as a consequence of the need to repeatedly check the strength constraints (at each step of the

optimization algorithm) to the final time  $t = t_{cr}$  while taking into account (1), the durability conditions are taken into account by only one (inverse) integration of the equations of the mathematical model of corrosion.

The application of this approach to solving problems of optimization of structures, taking into account the simultaneous action of force loads and the impact of the accumulation of corrosion damage to the surface, allows to significantly reduce the amount of calculations to obtain the optimal design.

Since the components of the shell wall thickness are functions of the coordinate  $s$  along the meridian, for weight optimization it is expedient to use the methods of the theory of optimal control in the form of the necessary conditions of optimality of the maximum principle L.S. Pontryagin [20].

The extended Hamiltonian and the system for conjugate functions with boundary conditions of transversality are presented in the form:

$$\begin{aligned} \text{(a)} \quad G^* &= G + \xi_1 \cdot (\sigma_{\text{equ}}|_{z=-d(s)/2} - [\sigma]) + \xi_2 \cdot (\sigma_{\text{equ}}|_{z=+d(s)/2} - [\sigma]); \\ \text{(b)} \quad \frac{d\lambda_i}{ds} &= -\frac{\partial G}{\partial u_i} - \xi_1(s) \frac{\partial \sigma_{\text{equ}}|_{z=-d(s)/2}}{\partial u_i} - \xi_2(s) \frac{\partial \sigma_{\text{equ}}|_{z=+d(s)/2}}{\partial u_i}, \quad i = \overline{1, n}; \\ \text{(c)} \quad \bar{\lambda}(s_e) &= \sum_{j=1}^{P_e} c_j \text{grad} \Psi_{ej}(\bar{u}(s_e)), \quad e = 0 \vee e = L, \end{aligned} \quad (36)$$

where  $G = -2\pi r(s)d(s, t_{cr}) + \sum_{i=1}^n \lambda_i(s) \varphi_i(\bar{u}(s), h(s), H(s), s)$ ,  $\lambda_i(s)$  are conjugate functions that satisfy the system of equations (36, b) with boundary conditions of transversality (36, c);  $\psi_{ej}$  are determined from (31);  $\xi_1(s), \xi_2(s)$  are Lagrange functions; and optimal control is found from the condition of the Hamiltonian maximum

$$G^*(\bar{u}^*(s), \bar{\lambda}^*(s), h^*(s), H^*(s), s) = \sup_{h, H} G^*(\bar{u}^*(s), \bar{\lambda}^*(s), h(s), H(s), s), \quad (37)$$

which, in principle, allows you to establish dependencies

$$h_{\text{opt}}(s) = h^*(\bar{u}^*(s), \bar{\lambda}^*(s), s); \quad H_{\text{opt}}(s) = H^*(\bar{u}^*(s), \bar{\lambda}^*(s), s). \quad (38)$$

The fulfillment of the necessary conditions of optimality (36), (37) is carried out by the method of successive approximations [21].

For the given initial values  $H^k(s), h^k(s)$  of the  $k$ -th step of approximations, the solution of equations (33), (34) and (36, b), (36, c) allows to determine  $\bar{u}^k(s), \bar{\lambda}^k(s)$  and then, after substituting them in (37), determine the optimal control (38)  $d_{\text{opt}}(s) = h_{\text{opt}}(s) + H_{\text{opt}}(s)$  for all  $s_0 \leq s \leq s_L$  nodes of integration of the system of equations (33), (34).

Finding the extremum (37) (for each of the nodes  $s^*$ ) is carried out by solving a sequence of problems of finite-dimensional (in this case two-

dimensional) optimization of variables  $H^k(s^*)$ ,  $h^k(s^*)$  using the method of generalized Lagrange multipliers to determine  $\xi_1(s)$ ,  $\xi_2(s)$ .

The thus obtained functions of the components of the shell wall thickness, which are optimal when  $t = t_{cr}$ , are further used as initial data for increasing the rational "sacrificial" layer of material by integrating in reverse the equations of the selected corrosion model (2).

The obtained optimal distribution of the shell material will have an irregular nature of changes in the outer and inner components of the shell wall thickness as a function  $d_{opt}(s)$  and therefore the practical implementation of such a project can be quite complex and technologically costly. At the same time, the distribution of material obtained by solving the correctly formulated problem of optimal design can be considered not only as a standard, comparison with which allows to assess the degree of perfection of the actual design, but also as a basis for realization technological conditions.

#### 4. Optimization of cylindrical tank parameters

The possibilities of the proposed approach are illustrated by the example of the problem of reducing the material consumption of a cylindrical tank for storage of petroleum products, taking into account the evolution of corrosion damage to the surface of the material and technological requirements. The tank, in addition to the load of hydrostatic pressure of the liquid, is under the influence of aggressive internal environment of petroleum products and external atmospheric corrosion.

The problem of finishing the project of continuously variable stiffness under the conditions of manufacturability also arises due to the fact that in practice large-capacity reservoir is welded from a set of short cylindrical shells of wall length  $l_j$  and thickness  $d_j$ ,  $j = \overline{1, k}$ . In this case its inner surface remains smooth, and the outer changes stepwise.

To solve this problem, we propose a method for the best approximation of continuous controls of a step-variable function by solving the auxiliary finite-dimensional problem of finding the minimum deviation function of shell material volumes with optimal continuous and rational step-variable wall thickness

$$\Delta V = \min_{hj, lj \in D_k} (V_{var} - V_c)^2. \quad (39)$$

Here for the rotation shells  $V_{var} = V_{opt}$  is determined from (32);  $V_c$  for the shell of step-variable wall thickness has the form  $V_c = 2\pi \sum_{j=1}^k l_j d_j r_j$ .

To determine the rational value of the wall thickness  $d_j$  of the  $j$ -th section of the shell is used analog to the principle of discrete equivalence [22], when for each of the sections the length  $l_j^1 \leq l_j \leq l_j^2$  of the wall thickness of the shell is determined in the form  $d_j = \max_{s_i \in [l_j, l_{j+1}]} d_{opt}(s_i)$ , where  $d_{opt}(s)$  is the result of

solving problem (32) taking into account corrosion damage (1) and dependences (35), (13), (38). Here  $l_j^1, l_j^2$  the lower and upper limits on the length of the sections of piecewise constant stiffness;  $\sum_{j=1}^k l_j = L = s_L - s_0$ ,  $d_i \geq d_0$ .

Numerical results are obtained for the case of computer simulation of corrosion degradation of the surface and the optimal design of the reservoir with such parameters:  $R=12\text{m}$ ;  $L=11.65\text{m}$ ;  $q_r = q(l-x)$ ;  $q = 0.741 \cdot 10^4 \text{ N/m}^3$ ;  $E=2 \cdot 10^{11} \text{ N/m}^2$ ;  $\mu=0.3$ ;  $[\sigma]=1.6 \cdot 10^8 \text{ N/m}^2$ .

Graphic images of the corrosion damage to the surface of the cylindrical tank of constant rigidity (the outer side of the wall is on the left, and the inner is on the right) are shown in Fig. 3: in Fig. 3(a) according to the model of corrosion damage V.M. Dolynsky for  $V(t)=V_0=0,15 \cdot 10^{-2}$ ;  $K=0,2 \cdot 10^{-5}$  and in Fig. 3(b) according to the model of E.M. Gutman for  $\varphi(t)=V_0=0,15 \cdot 10^{-2}$ ;  $\gamma=0,1 \cdot 10^{-6}$ , where the darkened shows the corrosion-damaged part of the material.

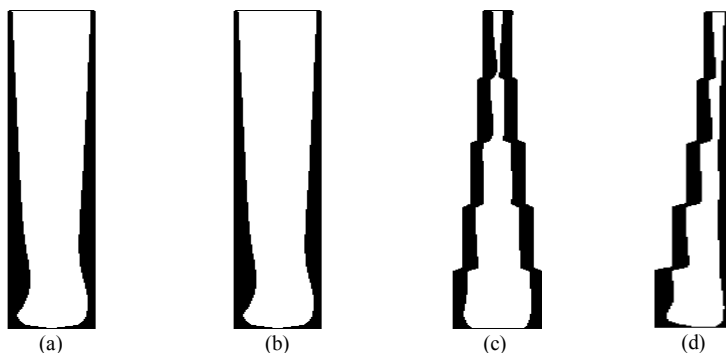


Fig. 3. Image of corrosion damage to the surface cylindrical tank

Pictures of corrosion damage to the surface of the wall of the cylindrical tank of a rational stepped bilateral configuration (for  $k=5$ ) according to model (2, a) is presented in Fig. 3(c). and one-sided change in wall thickness is presented in Fig. 3(d).

The optimal values  $d_j, l_j$  of the step change of the one-sided wall thickness (for  $k=8$ ) of the shell are given in table 1.

Table 1

Optimal parameters of step change the thickness of the tank wall

$j$	1	2	3	4	5	6	7	8
$d_i^{\text{opt}} \cdot 10^2, \text{ m}$	1.6	1.35	1.1	0.85	0.7	0.5	0.35	0.3
$l_i^{\text{opt}}, \text{ m}$	0.90	1.53	1.76	1.88	1.18	1.75	1.00	1.64

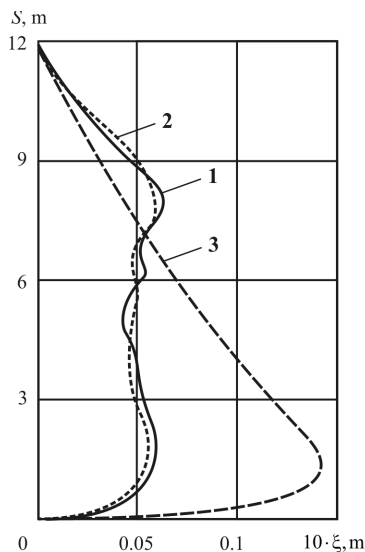


Fig. 4. Graphs of deflections of the lateral surface of the cylindrical reservoir

In Fig. 4 shows the graphs of the deflections of the walls of the reservoir made of the same amount of material: rational one-sided step-variable profile (line 1), optimal non-continuously variable stiffness (line 2) and constant thickness (line 3) for  $k=8$ .

Comparison of the stress-strain state of shells of the same weight with constant and optimal variable wall thickness shows that the selected step change in wall thickness reduces the maximum deflection by 2.1 times and the maximum value of stress intensity by 2.8 times.

Comparison of deflection functions (line 1 and line 2) for projects of optimal continuous and rational step rigidity indicates their insignificant ( $\leq 5\%$ ) difference, and the patterns of corrosion damage of rational step

projects (Fig. 3(c),(d)) in contrast to the corrosion damage of the tank of constant rigidity (Fig. 3(a),(b)) are more uniform, which at the same cost of material allows to achieve greater durability.

### Conclusions

Thus, in the given article the rather effective and enough general algorithm of computer modeling of corrosion damages of a surface of shell designs in the conditions of joint action of force loadings and the aggressive environment is developed. The main principles and stages of construction of the algorithm for the evolution of corrosion damage to the surfaces of shell elements in aggressive environments are presented, its approbation for the two most common mathematical models – the models of V.M. Dolinsky and E.M. Gutman. The proposed algorithm allows us to trace the evolution of corrosion damage to the shells of rotation from the initial moment of time to the moment of complete destruction due to corrosion degradation. Numerical results of corrosion degradation of the tank for storage of oil products are presented, which demonstrate the wide possibilities of the algorithm, its sufficient versatility and the possibility of generalization. An original algorithm for selecting the optimal parameters of the shells of rotation taking into account corrosion damage is built. The results of research can be used in calculations of durability and selection of optimal parameters of structural elements that are in conditions of simultaneous action of force load and aggressive environment.

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*Стаття надійшла 18.05.2022*

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### **ОПТИМАЛЬНЕ ПРОЕКТУВАННЯ ОБОЛОНКОВИХ КОНСТРУКЦІЙ З УРАХУВАННЯМ ЕВОЛЮЦІЇ КОРОЗІЙНОГО ПОШКОДЖЕННЯ**

Розроблено алгоритм комп'ютерного моделювання еволюції корозійного зношування матеріалу поверхні оболонкових елементів конструкцій в умовах спільної дії механічних зовнішніх навантажень та впливу агресивного середовища.

Побудована математична модель деформування оболонки обернення зі змінною в результаті корозії зовнішньою і внутрішньою складовими товщини стінки.

Запропонований алгоритм дозволяє прослідкувати в часі процес корозійної деградації матеріалу поверхні оболонки обернення у відповідності до довільних (існуючих) математичних моделей корозії з початкового моменту часу до повного руйнування внаслідок корозійної втрати матеріалу.

Розроблено оригінальний алгоритм вибору оптимальних параметрів оболонки обернення та дослідження їх довговічності з урахуванням корозійної деградації матеріалу поверхні, суть якого полягає в оптимізації параметрів конструкції в кінцевий момент часу її довговічності, подальшого зворотного двостороннього нарощування «жертвовного» шару матеріалу та раціонального доопрацювання проекту за технологічними вимогами.

Вирішення задачі вагової оптимізації оболонки за умов несиметричного двостороннього нерівномірного, в результаті корозійного ураження, матеріалу поверхні здійснюється з використанням необхідних умов оптимальності у формі принципу максимуму Л. С. Понтрягіна з фазовими обмеженнями. Проблема вдоволення технологічними вимогам в початковий момент часу формулюється як задача найкращої квадратичної апроксимації отриманих неперервних керувань у вигляді оптимально-змінної товщини стінки оболонки кусково-постійною функцією.

Приведені числові результати комп'ютерного моделювання корозійної деградації матеріалу поверхні та вибору оптимальних параметрів циліндричного резервуара для зберігання нафтопродуктів в умовах одночасної дії силового навантаження та впливу агресивного середовища.

**Ключові слова:** моделювання еволюції корозійного ураження, оболонки, міцність, агресивне середовище, оптимальне проектування.

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### **OPTIMAL DESIGN OF SHELL CONSTRUCTIONS TAKING INTO ACCOUNT THE EVOLUTION OF CORROSION DAMAGE**

An algorithm for computer modeling of the evolution of corrosion wear of the surface material of shell elements in the conditions of joint action of mechanical external loads and the impact of aggressive environments has been developed.



A mathematical model of deformation of shells of rotation with variable as a result of corrosion of external and internal components of wall thickness is constructed.

The proposed algorithm allows to trace in time the process of corrosion degradation of the surface material of the shells in accordance with arbitrary (existing) mathematical models of corrosion from the initial moment of time to complete destruction due to corrosion loss of material.

An original algorithm for selecting the optimal parameters of the rotation shells and the study of their durability taking into account the corrosion degradation of the surface material has been developed. The essence of the algorithm is to optimize the design parameters at the end of its durability, further reverse bilateral build-up of the "sacrificial" layer of material and rational refinement of the project according to technological requirements.

The solution of the problem of weight optimization of the shell under conditions of asymmetric bilateral non-uniformity to the surface material, as a result of corrosion damage, is carried out using the necessary conditions of optimality in the form of the principle of maximum L. S. Pontryagin with phase constraints. The problem of satisfying technological requirements at the initial moment of time is formulated as the problem of the best quadratic approximation of the obtained continuous controls in the form of optimally variable thickness of the shell wall with a piecewise constant function.

Numerical results of computer modeling of corrosion degradation of surface material and selection of optimal parameters of a cylindrical reservoir for storage of petroleum products under conditions of simultaneous action of force loading and influence of aggressive environment are given.

**Keywords:** modeling of corrosion damage evolution, shells, strength, aggressive environment, optimal design.

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### **ОПТИМАЛЬНОЕ ПРОЕКТИРОВАНИЕ ОБОЛОЧЕЧНЫХ КОНСТРУКЦИЙ С УЧЕТОМ ЭВОЛЮЦИИ КОРРОЗИОННОГО ПОВРЕЖДЕНИЯ**

Разработан алгоритм компьютерного моделирования эволюции коррозионного изнашивания материала поверхности оболочечных элементов конструкций в условиях совместного действия механических внешних нагрузок и агрессивной среды.

Построена математическая модель деформирования оболочек вращения с изменяющейся в результате коррозии наружной и внутренней составляющими толщины стенки.

Предлагаемый алгоритм позволяет проследить во времени процесс коррозионной деградации материала поверхности оболочек вращения в соответствии с произвольными (существующими) математическими моделями коррозии с начального момента времени до полного разрушения вследствие коррозионной потери материала.

Разработан оригинальный алгоритм выбора оптимальных параметров оболочек вращения и исследование их долговечности с учетом коррозионной деградации материала поверхности, суть которого заключается в оптимизации параметров конструкции в конечный момент времени ее долговечности, последующего обратного двустороннего наращивания «жертвенного» слоя материала и рациональной доработки проекта по технологическим требованиям.

Решение задачи весовой оптимизации оболочки принесимметричного двустороннего неравномерного, в результате коррозионного поражения, материала поверхности осуществляется с использованием необходимых условий оптимальности в форме принципа максимума Л.С. Понtryгина с фазовыми ограничениями. Проблема удовлетворения технологическим требованиям в начальный момент времени формулируется как задача наилучшей квадратичной аппроксимации полученных непрерывных управлений в виде оптимально-сменной толщины оболочки стенки кусочно-постоянной функцией.

Приведены числовые результаты компьютерного моделирования коррозионной деградации материала поверхности и выбор оптимальных параметров цилиндрического резервуара для хранения нефтепродуктов в условиях одновременного действия силовой нагрузки и воздействия агрессивной среды.

**Ключевые слова:** моделирование эволюции коррозионного поражения, оболочки, крепость, агрессивная среда, оптимальное проектирование.

УДК 539.3

*Дзюба А.П., Дзюба О.А., Левитіна Л.Д. Оптимальне проектування оболонкових конструкцій з урахуванням еволюції корозійного пошкодження // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, – 2022. – Вип. 108. – С. 17-34. – Англ.*

*Розроблено алгоритм комп'ютерного моделювання корозійного зношування поверхні матеріалу та оптимального проектування оболонкових конструкцій з урахуванням агресивного середовища.*

Табл. 1. Іл. 4. Бібліогр. 22 назв.

UDC 539.3

*Dzyuba A.P., Dzyuba A.A., Levitina L.D. Optimal design of shell constructions taking into account the evolution of corrosion damage // Streng of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA. – 2022. – Issue 108. – P. 17-34.*

*The algorithm for computer modeling of corrosion wear of the material surface and optimal design of shell structures taking into account the aggressive environment has been developed.*

Table 1. Fig. 4. Ref. 22.

УДК 539.3

*Дзюба А.Р., Дзюба А.А., Левитина Л.Д. Оптимальное проектирование оболочечных конструкций с учетом эволюции коррозионного повреждения // Сопротивление материалов и теория сооружений: науч.-техн. сборник. – К.: КНУСА, 2022. – Вип. 108. – С. 17-34. – Англ.*

*Разработан алгоритм компьютерного моделирования коррозионного износа поверхности материала и оптимального проектирования оболочечных конструкций с учетом агрессивной среды*

Табл. 1. Ил. 4. Библиогр. 22 назв.

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