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ENGINEERING METHOD OF CALCULATING LAMINATED TIMBER ELEMENTS REINFORCED WITH COMPOSITE TAPES

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Annotation. In today's world, construction requires environmentally friendly materials that cause minimal damage to the environment. At the same time, they must have great strength and be resistant to various types of external influences. Timber structures, which are made of renewable natural materials and have a relatively high strength, can rightfully be considered as such material. Despite the fact that timber itself has certain negative properties, namely the tendency to shrink and swell, rot, anisotropy of properties, which requires special attention in construction, in laminated timber structures (LTS) these disadvantages are already more manageable. Beams are one of the main laminated timber structures. Laminated timber beams reinforcement allows to significantly increasing their stiffness and strength.

The article proposes a methodology for calculating laminated timber elements of rectangular section reinforced with composite tapes. Refined formulas for determining the efficient geometric and mechanical characteristics are provided. The proposed method was compared with another analytical method of calculation and with the numerical method of modeling in the LIRA-CAD software complex using the finite element method. It was proved that this method of reinforcing laminated timber structures is promising and relevant.

Keywords: reinforcement, analytical method of calculation, laminated, finite element method, laminated timber structures, efficient geometric cross-section characteristics, efficient elasticity modulus.

Introduction. In today's world, construction requires environmentally friendly materials that cause minimal damage to the environment. At the same time, they must have great strength and be resistant to various types of external influences. Timber structures, which are made of renewable natural materials and have a relatively high strength, can rightfully be considered as such material.Despite the fact that timber itself has certain negative properties, namely the tendency to shrink and swell, rot, anisotropy of properties, which requires special attention in construction, in laminated timber structures (LTS) these disadvantages are already more manageable. Beams are one of the main laminated timber structures. Laminated timber beams reinforcement allows to significantly increase their stiffness and strength.

For a detailed analysis of the stress-strain state (STS) of laminated timber elements of rectangular section reinforced with composite tapes, taking into account the peculiarities of their work and structure, a calculation method by analogy with [1] is proposed, which consists in applying the efficient geometric characteristics of a rectangular section to the usual calculation formulas [2]. For the second group of limit states calculation of laminated timber elements of rectangular section, it is proposed to use the efficient elasticity modulus for the boards of the outer layer, in which the maximum stresses will be observed.

For the analysis of this method, a number of numerical studies were carried out on laminated timber beams reinforced with composite tape and ordinary laminated timber beams of the same strength class using analytical methods of calculation according to DBN [2], similar beams using the V.Shchuko`s engineering method of calculating reinforced timber structures [3], laminated timber beams according to the analytical method of calculation according to SNiP [4] and using the finite element method (FEM) in the LIRA-CAD program complex (PC) using volumetric and flat finite elements (FE).

Laminated timber beams of the same strength class at different spans and different evenly distributed loads calculations using the specified methods were compared with calculations of similar beams reinforced with composite tapes according to the specified methods, and showed that reinforcing the LTS of the rectangular section significantly increases the load-bearing capacity and stiffness of the elements according to all comparative methods.

It has been confirmed that the proposed method of laminated timber beams reinforced with composite tapes analytical calculation is appropriate for use in the calculation of both individual elements and complex systems made of them. According to this method, it is possible to take into account the thickness and mechanical characteristics of the boards` timber from which the cross-section of the element is made and reinforcement, which significantly increases the range of use of laminated timber cross-sections reinforced with composite tapes, even in the case of a boards of different cross-sectional strength classes combination.

Literature review. There is very little domestic experience in designing the LTS of a rectangular section reinforced with composite tapes. The studies, the results of which are given in the article [5] by the authors Gomon S. and Polishchuk M., in which they experimentally consider the technology of manufacturing laminated timber beams reinforced with rod reinforcement and composite tapes, are one of the few. Also, certain results of experimental researches are described in the article [6] by the authors Bashynskyi O., Bondarchuk T., Peleshko M.V., which presents three methods of reinforcing timber beams with tape reinforcement, which made it possible to almost double their bearing capacity.

Let us note that in the latest editions of the regulatory documents of the European Union (Eurocode-5 or EN 1995-1-1:2008 [7]), Ukraine (DBN V.2.6-161:2017 [2]) there are no methodological recommendations for designing and calculating LTS reinforced with composite tapes.

The conducted analysis of the presented research confirms the urgent need for the development of an engineering method for calculating laminated timber structures of rectangular cross-section reinforced with composite tapes and the use of PC LIRA-CAD for their calculation according to an engineering method of calculating laminated timber elements reinforced with composite reinforcement [1].

The purpose and tasks of research. The purpose of this work is to present the engineering methodology for calculating laminated timber structures reinforced with composite tapes.

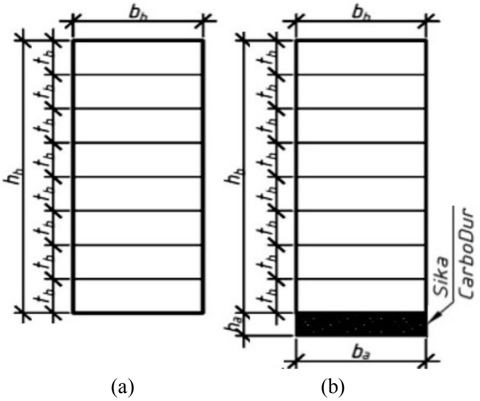
Laminated timber beams with a total cross-section $h_b \times b_b = 24 \times 10\,\text{cm}$ from 8 boards layers of strength class C35 and similar beams reinforced in the stretched zone with composite tapes of carbon fibers Sika CarboDur S 1012 with a cross-section $h_a \times b_a = 0.12 \times 10\,\text{cm}$ were chosen as the object of research. The researched beams rest on two hinged supports, loaded with a uniformly distributed load of intensity: 2 kN/m, 4 kN/m, 8 kN/m and have spans: 4 m, 6 m and 8 m.

To research these structures, their calculation was carried out using several methods:

1. Numerical studies of the studied objects modeling in PC LIRA-CAD.
2. Analytical laminated timber beams calculation according to the calculation method given in SNiP II-25-80 and similar beams reinforced with composite tapes [4].
3. Analytical calculation of laminated timber beams according to the calculation method given in DBN B.2.6-161:2017 and similar beams reinforced with composite tapes according to the proposed method.
4. Analysis of the results of laminated timber beams reinforced with composite tapes calculation methods.

Numerical studies of modeling of the researched objects in PC LIRA-CAD. In the LIRA-CAD software complex, laminated timber beams with a total cross-section $h_b \times b_b = 24 \times 10\,\text{cm}$ of 8 boards layers of strength class C35 (Fig. 1 (a)) with the following mechanical properties were modeled with three-dimensional FE No.36: elasticity modulus of timber along the fibers $E_1 = E_{0,\text{mean}} = 13000$ MPa, the elasticity modulus of timber across the fibers $E_2 = E_3 = E_{90,\text{mean}} = 430$ MPa, the shear modulus $G = G_{\text{mean}} = 810$MPa and similar beams reinforced with Sika CarboDur S 1012 composite tapes (Fig. 1 (b)), which are modeled by flat FE No.44, with mechanical characteristics: elasticity modulus along the fibers $E_1 = E_{0,\text{mean}} = 170\,000$ MPa.

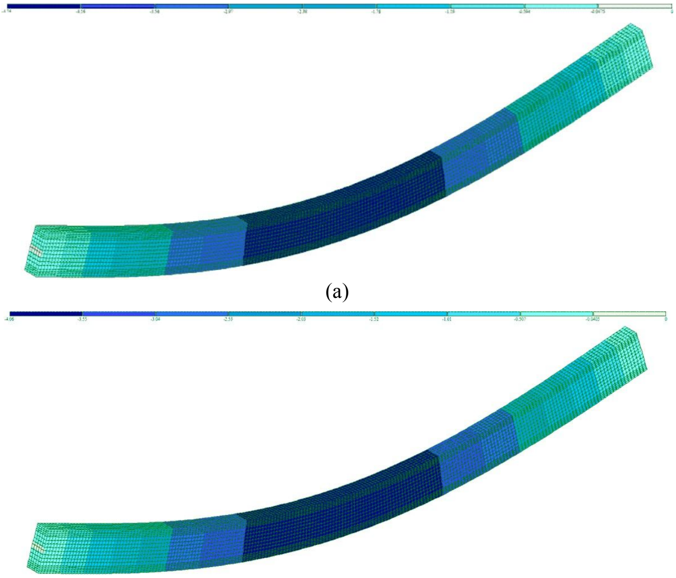
After the calculation, we obtain the isopolies of vertical deformations and normal stresses along the timber fibers based on the numerical studies results of FEM for laminated timber beams with and without composite tape reinforcement with spans: 4 m, with a uniformly



(a) (b)

Fig. 1. Geometric diagram of thebeam cross-section without reinforcement (a) and reinforced with a composite tape (b)

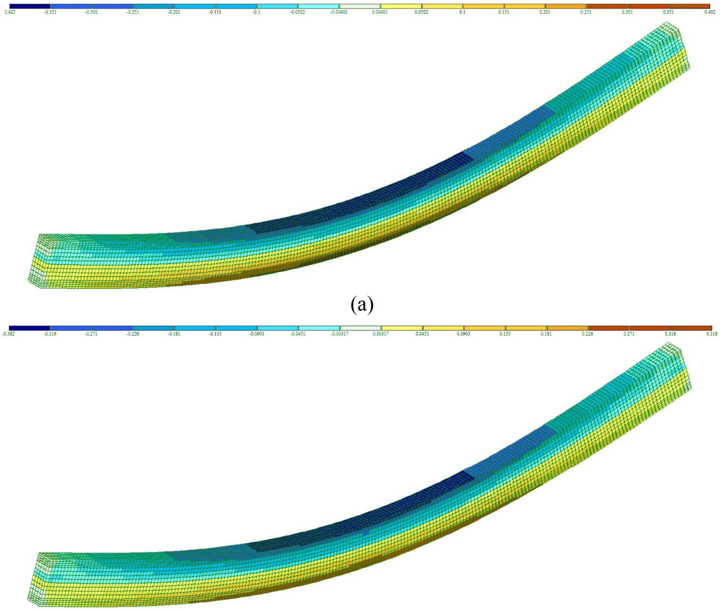
distributed load of 2 kN/m is presented in Fig. 2 – 3.



(a)

(b)

Fig. 2. Isopolies of vertical deformations of laminated timber beams without reinforcement (a) and reinforced beams (b) with a span of 4 m under a uniformly distributed load of 2 kN/m



(a)

(b)

Fig. 3. Isopolies of normal stresses in laminated timber beams without reinforcement (a) and in reinforced beams (b) with a span of 4 m under a uniformly distributed load of 2 kN/m

Full results of FEM numerical studies for laminated timber beams with and without composite tape reinforcement for investigated spans of 4 m, 6 m, 8 m and all variants of uniformly distributed load 2 kN/m, 4 kN/m, 8 kN/m presented in tables 1.

Table 1

Values of deflections and maximum normal stresses for laminated timber beams of the same strength class with and without reinforcement under a uniformly distributed load

Span, m		4		6		8	
Calculation methods	EI (W_x), kNcm^2 (cm^3)	w, mm	$\sigma_{m,d}$, kN/cm^2	w, mm	$\sigma_{m,d}$, kN/cm^2	w, mm	$\sigma_{m,d}$, kN/cm^2
uniformly distributed load - 2.0 kN/m							
FEM with volumetric FE No. 36	without reinforcement	4,76	0,402	25,1	0,901	72,3	1,60
FEM with volumetric FE No. 36 and flat FE No. 44	reinforced with composite tape	4,06	0,318	21	0,701	61,2	1,27
Percentage difference		17%	26%	20%	29%	18%	26%
uniformly distributed load - 4.0 kN/m							
FEM with volumetric FE No. 36	without reinforcement	9,51	0,804	50,1	1,8	145,0	3,2
FEM with volumetric FE No. 36 and flat FE No. 44	reinforced with composite tape	8,11	0,635	42,0	1,4	122,0	2,54
Percentage difference		17%	26%	20%	29%	18%	26%
uniformly distributed load - 8.0 kN/m							
FEM with volumetric FE No. 36	without reinforcement	19	1,61	100,0	3,6	289,0	6,4
FEM with volumetric FE No. 36 and flat FE No. 44	reinforced with composite tape	16,2	1,27	84	2,8	245,0	5,07
Percentage difference		17%	17%	27%	19%	29%	18%

For a more illustrative example, comparative diagrams of the maximum deflections w (mm) and the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the investigated beams under a uniform load of 2 kN/m, 4 kN/m, 8 kN/m were created, when calculating by the method of finite elements (Fig. 4 – 9.)

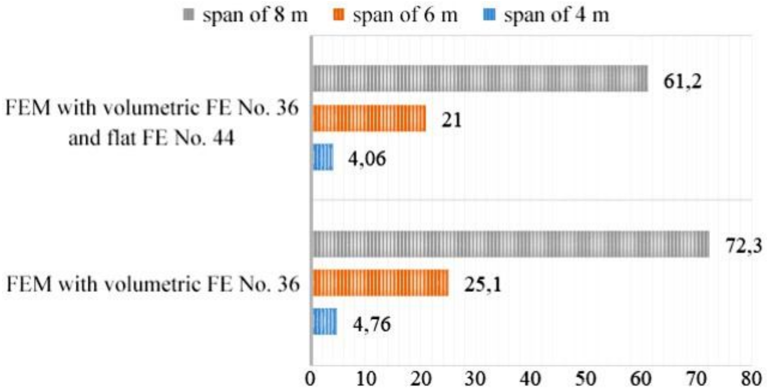


Fig. 4. Diagram of the maximum deflections w (mm) of the researched beams under a uniform load of 2 kN/m when calculated by the finite element method

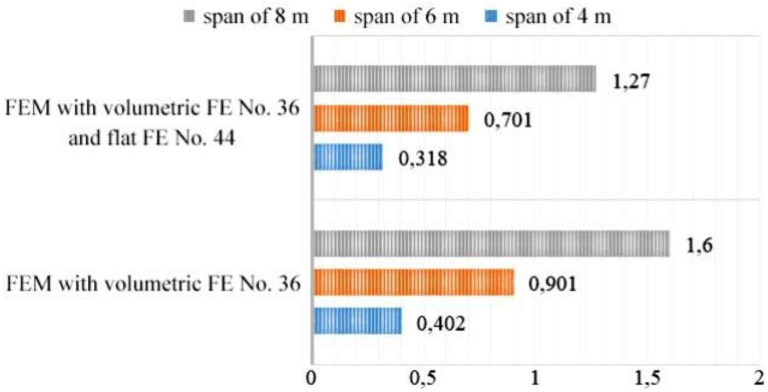


Fig. 5. Diagram of the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the researched beams under a uniform load of 2 kN/m when calculated by the finite element method

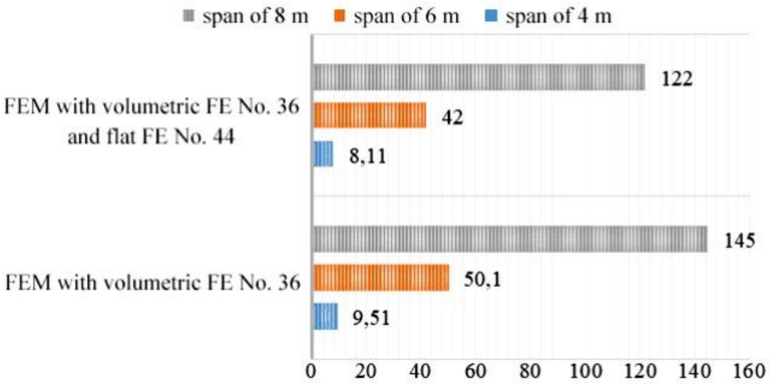


Fig. 6. Diagram of the maximum deflections w (mm) of the researched beams under a uniform load of 4 kN/m when calculated by the finite element method

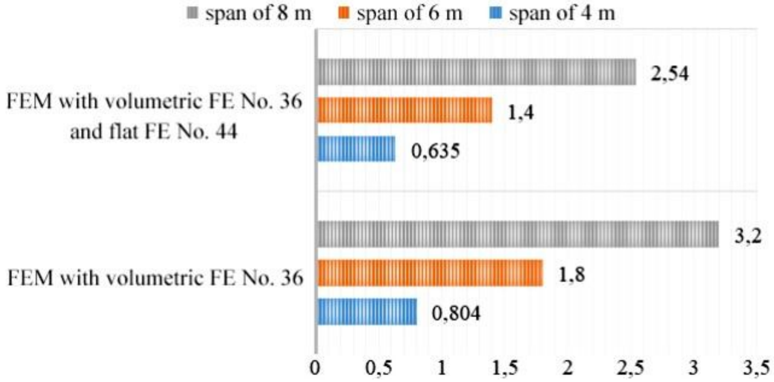


Fig. 7. Diagram of the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the researched beams under a uniform load of 4 kN/m when calculated by the finite element method

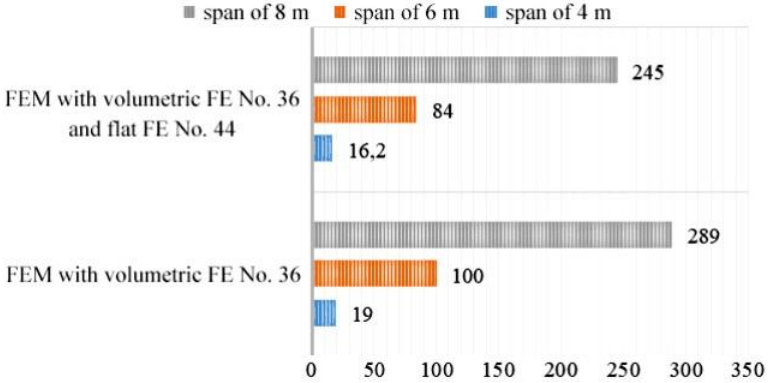


Fig. 8. Diagram of the maximum deflections w (mm) of the researched beams under a uniform load of 8 kN/m when calculated by the finite element method

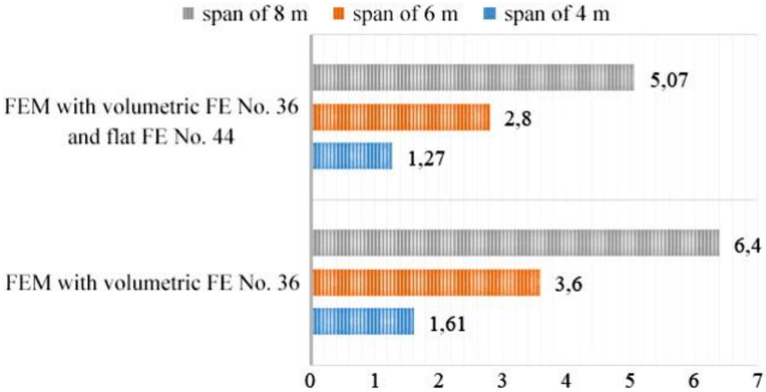


Fig. 9. Diagram of the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the researched beams under a uniform load of 8 kN/m when calculated by the finite element method

From the conducted numerical studies, the results of which are given in the table 1 and fig. 4 – 9 it is clear that according to the calculation by the finite element method in PC LIRA-CAD for laminated timber beams of the same strength class with reinforcement with composite tapes under a uniformly distributed load of 2.0 kN/m, the bearing capacity increases by more than 25% than for similar beams without reinforcement with composite tapes. And the movement of such beams when reinforced with tapes is reduced by more than 15%. For reinforced beams with a uniformly distributed load of 4.0 kN/m, the load-bearing capacity increases by more than 25% compared to similar beams without composite tape reinforcement. And the movement of such beams when reinforced with tapes is reduced by almost 20%. For reinforced beams with a uniformly distributed load of 8.0 kN/m, the load-bearing capacity also increases from 26 to 29%, compared to similar beams without composite tape reinforcement. And the movement of such beams when reinforced with tapes decreases by 17-19%.

Analytical calculation of laminated timber beams according to the calculation method given in SNiP II-25-80 and similar beams reinforced with composite tapes. Calculation of laminated timber elements of a rectangular cross-section strength according to the maximum normal stresses was carried out as for elements working in bending according to the formulas of SNiP II-25-80 [4]:

$$\frac{M}{W_{розр.}} \le R_u \tag{1}$$

resistance; R_u is the calculated bending resistance.

The calculation of laminated timber elements of rectangular cross-section according to the second group of limit states is performed according to the formulas of SNiP II-25-80 [4]:

$$w = \frac{f_0}{k}\left[1 + C\left(h/l\right)^2\right] \tag{2}$$

deformations; h – cross-section height; l – beam span; k – coefficient that takes into account the effect of a change in the height of the section, $k = 1$ for beams with a constant height of the section; C is a coefficient that takes into account shear deformations due to transverse force.

According to Shchuko's engineering calculation method [3], the calculation of reinforced laminated timber elements of a rectangular cross-section should be performed with the reduction of the geometric characteristics of the cross-section, taking into account the reinforcement. According to this method, the moment of inertia of reinforced structures operating in bending is determined according to the dependence:

$$I_{пр} = (b \cdot h^3)/12 + n \cdot F_a \cdot (h_0/2)^2 , \tag{3}$$

where $I_{пр}$ – efficient moment of inertia; b – cross-section width; h – cross-section height; n – the ratio of the elasticity modulus of reinforcement and timber; F_a – the cross-sectional area of the armature; h_0 – the distance from the armature gravity center to the beam gravity center.

Then the moment of resistance is determined:

$$W_{пр} = I_{пр} \big/ h_0 \,. \qquad (4)$$

Full calculation results for laminated timber beams with and without reinforcement with composite tapes for spans of 4 m, 6 m, 8 m that were researched and all variants of uniformly distributed load of 2 kN/m, 4 kN/m, 8 kN/m are presented in the table 2.

Table 2

Values of deflections and maximum normal stresses for laminated timber beams of the same strength class with and without reinforcement under a uniformly distributed load

Span, m		4		6		8	
Calculation methods	EI (W_x), kNcm^2 (cm^3)	w, mm	$\sigma_{m,d}$, kN/cm^2	w, mm	$\sigma_{m,d}$, kN/cm^2	w, mm	$\sigma_{m,d}$, kN/cm^2
uniformly distributed load - 2.0 kN/m							
Classical calculation method according to SNiP II-25-80	11520x10^3 (960,0) without reinforcement	6,2	0,42	30,2	0,94	94,2	1,67
Shchuko's engineering method	13803x103 (1144,5) reinforced with composite tape	5,32	0,35	26,93	0,79	85,13	1,39
Percentage difference		14%	17%	11%	16%	10%	17%
uniformly distributed load - 4.0 kN/m							
Classical calculation method according to SNiP II-25-80	11520x10^3 (960,0) without reinforcement	12,4	0,83	60,4	1,875	188,4	3,33
Shchuko's engineering method	13803x10^3 (1144,5) reinforced with composite tape	10,6	0,69	53,9	1,57	170,3	2,79
Percentage difference		15%	17%	11%	16%	10%	16%
uniformly distributed load - 8.0 kN/m							
Classical calculation method according to SNiP II-25-80	11520x10^3 (960,0) without reinforcement	24,7	1,67	120,8	3,75	376,8	6,67
Shchuko's engineering method	13803x10^3 (1144,5) reinforced with composite tape	21,3	1,39	107,8	3,15	340,5	5,59
Percentage difference		14%	17%	11%	16%	10%	16%

For a more illustrative example, comparative diagrams of the maximum deflections w (mm) and the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the investigated beams under a uniform load of 2 kN/m, 4 kN/m, 8 kN/m were created, when calculating by classical method of calculation according to SNiP II-25-80 and Shchuko's engineering method (Fig. 10 – 15).

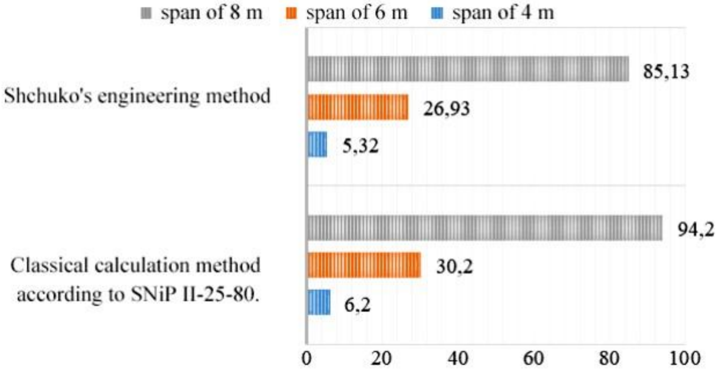


Fig. 10. Diagram of the maximum deflections *w* (mm) of the researchedbeams under a uniform load of 2 kN/m when calculating using the classic calculation method according to SNiP II-25-80 and Shchuko's engineering method

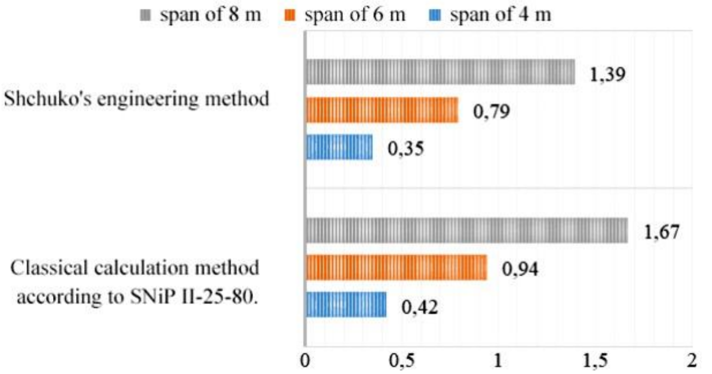


Fig. 11. Diagram of the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the researched beams under a uniform load of 2 kN/m when calculating using the classic calculation method according to SNiP II-25-80 and Shchuko's engineering method

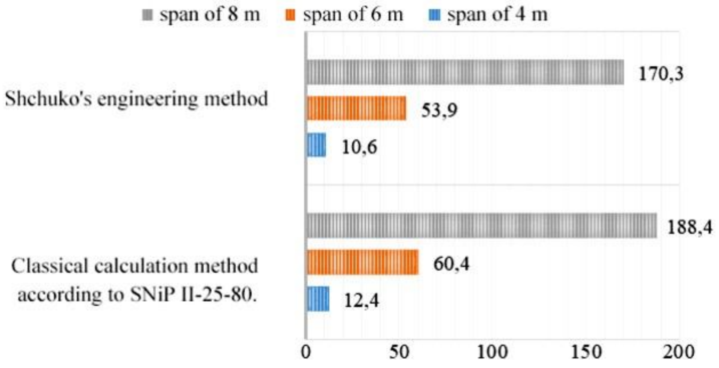


Fig. 12. Diagram of the maximum deflections *w* (mm) of the researched beams under a uniform load of 4 kN/m when calculating using the classic calculation method according to SNiP II-25-80 and Shchuko's engineering method

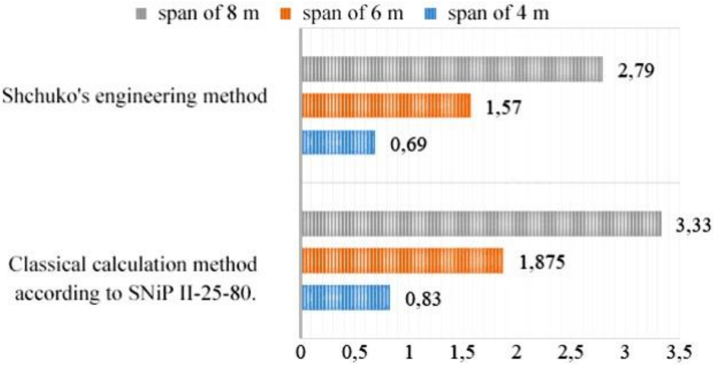


Fig. 13. Diagram of the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the researched beams under a uniform load of 4 kN/m when calculating using the classic calculation method according to SNiP II-25-80 and Shchuko's engineering method

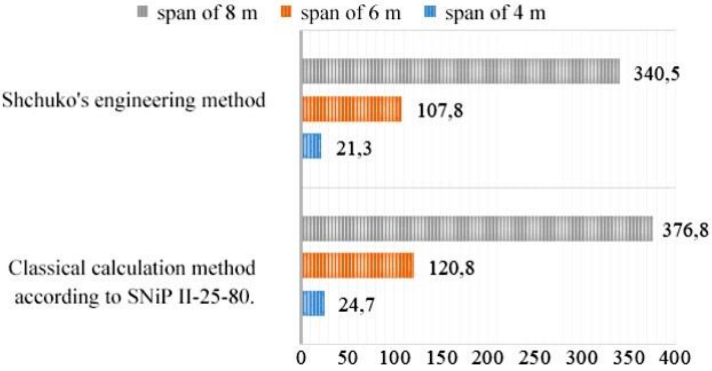


Fig. 14. Diagram of the maximum deflections w (mm) of the researched beams under a uniform load of 8 kN/m when calculating using the classic calculation method according to SNiP II-25-80 and Shchuko's engineering method

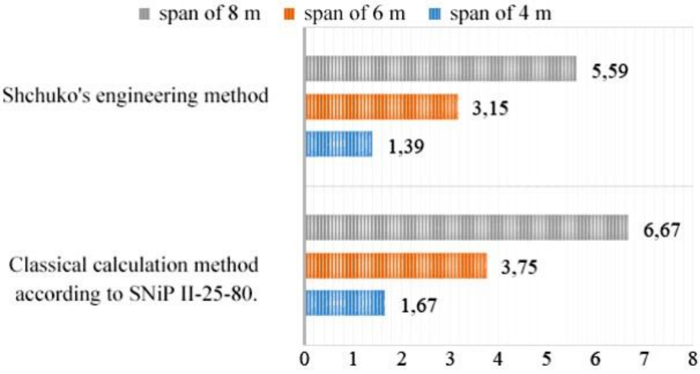


Fig. 15. Diagram of the maximum normal stresses $\sigma_{m,d}$, (kN/cm^2) of the researched beams under a uniform load of 8 kN/m when calculating using the classic calculation method according to SNiP II-25-80 and Shchuko's engineering method

From the conducted numerical studies, the results of which are given in the table 2 and fig. 10 – 15 it is clear that according to the Shchuko's engineering calculation method for laminated timber beams of the same strength class reinforced with composite tapes under a uniformly distributed load of 2.0 kN/m, the bearing capacity increases by more than 15% compared to similar beams without composite reinforcement tapes calculated according to the classic method of calculation according to SNiP II-25-80. And the movement of such beams when reinforced with tapes is reduced by more than 10%. For reinforced beams with a uniformly distributed load of 4.0 kN/m, the bearing capacity increases by more than 16% compared to similar beams without composite tape reinforcement. And the movement of such beams when reinforced with tapes is reduced by almost 15%. For reinforced beams with a uniformly distributed load of 8.0 kN/m, the bearing capacity also increases by 16% compared to similar beams without reinforcement with composite tapes. And the movement of such beams when reinforced with tapes is reduced to 13%.

Analytical calculation of laminated timber beams according to the calculation method given in DBN V.2.6-161:2017 and similar beams reinforced with composite tapes according to the proposed method. Calculation of the strength of laminated timber elements of a rectangular cross-section according to the maximum normal, working for bending in the plane of one axis of the section, according to the normal stresses from bending should be performed according to the formulas of DBN B.2.6-161:2017 [2]:

$$\frac{\sigma_{m,y,d}}{f_{m,y,d}} \le 1, \tag{5}$$

where $\sigma_{m,y,d}$ – calculated bending stress, determined by formula (6); $f_{m,y,d}$ – calculated value of bending strength.

The calculated bending stress is determined by the formula:

$$\sigma_{m,y,d} = \frac{M_{y,d}}{W_{y,d}}, \tag{6}$$

where $M_{y,d}$ is the calculated bending moment; $W_{y,d}$ is the calculated resistance moment of the cross-section.

The calculation of laminated timber elements of rectangular cross-section according to the second group of limit states is performed according to the formulas of DBN B.2.6-161:2017 [2]:

$$w = \frac{5 \cdot q \cdot l_{ef}^4}{32 \cdot E_{0,mean} \cdot b \cdot h^3} \cdot \left[1 + 0,96 \cdot \frac{E_{0,mean}}{G_{0,mean}} \cdot \left(h/l_{ef}\right)^2\right], \tag{7}$$

where w is beam deflection; q – evenly distributed load along the length of the hinged beam; h – cross-section height; b – cross-section width; l_{ef} – calculated beam span; $E_{0,mean}$ – the average elasticity modulus for the material along the fibers; $G_{0,mean}$ – the average shear modulus.

Taking into account the structure and features of laminated timber elements reinforced with composite reinforcement of a rectangular cross-section work, for a detailed analysis of the stress-strain state, it is necessary to obtain the efficient characteristics only along the fibers (x-axis).

The efficient cross-sectional area for timber:

$$A_{x,ef} = A_{x,a} \cdot \frac{E_{x,a}}{E_{x,b}}, \tag{8}$$

where $A_{x,a}$ is the cross-sectional area of the reinforcement; $E_{x,b}$ is the elasticity modulus of the boards relative to the x axis, along the fibers; $E_{x,a}$ is the elasticity modulus of the reinforcement along the fibers.

The efficient moment of inertia of the section to the timber:

$$I_{x,ef} = I_{x,b} + I_{x,a} \cdot \frac{E_{x,a}}{E_{x,b}}, \tag{9}$$

where $I_{x,b}$ – the moment of inertia of the beam cross-section made of laminated timber relative to the neutral axis; $I_{x,a}$ – the moment of inertia of the reinforcement cross-section relative to the neutral axis; $E_{x,b}$ – elasticity modulus of the boards relative to the x axis, along the fibers; $E_{x,a}$ – the elasticity modulus of the reinforcement along the fibers.

The cross-section efficient moment of resistance to the timber:

$$W_{x,ef} = \frac{I_{x,ef}}{h_z}, \tag{10}$$

in which $I_{x,ef}$ – the efficient moment of inertia of the cross-section perpendicular to the x axis, which should be determined by formula (2); h_z – the distance from the center of cross section gravity to the extreme fiber in which the stress is determined.

For the calculation of laminated timber elements reinforced with composite reinforcement of a rectangular cross-section according to the second limit state (serviceability), it is necessary to determine the efficient elasticity modulus of the cross-section to the timber.

The efficient elasticity modulus of the studied element is determined from the condition:

$$I_{x,ef} \cdot E_{x,b} = I_{x,b} \cdot E_{x,ef}, \tag{11}$$

where $I_{x,ef}$ – the efficient moment of inertia of the cross-section perpendicular to the x axis, which should be determined by formula (9); $E_{x,b}$ – elasticity modulus of timber along the fibers; $I_{x,b}$– the moment of inertia of the cross-section of the beam made of laminated timber relative to neutral; $E_{x,ef}$ – the efficient elasticity modulus of an element made of laminated timber reinforced with a composite tape along the stretched fibers.

From formula (11) we obtain the formula for determining the efficient elasticity modulus of an element made of laminated timber reinforced with a composite tape along the stretched fibers:

$$E_{x,ef} = \frac{I_{x,ef} \cdot E_{x,b}}{I_{x,b}}; \tag{12}$$

The full calculation results for laminated timber beams with and without reinforcement with composite tapes for spans of 4 m, 6 m, 8 m, which were researched and all variants of uniformly distributed load of 2 kN/m, 4 kN/m, 8 kN/m are presented in the table 3.

Table 3

Values of deflections and maximum normal stresses for laminated timber beams of the same strength class with and without reinforcement under a uniformly distributed load

Span, m		4		6		8	
Calculation methods	EI (W_x), kNcm^2 (cm^3)	w, mm	$\sigma_{m,d}$, kN/cm^2	w, mm	$\sigma_{m,d}$, kN/cm^2	w, mm	$\sigma_{m,d}$, kN/cm^2
uniformly distributed load - 2.0 kN/m							
Classical calculation method according to DBN B.2.6-161:2017	14976x10^3 (960,0) without reinforcement	4,45	0,42	22,5	0,94	71,2	1,67
The proposed method	21429x10^3 (1142,7) reinforced with composite tape	3,11	0,35	15,75	0,78	49,78	1,4
Percentage difference		30%	17%	30%	17%	30%	17%
uniformly distributed load - 4.0 kN/m							
Classical calculation method according to DBN B.2.6-161:2017	14976x10^3 (960,0) without reinforcement	8,90	0,84	45,1	1,88	142,5	3,34
The proposed method	21429x10^3 (1142,7) reinforced with composite tape	6,2	0,7	31,5	1,58	99,6	2,8
Percentage difference		30%	17%	30%	15%	30%	16%
uniformly distributed load - 8.0 kN/m							
Classical calculation method according to DBN B.2.6-161:2017	14976x10^3 (960,0) without reinforcement	17,8	1,67	90,1	3,75	284,9	6,67
The proposed method	21429x10^3 (1142,7) reinforced with composite tape	12,4	1,40	63,0	3,15	199,1	5,60
Percentage difference		30%	16%	30%	16%	30%	16%

For a more illustrative example, comparative diagrams of the maximum deflections w (mm) and the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the investigated beams under a uniform load of 2 kN/m, 4 kN/m, 8 kN/m were created, when calculating the classical calculation method according to DBN B.2.6-161:2017 and the proposed engineering method, which consists in adding the efficient geometric and mechanical characteristics to the classical calculation formulas (Fig. 16 – 21).

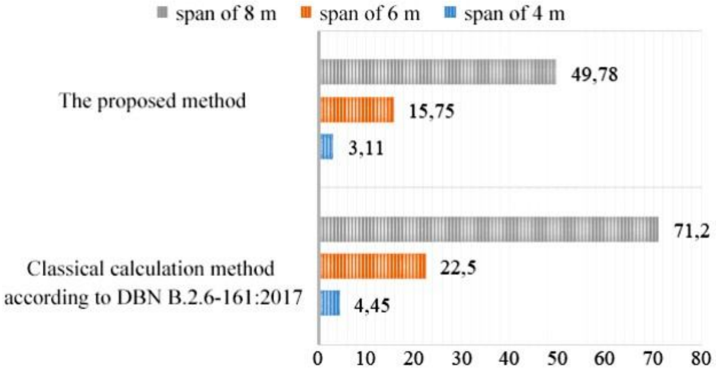


Fig. 16. Diagram of the maximum deflections *w* (mm) of the researched beams under a uniform load of 2 kN/m when calculating using the classic calculation method according to DBN B.2.6-161:2017 and the proposed engineering method

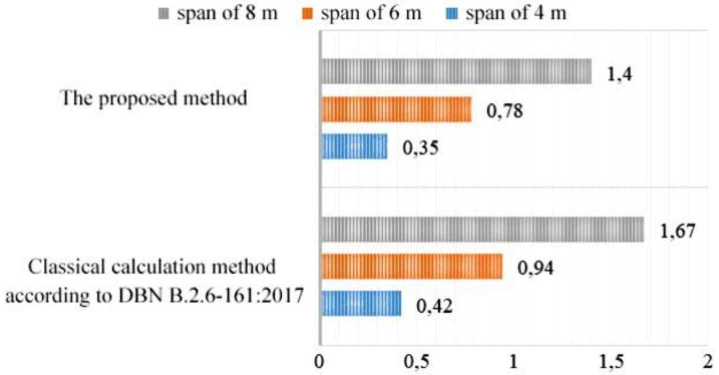


Fig. 17. Diagram of the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the researched beams under a uniform load of 2 kN/m when calculating using the classical method of calculation according to DBN B.2.6-161:2017 and the proposed engineering method

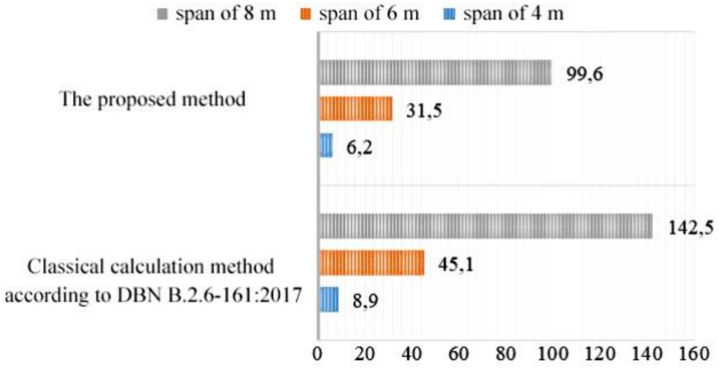


Fig. 18. Diagram of the maximum deflections *w* (mm) of the researched beams under a uniform load of 4 kN/m when calculating using the classic calculation method according to DBN B.2.6-161:2017 and the proposed engineering method

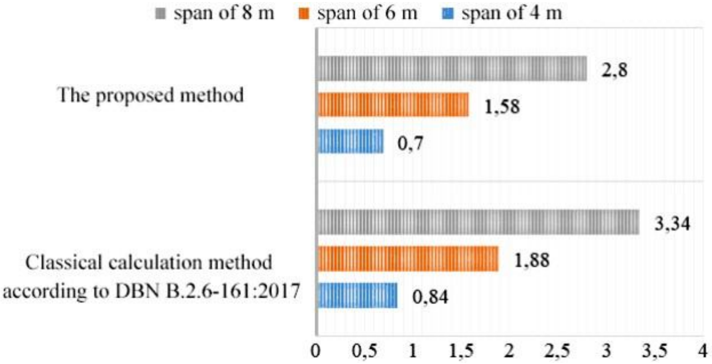


Fig. 19. Diagram of the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the researched beams under a uniform load of 4 kN/m when calculating using the classical method of calculation according to DBN B.2.6-161:2017 and the proposed engineering method

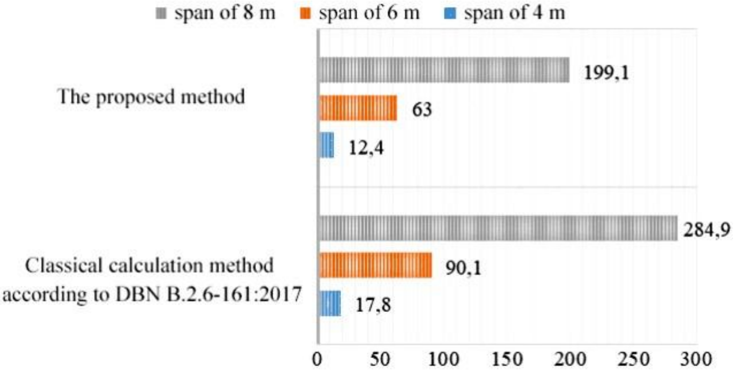


Fig. 20. Diagram of the maximum deflections w (mm) of the researched beams under a uniform load of 8 kN/m when calculating using the classic calculation method according to DBN B.2.6-161:2017 and the proposed engineering method

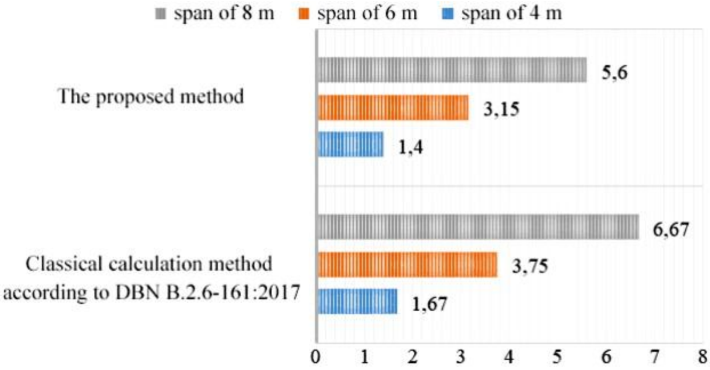


Fig. 21. Diagram of the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the researched beams under a uniform load of 8 kN/m when calculating using the classical method of calculation according to DBN B.2.6-161:2017 and the proposed engineering method

From the conducted numerical studies, the results of which are given in the table 3 and fig. 16 – 21, it is clear that according to the proposed engineering method of calculation, which consists in the combination of classical calculation formulas with the efficient geometric and mechanical characteristics for laminated timber beams of the same strength class reinforced with composite tapes under a uniformly distributed load of 2.0 kN/m, the bearing capacity increases by more than 16% compared to similar beams without reinforcement with composite tapes according to the classic method of calculation according to DBN B.2.6-161:2017. And the movement of such beams when reinforced with tapes is reduced by more than 30%. For reinforced beams with a uniformly distributed load of 4.0 kN/m, the load-bearing capacity increases by more than 15% compared to similar beams without composite tape reinforcement. And the movement of such beams when reinforced with tapes is reduced by almost 30%. For reinforced beams with a uniformly distributed load of 8.0 kN/m, the bearing capacity also increases by 16% compared to similar beams without reinforcement with composite tapes. And the movement of such beams when reinforced with tapes is reduced to 30%.

Analysis of the results of methods of calculation of laminated timber beams reinforced with composite tapes. From the above calculations comparing laminated timber beams of the same strength class of different spans at different values of uniformly distributed load and similar beams reinforced with composite reinforcement, it can be concluded that reinforcement with composite tapes is relevant and appropriate for its use in construction.

Let us analyze the results of calculation methods for laminated timber beams reinforced with composite tapes. Table 4 show the summarized results of the considered methods for calculating the elements of LTS with a rectangular section reinforced with composite tapes.

Table 4

Values of deflections and maximum normal stresses for laminated timber beams of the same strength class with and without reinforcement under a uniformly distributed load

Span, m		4		6		8	
Calculation methods	EI (W_x), kNcm^2 (cm^3)	w, mm	$\sigma_{m,d}$, kN/cm^2	w, mm	$\sigma_{m,d}$, kN/cm^2	w, mm	$\sigma_{m,d}$, kN/cm^2
uniformly distributed load - 2.0 kN/m							
FEM with volumetric FE No. 36 and flat FE No. 44	reinforced with composite tape	4,06	0,318	21	0,701	61,2	1,27
Shchuko's engineering method	13803x10^3 (1144,5) reinforced with composite tape	5,32	0,35	26,93	0,79	85,13	1,39
The proposed method	21429x10^3 (1142,7) without reinforcement	3,11	0,35	15,75	0,78	49,78	1,4

Continuaton of table 4

uniformly distributed load - 4.0 kN/m							
FEM with volumetric FE No. 36 and flat FE No. 44	reinforced with composite tape	8,11	0,635	42,0	1,4	122,0	2,54
Shchuko's engineering method	$13803x10^3$ (1144,5) reinforced with composite tape	10,6	0,69	53,9	1,57	170,3	2,79
The proposed method	$21429x10^3$ (1142,7) without reinforcement	6,2	0,7	31,5	1,58	99,6	2,8
uniformly distributed load - 8.0 kN/m							
FEM with volumetric FE No. 36 and flat FE No. 44	reinforced with composite tape	16,2	1,27	и84	2,8	245,0	5,07
Shchuko's engineering method	$13803x10^3$ (1144,5) reinforced with composite tape	21,3	1,39	107,8	3,15	340,5	5,59
The proposed method	$21429x10^3$ (1142,7) without reinforcement	12,4	1,40	63,0	3,15	199,1	5,60

For a more illustrative example, comparative diagrams of maximum deflections w (mm) and maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the investigated beams reinforced with composite reinforcement under a uniform load of 2 kN/m, 4 kN/m, 8 kN/m were created, when calculating by the finite element method in PC LIRA_SAPR, Shchuko's engineering method, and the proposed engineering method, which consists in adding the efficient geometric and mechanical characteristics to the classical calculation formulas (Fig. 22 – 27).

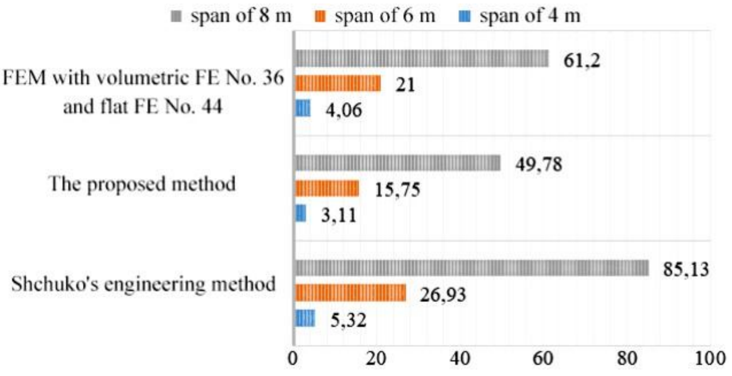


Fig. 22. Diagram of the maximum deflections w (mm) of the investigated beams reinforced with composite tapes under a uniform load of 2 kN/m with different calculation methods

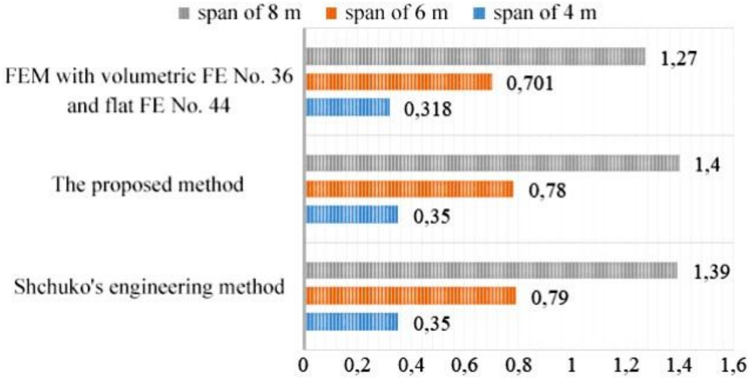


Fig. 23. Diagram of the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the examined beams reinforced with composite tapes under a uniform load of 2 kN/m with different calculation methods

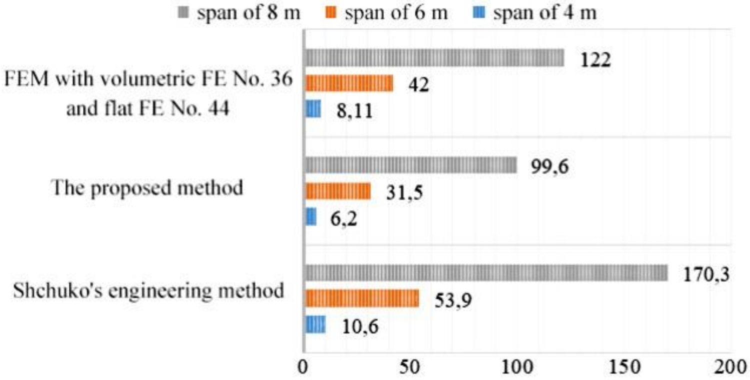


Fig. 24. Diagram of the maximum deflections w (mm) of the investigated beams reinforced with composite tapes under a uniform load of 4 kN/m with different calculation methods

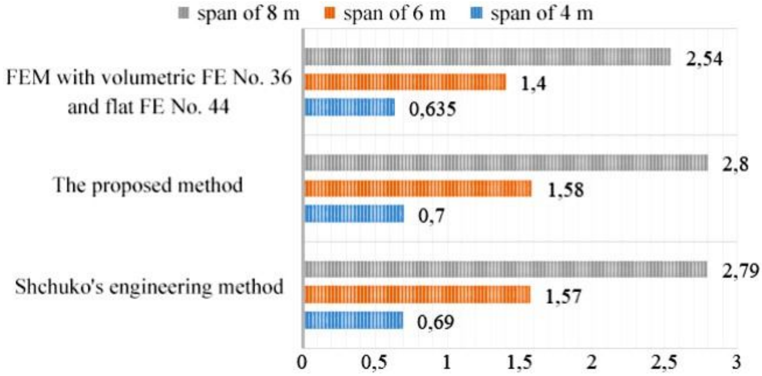


Рис. 25. Diagram of the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the investigated beams reinforced with composite tapes from a uniform load of 4 kN/m with different calculation methods

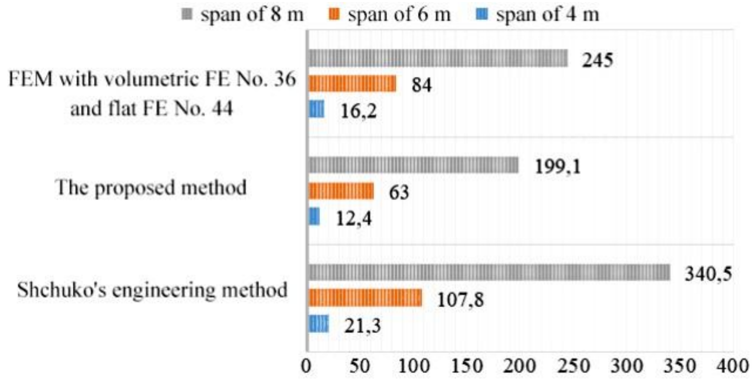


Fig. 26. Diagram of the maximum deflections *w* (mm) of the investigated beams reinforced with composite tapes under a uniform load of 8 kN/m with different calculation methods

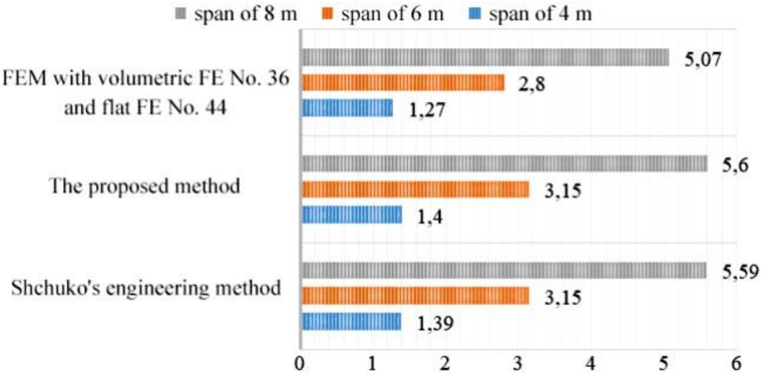


Fig. 27. Diagram of the maximum normal stresses $\sigma_{m,d}$ (kN/cm^2) of the investigated beams reinforced with composite tapes under a uniform load of 8 kN/m with different calculation methods

Conclusions. From the obtained results, it can be seen that the analytical calculation of laminated timber beams according to the calculation method given in SNiP II-25-80 [4] gives more than 25% of the margin of deflections in comparison with the similar analytical calculation given in DBN B.2.6-161:2017 [2], and the calculation of beams reinforced with composite tapes according to Shchuko's calculation method [3] gives deflections on average 41% larger than the calculation according to the proposed method, with almost identical maximum normal stresses.

As can be seen from the obtained results, calculations of laminated timber beams reinforced with composite reinforcement using the finite element method in PC LIRA-CAD gives a sufficiently high coincidence of the maximum normal stresses (within 5-10%) with the proposed analytical method of calculation, however, deflections in reinforced laminated timber beams according to the finite element method is almost 30% larger than according to the proposed analytical method of calculation. This indicates that the method of modeling composite tapes with flat FE No. 44 in PC LIRA-CAD requires more detailed research.

The use of composite tape reinforcement of laminated timber elements should increase the reliability of these structures and expand the range of their application.

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Михайловський Д.В., Комар О.А., Комар М.А.

ІНЖЕНЕРНА МЕТОДИКА РОЗРАХУНКУ ЕЛЕМЕНТІВ З КЛЕЄНОЇ ДЕРЕВИНИ АРМОВАНИХ КОМПОЗИТНИМИ СТРІЧКАМИ

Актуальність. В сучасному світі будівництво потребує екологічно чистих матеріалів, що наносять мінімальної шкоди навколишньому середовищу. Водночас вони повинні мати велику міцність та бути стійкими до різного типу зовнішніх впливів. Таким матеріалом, по праву, можна вважати конструкції з деревини, які виготовляються з відновлювальних природніх матеріалів і мають порівняно високу міцність. Попри те, що сама деревина має певні негативні властивості, а саме – схильність до усушки та розбухання, гниття, анізотропію властивостей, що потребує особливої уваги на будівництві, в конструкціях з клеєної деревини (ККД) данні недоліки вже більш керовані. Балки є однією з основних конструкцій з клеєної деревини. Армування балок з клеєної деревини дозволяє значно підвищити їхню жорсткість і міцність. **Мета роботи.** У статті запропоновано методику розрахунку елементів прямокутного перерізу з клеєної деревини армованих композитними стрічками. Наведено уточнені формули для визначення приведених геометричних і механічних характеристик. Порівняно запропоновану методику з іншою аналітичною методикою розрахунку та з чисельною методикою моделювання в програмному комплексі

ЛІРА-САПР методом скінченних елементів. **Результати.** Підтверджено, що запропонована методика аналітичного розрахунку балок з клеєної деревини, підсилених композитними стрічками, є доцільною для застосування при розрахунку як окремих елементів, так і складних систем з них. За даною методикою можливе врахування товщини та механічних характеристик деревини дошок, з яких складений поперечний переріз елементу та армування, що значно збільшує діапазон використання перерізів з клеєної деревини, армованих композитними стрічками, навіть за умови комбінації дошок різних класів міцності у перерізі.

Ключові слова: армування, аналітична методика розрахунку, клеєна деревина, метод скінченних елементів, конструкції з клеєної деревини, приведені геометричні характеристики перерізу, приведений модуль пружності.

Mykhailovskyi D.V., Komar O.A., Komar M.A.

ENGINEERING METHOD OF CALCULATING LAMINATED TIMBER ELEMENTS REINFORCED WITH COMPOSITE TAPE

Annotation. In today's world, construction requires environmentally friendly materials that cause minimal damage to the environment. At the same time, they must have great strength and be resistant to various types of external influences. Timber structures, which are made of renewable natural materials and have a relatively high strength, can rightfully be considered as such material. Despite the fact that timber itself has certain negative properties, namely the tendency to shrink and swell, rot, anisotropy of properties, which requires special attention in construction, in laminated timber structures (LTS) these disadvantages are already more manageable. Beams are one of the main laminated timber structures. Laminated timber beams reinforcement allows to significantly increasing their stiffness and strength.

The article proposes a methodology for calculating laminated timber elements of rectangular section reinforced with composite tapes. Refined formulas for determining the efficient geometric and mechanical characteristics are provided. The proposed method was compared with another analytical method of calculation and with the numerical method of modeling in the LIRA-CAD software complex using the finite element method. It was proved that this method of reinforcing laminated timber structures is promising and relevant.

Keywords: reinforcement, analytical method of calculation, laminated, finite element method, laminated timber structures, efficient geometric cross-section characteristics, efficient elasticity modulus.

Михайловський Д.В., Комар О.А., Комар М.А.

ИНЖЕНЕРНАЯ МЕТОДИКА РАСЧЕТА ЭЛЕМЕНТОВ ИЗ КЛЕЕННОЙ ДРЕВЕСИНЫ, АРМИРОВАННЫХ КОМПОЗИТНЫМИ ЛЕНТАМИ

Актуальность. В современном мире строительство нуждается в экологически чистых материалах, наносящих минимальный ущерб окружающей среде. В то же время, они должны иметь большую прочность и быть устойчивыми к разным типам внешних воздействий. Таким материалом, по праву, можно считать конструкции из древесины, которые производятся из восстановительных природных материалов и имеют сравнительно высокую прочность. Несмотря на то, что сама древесина обладает определенными негативными свойствами, а именно: склонностью к усушке и разбуханию, гниению, анизотропии свойств, что требует особого внимания в строительстве, в конструкциях из клееной древесины (ККД) данные недостатки поправимы. Балки являются одной из основных конструкций из клееной древесины. Армирование балок из клееного дерева позволяет значительно повысить их жесткость и прочность. **Цель работы.** В статье предложена методика расчета элементов прямоугольного сечения из клееной древесины, армированных композитными лентами. Представлены уточненные формулы для определения приведенных геометрических и механических характеристик. Проведены сравнения предложенной методики с другой аналитической методикой расчета и с численной методикой моделирования в программном комплексе ЛИРА-САПР методом конечных элементов. **Результаты.** Подтверждено, что предложенная методика аналитического расчета балок из клееной древесины, усиленных композитными лентами, целесообразна для применения при расчете, как отдельных элементов, так и сложных систем, состоящих из них. По данной методике возможен учет толщины и механических характеристик древесины досок, из которых составлено поперечное сечение элемента и армирования, что значительно увеличивает диапазон использования сечений из клееной древесины, армированных композитными лентами, даже при комбинации досок различных классов прочности в сечении.

Ключевые слова: армирование, аналитическая методика расчета, клееная древесина, метод конечных элементов, конструкции из клееной древесины, приведены геометрические характеристики сечения, приведен модуль упругости.

УДК 624.011
Михайловський Д.В., Комар О.А., Комар М.А. **Інженерна методика розрахунку елементів з клеєної деревини, армованих композитними стрічками** / Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2022. – Вип. 109. – С. 239-262. – Англ.
Запропоновано методику розрахунку елементів прямокутного перерізу з клеєної деревини, армованих композитними стрічками, уточнені формули для визначення приведених геометричних і механічних характеристик, порівняно запропоновану методику із іншою аналітичною методикою розрахунку та з чисельною методикою моделювання в програмному комплексі ЛІРА-САПР методом скінченних елементів.
Табл. 4. Іл. 27. Бібліогр. 7 назв.

УДК 624.011
Mykhailovskyi D.V., Komar O.A., Komar M.A. **Engineering method of calculating laminated timber elements reinforced with composite tapes** / Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2022. – Issue 109. – P. 239-262.
A method of calculating elements of rectangular section made of glued wood reinforced with composite tapes is proposed, formulas for determining the given geometric and mechanical characteristics are specified, the proposed method is compared with another analytical method of calculation and with a numerical method of modeling in the LIRA-CAD software complex using the finite element method.
Tabl. 4. Fig. 27. Ref. 7.

УДК 624.011
Михайловський Д.В., Комар О.А., Комар М.А. **Инженерная методика расчета элементов из клеенной древесины, армированных композитными лентами** / Сопротивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУСА, 2022. – Вип. 109. – С. 239-262. – Англ.
Предложена методика расчета элементов прямоугольного сечения из клееной древесины, армированных композитными лентами, уточнены формулы для определения приведенных геометрических и механических характеристик;проведено сравнение предложенной методика с другой аналитической методикой расчета и численной методикой моделирования в программном комплексе ЛИРА-САПР методом конечных элементов.
Табл. 4. Ил. 27. Библиогр. 7 назв.

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