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COMPARATIVE ANALYSIS OF NONLINEAR DEFORMATION AND BUCKLING OF THIN ELASTIC SHELLS OF STEP-VARIABLE THICKNESS

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A comparative analysis of finite element models and methods for solving complex problems of geometrically nonlinear deformation, buckling and post-buckling behavior of thin shells of stepwise variable thickness is carried out. An approach based on the use of the moment scheme of finite elements is considered. The features of using the software suite LIRA and integrated software system SCAD for solving the assigned problems are also provided. Thin and medium thickness shells are considered. They can have different geometric features in thickness and be under the action of static thermomechanical loads. A technique for solving these problems with the help of an efficient refined approach is presented. The technique is based on the general methodological positions of the three-dimensional theory of thermoelasticity and the use of the finite element moment scheme. With this approach, the approximation through the shell thickness is carried out by a single universal spatial finite element. The element can be modified in different portions of the shell with a step-variable thickness. It can be located eccentrically relative to the middle surface of the casing and can change its dimensions in the direction of the shell thickness. Such a unified approach made it possible to create a unified designed finite element model of a shell of an inhomogeneous geometric structure under the combined action of a thermomechanical load. A comparative analysis of the application of three finite element approaches for problems of geometrically nonlinear deformation and buckling of shells of stepwise variable thickness is carried out.

Key words: flexible shell, step-variable thickness, thin inhomogeneous shell, universal spatial finite element, finite element moment scheme, geometrically nonlinear deformation, buckling, post-buckling behavior, thermomechanical load.

Introduction

Shells as flexible thin-walled elements of increased strength are widely used in various engineering structures in many industries. Therefore, the problems of analyzing the behavior of thin elastic shells have a long history and currently continue to arouse great and constant interest. In recent decades, the number of works on the subject has increased significantly [1–20]. Much attention is paid to the study of elastic shells of stepwise-variable thickness, in particular, thin shells reinforced with ribs [1-6, 8-9, 12, 15, 16]. Much less research is devoted to the consideration of shells with different types of weakening [1-2, 4, 5, 12, 19, 20]. Shell structures are usually subjected to various operational loads, including thermomechanical ones. It should be noted that common algorithms for studying the nonlinear deformation and stability of shell structures with stepwise thickness are not sufficiently developed in the known software suites. Due to their complexity and possible ambiguity of the obtained results, obtaining solutions for this class of problems is difficult to implement in the form of a standard computational procedure.

The development of approaches to solving this problem took place in parallel with the progress in the field of computer technology. Modern methods for studying the strength, stability and vibrations of the responsible shell elements of structures under complex thermomechanical loads are presented, in particular, in [1-9].The most significant developments are the creation of a finite element method for studying the processes of nonlinear deformation, buckling, post-buckling behavior and oscillations of a wide class of thin and medium-thickness elastic shells of complex shape and structure under the complex mechanical and thermal loads. The method is based on the unified methodological positions of the 3D geometrically nonlinear theory of thermoelasticity and the use of the finite element moment scheme (FEMS).

1. Technique for solving geometrically nonlinear problems of deformation and buckling of inhomogeneous shells using the FEMS

A finite element method for studying geometrically nonlinear deformation, buckling, post-buckling behavior and vibrations of elastic shells of various shapes and structures under the static action of thermomechanical loads has been developed from the unified standpoint of the 3D geometrically nonlinear theory of thermoelasticity [1, 2]. A model of a linearly elastic continuous medium subject to the generalized Duhamel-Neumann law has been used. Large displacements but small deformations are assumed.

The finite element moment scheme developed and theoretically substantiated by A.S. Sakharov [21] has been applied. The FEMS is extended to problems of geometrically nonlinear deformation of thin shells of stepwise-variable thickness under the action of thermomechanical loads [1-2, 5-6]. Approximations of displacements and deformations within finite elements (FE) are coordinated according to the FEMS. This approach guarantees the correct consideration of rigid body motion, which increases the convergence and accuracy of the solutions obtained for sparse meshes. The temperature field in the volume of the shell is considered to be a known function of coordinates independent of the stress-strain state [1-2]. The temperature distribution over the thickness of the shell because of its thinness is assumed to be linear.

A thin shell is considered as a three-dimensional body and is modeled in thickness by one isoparametric spatial FE with multilinear shape functions. The FE is universal. It can be eccentrically displaced relative to the middle surface of the shell casing, can change its dimensions in the thickness direction, modeling ribs and cavities. The casing of a shell is understood as the body of the shell without stepped features. The universal FE has additional variable parameters. Varying the values of these additional FEs' parameters makes it possible to model a wide class of shells with geometric features along the thickness according to a unified methodology. Thanks to this approach, the volume of finite elements with ribs, cover plates, channels and cavities, located eccentrically with respect to the middle surface of the shell, is accurately modeled. The examples of modeling shells of stepwise variable thickness by the universal FE are schematically shown in Fig. 1 (for the shell with 'ribs') and Fig. 2 (for the shell with 'cavities').

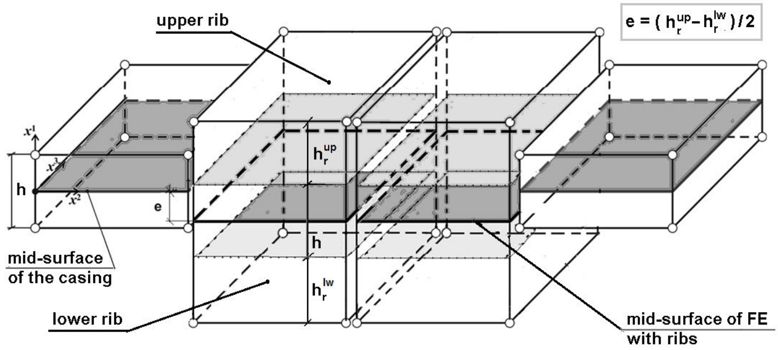


Fig. 1. Modeling a shell portion with 'ribs' by the universal FE

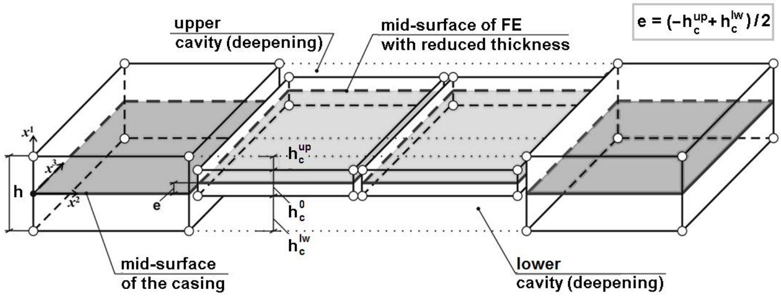


Fig. 2. Modeling a shell portion with 'cavities' by the universal FE

Features of the stress-strain state of a thin inhomogeneous shell are taken into account by using two non-classical hypotheses: (i) the static hypothesis which assumes that the compressive stresses in the fibers throughout the shell thickness are constant; and (ii) the kinematic hypothesis of deformed straight line. The use of the first hypothesis does not deprive the stress state of an inhomogeneous shell of its three-dimensional properties. The latter hypothesis makes it possible to perform the docking of spatial FEs in the process of deformation without violating the compatibility in terms of coordinates and displacements, as well as to simulate in a natural way sharp bends in the mid-surface, slopes of the walls of ribs, recesses and holes. In the thickness direction, the distribution of displacements is assumed to be linear as it is accepted in the theory of thin shells [22]. The nodal displacements on the bounding surfaces of the finite element shell model, defined in the global Cartesian coordinate system, are taken as unknown. To improve the convergence of solutions for thin shells, the set of displacements of nodal points on the middle surface of the FE and the difference of nodal displacements on its bounding surfaces are usually taken as resolving functions [1, 2]. The algorithm for constructing a system of resolving equations for a finite element model of a shell with a stepped thickness always requires transformation of the corresponding matrices of the modified FE relative to the reference surface [1, 2]. The mid-surface of the shell's casing is taken as a single reference surface.

The implementation of the nonlinear stability problem is performed by a step-by-step algorithm that combines the parameter continuation method and the Newton-Kantorovich iterative procedure. Automated procedures have been developed for solving problems of nonlinear deformation, buckling, and post-buckling behavior of inhomogeneous shells with determination of branching points $\bar{q}^*$, the upper $\bar{q}_{cr}^{up}$ and lower $\bar{q}_{cr}^{lw}$ critical loads [1, 2].

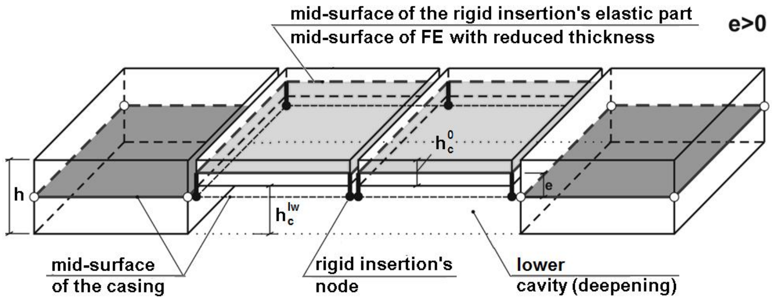
2. The analysis of geometrically nonlinear buckling problems for shells with stepped thickness using LIRA and SCAD software

Two types of flat shell FE are applied to determine the stress-strain state of thin shells and plates with the help of the software package (SP) LIRA [23, 24] and the integrated software system (ISS) SCAD [25, 26]: (i) the triangular three-node FE No.342 and (ii) the quadrangular four-node FE No. 344. Finite elements have a constant thickness. FE nodes located on its middle surface have 6 degrees of freedom: three displacements u_1, u_2, u_3 and three rotations $\alpha_1, \alpha_2, \alpha_3$ relative to local Cartesian axes x_i .

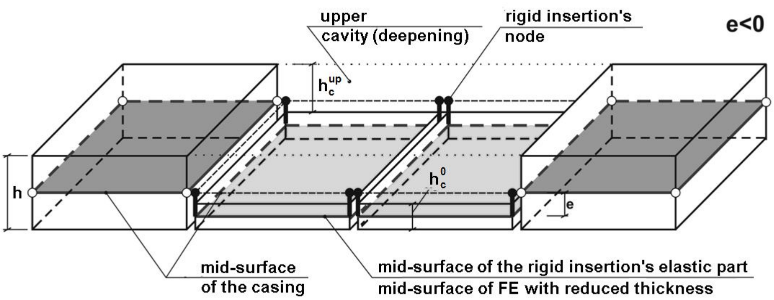
2.1. Modeling of shell fragments with step-variable thickness in SP LIRA and ISS SCAD. Special elements are used in SP LIRA and ISS SCAD, when modeling the geometric features of shells in the form of eccentrically located elements of stepwise-constant thickness (ribs, overlays, channels and cavities). Two types of such elements are used in SP LIRA. These are the so-called 'absolutely rigid insertions' (ARIs) and 'absolutely rigid bodies' (ARBs) [23, 24]. ISS SCAD uses 'absolutely rigid (solid) bodies' (ARBs) [25, 26]. In all cases, this is an artificial technique. It is used to approximate the step change in the thickness of the shell and consider the eccentric location of its elements. The purpose of introducing these special elements is to set the kinematic connection of the corresponding nodal displacements.

2.1.1. 'Absolutely rigid insertions' of SP LIRA are used for attaching special elements nodes to the main structural nodes located on its middle surface in areas of the stepwise-constant thickness. Modeling of the shift (eccentricity) of the 'elastic part' of the FE is carried out with the help of the ARI too.

The 'elastic part' of the rigid insertion is understood as the FE of the corresponding constant thickness, shifted relative to the middle surface of the structure. The nodes of the 'rigid insertion' are tied to the middle surface of the original shell using kinematic relations. The use of the ARIs in the calculation model is schematically shown in Fig. 3 on the example of a shell portion with 'lower' and 'upper' deepening (cavities). A similar technique is used when modeling ribs and cover plates.



(a)



(b)

Fig 3. Modeling of step-variable shell thickness by 'absolutely rigid insertions' in SP LIRA (a) in a portion with a 'lower cavity'; (b) in a portion with the 'upper cavity'

2.1.2. 'Absolutely rigid bodies' of SP LIRA and of ISS SCAD are special (conditional) FEs of high rigidity. They are additionally introduced into the calculation model to connect the nodes of the middle surfaces of the casing and the eccentric element.

In general, the ARB can only be conditionally attributed to the concept of a finite element, since it, in fact, does not have the classical attributes of a FE (basis functions, finite element area, etc.) [24]. However, from an implementation point of view, the ARB fits into a finite element procedure. The ARB is a rigid connection between nodes of eccentrically located elements when modeling displacements. This FE has no number in SP LIRA and it has No. 100 in ISS SCAD. The use of the ARBs in the calculation model is schematically shown in Fig. 4 on the example of a shell portion with 'lower' and 'upper' deepening (cavities). A similar technique is used when modeling ribs and cover plates.

2.2. Algorithms for solving geometrically nonlinear problems in SP LIRA and ISS SCAD

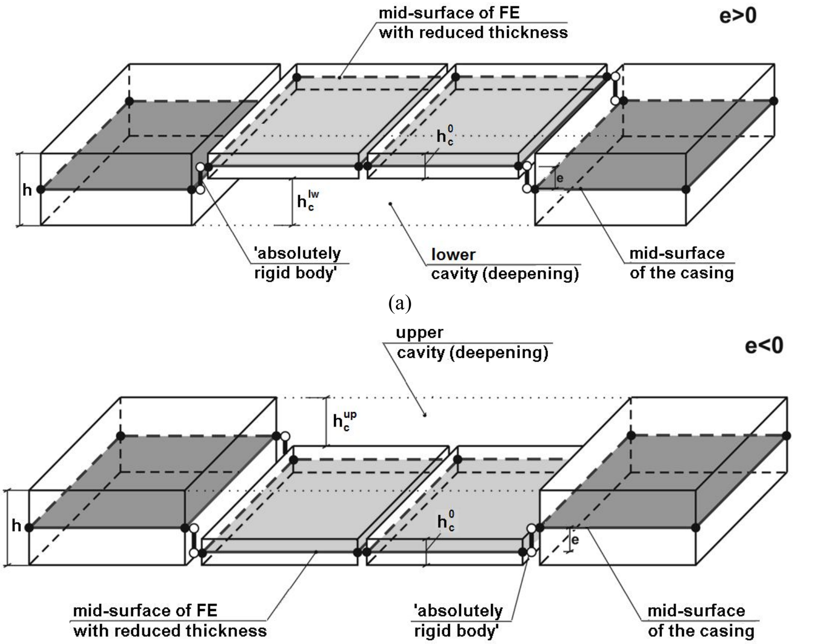
In both software suites three step algorithms are implemented.

2.2.1. SP LIRA implements the following algorithms for solving the geometrically nonlinear stability problem:

(i) sequential loading method ('simple stepping'). The number of steps and the value of the load step are set by a user. It has the designation in the figures as '1. SL';

(ii) sequential loading method (‘with automatic step selection’). The number of steps and its value are automatically selected by the algorithm (‘2. SLA’);

(iii) Newton-Raphson method (‘step by step with the search for new shapes of equilibrium’) implements the method of compensating loads. The buckling moment is fixed and the transition to a new stable branch of equilibrium is performed (‘3. N–R’).



(a)

(b)

Fig 4. Modeling of step-variable shell thickness by ‘absolutely rigid bodies’ in SP LIRA and ISS SCAD (a) in a portion with a ‘lower cavity’; (b) in the portion with the ‘upper cavity’

The first two algorithms use a simple modification of the sequential loading method, where the calculation is performed until the system stiffness matrix degenerates. The branch points and the upper critical load do not differ. The solution of the problem of nonlinear deformation is realized either up to the branching point ($\overline{q}\,*$) or up to the point of the upper critical load ($\overline{q}_{cr}^{up}$). The third algorithm implements the transition to a new stable branch. As studies have shown [4] it leads to a significant error.

2.2.2. The SCAD software also implements step-by-step algorithms for solving a geometrically nonlinear stability problem:

(i) sequential loading analysis (‘simple stepping’). The number of steps and the value of each step of the load are set by a user (‘1. SL’);

(ii) Newton-Kantorovich method (‘step-by-step with refinement’). The number of steps, the value of each step and the number of iterations are set (‘2. N–K’);

(iii) Newton-Raphson method ('step iterative'). The number of steps, the value of each step, the number of iterations are set ('3. N–R').

The ISS SCAD does not analyze the possible occurrence of a branch point.

3. Numerical examples

Documentation for SP LIRA and ISS SCAD has insufficiently complete descriptions of nonlinear algorithms. Therefore, the problems have been solved using each of the three programs.

3.1. A smooth spherical panel of constant thickness is considered. All non-linear calculation algorithms have been investigated in order to evaluate them and select the most suitable one. The obtained solutions have been compared with the solutions obtained using the FEMS and those presented in the article [12].

We consider the panel that is square in plan (in the $x^{2'}x^{3'}$ plane of the global Cartesian coordinate system), hinged along the contour and loaded with uniform normal pressure of intensity q. The research results are presented using dimensionless parameters $\overline{q}=a^4q/(Eh^4)$, $\overline{u}^{1'}=u^{1'}/h$. The curvature of the panel is determined by the parameter $K=2a^2/(Rh)=32$. The following values of the design parameters have been accepted: $a=60h$ – panel size in plan, $R=225h$ – radius, $h=1$ cm – thickness, $E=2.1\cdot10^6$ kg/cm^2, $\nu=0.3$. The design fragment is a quarter of the panel with a finite element mesh of 30×30 elements.

Fig. 5 and Table show the comparison between results obtained using the described finite element algorithms (FEMS, SP LIRA, ISS SCAD) and results presented in [12] that are considered benchmark. As can be seen in Fig. 5 (a), all results of the comparison give complete agreement of the curves '$\overline{q}$ - $\overline{u}^{1'}$' of the panel center in the sub-buckling domain.

Both variants of the sequential loading analysis (the '1. SL' and '1. SLA' methods of the SP LIRA) give a good match in terms of the value of the upper critical load $\overline{q}_{cr}^{up}$, where the solution of the problem ends (Fig. 5 (c), the point '*'). Newton-Raphson method ('3. H–P') implements the transition to a new stable equilibrium branch with a large error (Fig. 5 (a)). The sequential loading algorithm with automatic step selection ('2. PNA') leads to acceptable results.

All SCAD algorithms allow switching to a new stable branch of equilibrium. Sequential loading approach ('1. PS') performs the transition to the post buckling branch with a large error that occurs when calculating the value $\overline{q}_{cr}^{up}$ (Fig. 5 (d)). This problem is solved quite accurately by the algorithms using Newton-Kantorovich method ('2. N-K') and Newton-Raphson method ('3. H–R') (Fig. 5 (a), (d)).

The equilibrium shapes of the deformed panels in the subcritical and post buckling domains have a simple form and are in good agreement when using all the software suites. The Fig. 5 (b) shows the shapes of the middle surface of the shells in the vicinity of the upper critical load.

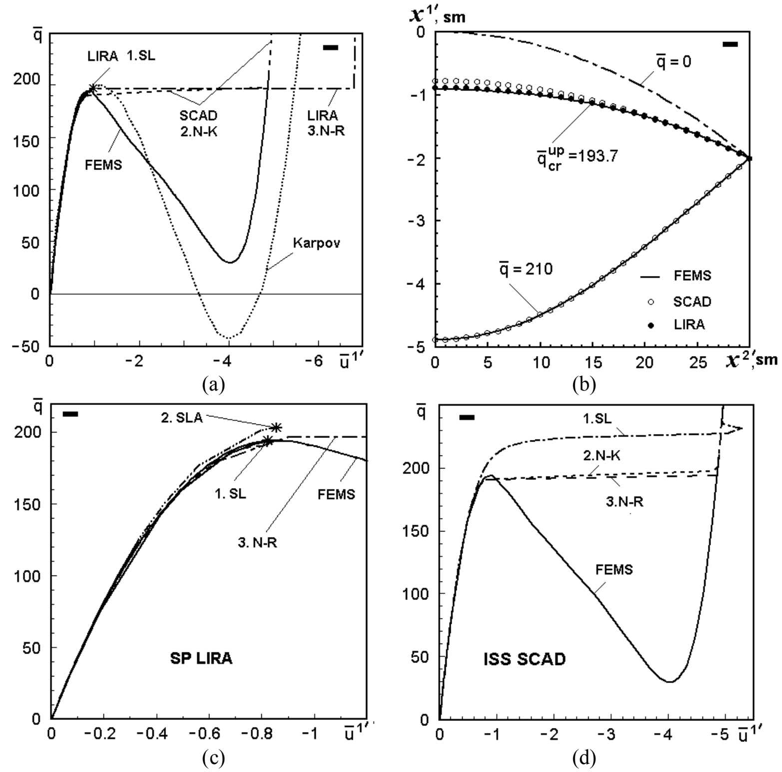


Fig. 5

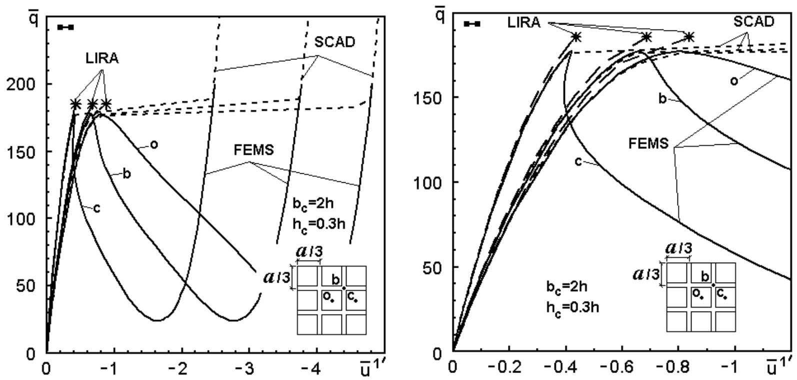
Table

Solution method (Algorithm)		$\overline{q}_{cr}^{up}$	Δ, %	$\overline{u}_{cr}^{1'up}$	Δ, %
Karpov V.V. [12]		200	0	1.1250	0
FEMS	N–K [1,2]	193.7	-3.15	0.9125	-18.89
	$\overline{q}*$	192.6		0.8888	
SP LIRA	1. SL ($\overline{q}*$)	194.1	-2.95	0.8796	-21.81
	2. SLA ($\overline{q}*$)	202.8	1.40	0.8580	-23.73
	3. N–R ($\overline{q}*$)	196.4	-1.80	0.9013	-19.88
ISS SCAD	1. SL	-	-	-	-
	2. N–K	190.2	-4.90	0.7729	-31.30
	3. N–R	190.2	-4.90	0.7730	-31.29

3.2. Analysis of the stability of shells with stepped-variable thickness is illustrated using the problem of deformation of a panel weakened by four criss-crossed channels [4]. The shell with the same parameters as in the previous problem is considered. The channels are located symmetrically on the inner and outer sides of the shell. Two types of channels are considered. Narrow channels have a width $b_c = 2h$ and a total depth $h_c = 0.3h$. Wide channels have such parameters: $b_c = 6h$, $h_c = 0.7h$. The design fragment is a quarter of the panel with a mesh of 30×30 finite elements.

To approximate channels, 'absolutely rigid insertions' are used in SP LIRA, and 'absolutely rigid bodies' in ISS SCAD. Method '2. SLA' in SP LIRA is used to solve the nonlinear stability problem. Newton-Kantorovich method '2. N-K' is used in ISS SCAD. Comparison of the results with the solution obtained using FEMS is carried out.

All curves 'load-displacement' obtained using FEMS, SP LIRA and ISS SCAD completely coincide in the subcritical region and in the zone of the upper critical load (Fig. 6, Fig. 7).



(a) (b)

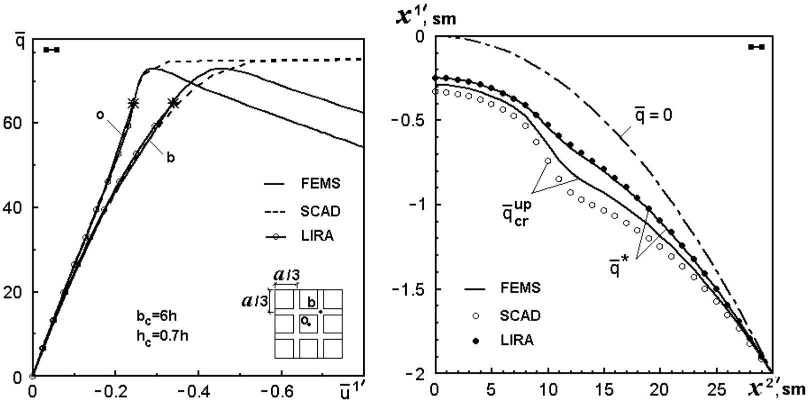
Fig. 6

Changing the width and depth of the channels significantly affects the results. For panels with narrow channels (see Fig. 6), the discrepancy between the value obtained by FEMS and SP LIRA is 4.45% while in the case of using ISS SCAD it is -0.73%. For panels with wide channels (see Fig. 7), the discrepancy is 2.43% when ISS SCAD is used. SP LIRA stopped calculations at the branch point $\bar{q}*$. The branch point $\bar{q}*$ has been also detected by FEMS. The discrepancy of the results at the branch point is -1.83%. In this case, the SP LIRA takes the branch point as the upper critical load. This load is -11.2% lesser than critical one. The difference between the deformation shapes is shown in Fig. 7*b* for the panel with wide channels.

Conclusions

Three methods for studying geometrically nonlinear deformation, buckling, and post-buckling behavior of thin elastic shells of complex shape and structure under the action of static loads are presented and analyzed. The methods are intended for the calculation of shell structures, which may have ribs, cover plates, channels, cavities, holes, breaks of the middle surface. An approach based on the use of the finite elements moment scheme is considered.

The features of using the software suites LIRA and SCAD for solving the assigned problems are provided too.



(a) (b)

Fig. 7

The performed investigations of nonlinear deformation and buckling of inhomogeneous shells confirm the reliability of the solutions obtained by the FEMS, software suites LIRA and SCAD.

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ПОРІВНЯЛЬНИЙ АНАЛІЗ НЕЛІНІЙНОГО ДЕФОРМУВАННЯ І СТІЙКОСТІ ТОНКИХ ПРУЖНИХ ОБОЛОНОК СТУПІНЧАСТО-ЗМІННОЇ ТОВЩИНИ

Проведено порівняльний аналіз скінчено-елементних моделей і методів розв'язання складних задач геометрично нелінійної деформації та втрати стійкості тонких оболонок ступінчасто-змінної товщини. Розглянуто підхід, що спирається на використанні моментної схеми скінченних елементів. Також наведено особливості використання програм ЛІРА та SCAD для вирішення поставлених задач. Розглядаються тонкі та середньої товщини оболонки. Вони можуть мати різні геометричні особливості за товщиною і перебувати під дією статичних термосилових навантажень. Наведено методику розв'язування цих проблем за допомогою ефективного уточненого підходу. Методика базується на загальних методологічних положеннях тривимірної теорії термопружності та використанні моментної схеми скінченних елементів. При такому підході апроксимація по товщині оболонки здійснюється одним універсальним просторовим скінченним елементом. Елемент може модифікуватися на різних ділянках оболонки зі ступінчасто-змінною товщиною. Він може розташовуватися ексцентрично відносно середньої поверхні обшивки і змінювати свої розміри в напрямку товщини оболонки. Такий уніфікований підхід дозволив створити єдину скінчено-елементну модель оболонки неоднорідної геометричної структури при спільній дії термосилового навантаження. Проведено порівняльний аналіз застосування трьох скінченних елементів для задач геометрично нелінійного деформування та втрати стійкості оболонок ступінчасто-змінної товщини.

Ключові слова: гнучка оболонка, ступінчасто-змінна товщина, тонка неоднорідна оболонка, універсальний просторовий скінченний елемент, момент на схема скінченних елементів, геометрично нелінійне деформування, стійкість, втрата стійкості, закритична поведінка, термосилове навантаження.

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Кривенко О.П., Ворона Ю.В. **Порівняльний аналіз нелінійного деформування та стійкості тонких пружних оболонок ступінчасто-змінної товщини** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2022. – Вип. 108. – С. 107-118. – Англ.
Представлено та проаналізовано три методи дослідження геометрично нелінійного деформування, втрати стійкості та закритичної поведінки тонких пружних оболонок складної форми та структури під дією статичних навантажень.
Табл. 1. Іл. 7. Бібліогр. 26 назв.

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Krivenko O.P., Vorona Yu.V. **Comparative Analysis of Nonlinear Deformation and Buckling of Thin Elastic Shells of Step-Variable Thickness** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2022. – Issue 108. – P. 107-118.
Three methods for studying geometrically nonlinear deformation, buckling, and post-buckling behavior of thin elastic shells of complex shape and structure under the action of static loads are presented and analyzed.
Tabl. 1. Fig. 7. Ref. 26.

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