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NUMERICAL AND EXPERIMENTAL MODELING OF THE BEHAVIOR OF FLEXIBLE SHELL ELEMENTS OF STRUCTURES

A.P. Dzyuba,
Doctor of Technical Science

I.A. Safronova,
Candidate of Physical and Mathematical Sciences

L.D. Levitina

Oles Honchar Dnipro National University,
72, Gagarina Av, Dnipro, Ukraine, 49010

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For the problems of axisymmetric nonlinear bending of thin shells of rotation with a complex meridian shape (such as thin annular plates, corrugated membranes of a sinusoidal profile and bellows), the comparative analysis of the application of two mathematical models of deformation of flexible shell elements was carried out. The numerical results are obtained by direct integration of boundary value problems of shell mechanics, by the finite element method and experimental research. The optimal design of a flexible corrugated membrane with a sinusoidal profile of the highest sensitivity are realized by using the necessary optimality conditions of the principle of maximum L. S. Pontryagin. The results are presented in the form of tables, photos and graphs.
Keywords: flexible shells of rotation, corrugated membranes, bellows, numerical analysis, experimental studies.

Introduction

The problem of developing effective approaches, models, methods and algorithms for solving nonlinear boundary value problems of shell mechanics, which are widely used in many structures of the aerospace, chemical, oil and gas and other industries, and the selection of their optimal parameters belongs to one of the most urgent problems of the mechanics of deformable solids [1, 2, 3, 4].

Bellows, as compensators of thermal movements of pipelines, shell elements in the form of membranes, as sensitive elements of measuring devices, and other similar elements refer to shells with a relatively small wall thickness, irregular stiffness parameters, and a complex meridian shape [5]. In the process of deformation of such shells, significant displacements are typical. Therefore, the corresponding boundary value problem is significantly nonlinear, and it is necessary to use efficient iterative numerical algorithms for its solution.

Reducing computational costs for solving such nonlinear problems is also important when solving problems of optimal design of complex multiparameter structures [6 – 8]. As known, the number of iterations of the search optimization algorithm is often quite significant. Given that the direct calculation of such optimization objects, the results of which are necessary at each step of the search to calculate the objective function and constraints, and in some cases also their derivatives, is also quite time-consuming, so the very possibility of correctly solving the optimization problem can be is called into question. At the same time, there is an obvious need to develop effective approaches, methods and algorithms for solving such boundary value problems, as well as to accelerate their convergence in order to reduce the corresponding computational costs.

Most of the known algorithms for solving nonlinear problems of the mechanics of inhomogeneous shells are related to the reduction of the original nonlinear boundary value problem to a Cauchy problem with unknown initial conditions. The conditions are further clarified from the conditions for fulfilling the boundary conditions on the right border of integration using the shooting methods, Newton – Kantorovich and other approaches.

The essence of another group of methods, which include, in particular, methods of variable parameters of elasticity, consists in the linearization of the original nonlinear system of equations and the subsequent solution of a set of corresponding linear boundary value problems [9, 10]. In [11], for this purpose, the method of successive loads is used, in [12] – continuation by parameter, in [13] – the method of disturbances taking into account the differences between the geometry of the undeformed and deformed states of the body under study.

1. Formulation of the problem. The analysis of the problem as a whole and the main results achieved in this field and methods for calculating the state of shells with nonlinear parameters, reflected in a fairly significant number of reviews, monographs and scientific articles, indicates that the construction and effective application of numerical algorithms for solving nonlinear boundary value problems often includes elements of a certain "art", and their implementation, as a rule, is quite labor-intensive and, in addition, does not always allow obtaining reliable solutions. At the same time, the question of the appropriateness of choosing one or another method still remains debatable due to the lack of reliable and practically convenient criteria for evaluating the convergence of existing methods of successive approximations for solving nonlinear boundary value problems of the theory of shells. A number of aspects of this problem are still insufficiently researched, and known models and algorithms for accelerating convergence when used in practice often turn out to be insufficiently efficient and computationally labor-intensive.

This article is devoted to computer and experimental modeling of the behavior of flexible elements of shell structures, and the development of effective algorithms for solving emerging nonlinear boundary value problems of their calculation and optimization of parameters. At the same time, the probability of the results obtained using different approaches to the construction of a nonlinear theory is established. Their comparative analysis, error estimation, which in this case is given by calculation according to linear and corresponding nonlinear theories, is carried out. The results of calculated data and experimental studies of the behavior of real structural elements are compared.

2. Boundary value problems of nonlinear deformation of axisymmetric shells of rotation. The nonlinear moment theory of shells is used. It is believed that the shells are isotropic, elastic and thin-walled. The equations of the stress-strain state of such shells of rotation, in the presence of large displacements during axisymmetric deformation, are presented in the form of corresponding boundary value problems for systems of ordinary differential equations with variable coefficients

$$\frac{d\overline{z}}{ds} = A\left(\overline{z}(s), s\right)\times\overline{z} + B\left(\overline{z}(s), s\right), \tag{1}$$

$$f_j\left(\overline{z}\left(s_p\right)\right) = 0, \quad j = \overline{1,m}, \tag{2}$$

it's correspond to different options for fixing the ends of the rotation shell at the initial $s_p=s_0$ and final $s_p = s_l$ points of the shell meridian. $\overline{z}^T(s) = \{\xi, \vartheta_r, N_r r, M_r r, \zeta\}$ – is a vector of shell state variables here.

Nonlinear components (of the second order of smallness) of the rotation angle of the normal of the median surface and their influence on the remaining components of the stress-strain state are taken into account in comparison with the linear formulation in the equations of state of the shells of rotation with an arbitrary shape of the meridian, in accordance with [14]. And system (1) is served as:

$$\frac{d\xi}{ds} = -\frac{\mu}{r}(\cos\vartheta_r\cos\theta - \sin\vartheta_r\sin\theta)\xi - \vartheta_r\sin\theta + \frac{1}{Kr}\cos^2(\theta+\vartheta_r)(N_r r) +$$
$$+\frac{1}{Kr}\sin(\theta+\vartheta_r)\cos(\theta+\vartheta_r)\frac{F(s)}{2\pi} + \cos(\theta+\vartheta_r) - \cos\theta + \vartheta_r\sin\theta,$$
$$\frac{d\vartheta_r}{ds} = \frac{1}{Dr}(M_r r) - \vartheta_r\frac{\mu}{r}\cos\theta + \frac{\mu}{r}\left(-\sin(\theta+\vartheta_r) + \sin\theta + \vartheta_r\cos\theta\right),$$
$$\frac{d(N_r r)}{ds} = \frac{K\left(1-\mu^2\right)}{r}\xi + \frac{\mu}{r}\cos(\theta+\vartheta_r)(N_r r) + \frac{\mu}{r}\sin(\theta+\vartheta_r)\frac{F(s)}{2\pi} - q_r r,$$
$$\frac{d(M_r r)}{ds} = \vartheta_r\frac{D\left(1-\mu^2\right)}{r}\cos(\theta+\vartheta_r)\cos\theta + \sin(\theta+\vartheta_r)(N_r r) +$$
$$+\frac{\mu}{r}\cos(\theta+\vartheta_r)(M_r r) - \cos(\theta+\vartheta_r)\frac{F(s)}{2\pi} +$$
$$+\frac{D\left(1-\mu^2\right)}{r}\cos(\theta+\vartheta_r)\left(\sin(\theta+\vartheta_r) - \sin\theta - \vartheta_r\cos\theta\right),$$
$$\frac{d\zeta}{ds} = -\frac{\mu}{r}\sin(\theta+\vartheta_r)\xi + \vartheta_r\cos\theta + \frac{1}{2Kr}\sin\left(2(\theta+\vartheta_r)\right)(N_r r) +$$
$$+\frac{1}{Kr}\sin^2(\theta+\vartheta_r)\frac{F(s)}{2\pi} + \sin(\theta+\vartheta_r) - \sin\theta - \vartheta_r\cos\theta, \tag{3}$$

where $s_0 \le s \le s_l$ is the independent variable along the meridian; $r(s)$ is the radius of a parallel circle; $\theta(s)$ is the angle between the axis of rotation and the normal to the undeformed surface; ϑ_r is the angle of rotation of the normal to the middle surface during deformation; ξ, ς is radial and axial movement; N_r is tensile force; M_r is bending moment; $F(s) = P_0 + 2\pi \int_{s_0}^{s} (q_n \cos\theta - q_\tau \sin\theta) r ds$; P_0 is axial load on the end of the shell; $q_\tau(s), q_n(s)$ is tangent and normal to the surface components of the external distributed load; $q_r(s) = q_\tau \cos\theta + q_n \sin\theta$, $K = Eh/(1-\mu^2)$, $D = Eh^3/(12(1-\mu^2))$ is tensile stiffness and cylindrical stiffness, respectively; E, μ is modulus of elasticity and Poisson's ratio; $h(s)$ is the variable (in the general case) thickness of the shell wall along the meridian.

Here, unlike [14], in the equations of nonlinear deformation (3), the substitution $\theta^+(s) = \theta(s) + \vartheta_r(s)$ is made, where $\theta^+(s)$ is the angle between the axis of rotation and the normal to the deformed surface.

The boundary conditions for the main variables $\{\xi, \vartheta_r, N_r, M_r, \zeta\}$ of the system (3) in accordance with the conditions for fixing the contours of the shell are taken as follows:

a) rigid clamping: $\xi = \vartheta_r = \zeta = 0$;
b) hinged fastening: $\xi = \zeta = M_r = 0$; (4)
c) free edge: $N_r = M_r = 0$ or $N_r = N_0$, $M_r = M_0$,

where N_0, M_0 are the given marginal forces.

Note that when both edges of the rotation shell are rigidly clamped or hinged, it is additionally necessary to reveal the static uncertainty by one of the known methods. It should be noted also that the solution of system (3) is significantly complicated by the fact that the value of one of the variables ϑ_r is a nonlinear component of the system coefficients.

In accordance with the approach proposed in [15, 16], in contrast to the linear theory, second-order variables are taken into account in the equilibrium equations and relations for deformations, and the system of equations of the nonlinear boundary value problem for the annular plate with respect to the vector of variables $\bar{z}^T(s) = \{N_r, \xi, Q_r, M_r, \vartheta_r, \zeta\}$ is taken in the form

$$\frac{d\xi}{ds} = -\frac{1}{K} N_r - \frac{\mu}{s}\xi - \frac{1}{2}\vartheta_r^2;$$

$$\frac{d\vartheta_r}{ds} = \frac{1}{D} M_r - \frac{\mu}{s}\vartheta_r;$$

$$\frac{dN_r}{ds} = -\frac{1-\mu}{s} N_r + \frac{(1-\mu^2)K}{s^2}\xi;$$

$$\frac{dQ_r}{ds} = -\frac{1}{s} Q_r - \frac{(1-\mu^2)K}{s^2}\xi\vartheta_r - \frac{1}{D} N_r M_r + q_n;$$

$$\frac{dM_r}{ds} = -Q_r - \frac{1-\mu}{s} M_r + \frac{\left(1-\mu^2\right)D}{s^2} \vartheta_r;$$

$$\frac{d\zeta}{ds} = -\vartheta_r, \qquad (5)$$

where in addition to the previously introduced designations; $a \le s \le b$; a, b are the inner and outer radii of the annular plate; Q_r is transverse force.

System (5) is supplemented by boundary conditions for fixing the contours of the plate in the form (4), where p. c) for the free edge has the form $N_r = M_r = Q_r = 0$ or $N_r = N_0$, $M_r = M_0$, $Q_r = Q_0$.

3. Linearization of nonlinear boundary value problems for the calculation of envelopes of rotation. Solving the nonlinear boundary value problem (1) with the corresponding boundary conditions (2) is complicated by the fact that the coefficients are significantly nonlinear from the vector $\overline{z}(s)$ components. Therefore, it is proposed to linearize the system of equations (1) in such a way that one part of the nonlinear components refers to the matrix of coefficients $A(\overline{z}(s), s)$ (as an analogue of the method of variable parameters of elasticity), and other part – to the column of free components $B(\overline{z}(s), s)$ (as an analogue of the method of additional loads). For the integration of linear boundary value problems at each step of the iterative process of refinement of nonlinear components, the sweep method with orthogonalization according to S. K. Godunov [10] is used.

The matrices $A(\overline{z}(s), s)$ and $B(\overline{z}(s), s)$ of the boundary value problem (3), (4) take the form when performing the above transformations directly:

$$A(\overline{z}(s), s) = \begin{bmatrix} -\frac{\mu}{r}\cos\theta^+ & 0 & \frac{1}{Kr}\cos^2\theta^+ & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{Dr} & 0 \\ \frac{K\left(1-\mu^2\right)}{r} & 0 & \frac{\mu}{r}\cos\theta^+ & 0 & 0 \\ 0 & 0 & \sin\theta^+ & \frac{\mu}{r}\cos\theta^+ & 0 \\ -\frac{\mu}{r}\sin\theta^+ & 0 & \frac{1}{2Kr}\sin\left(2\theta^+\right) & 0 & 0 \end{bmatrix}, \qquad (6)$$

$$B\left(\overline{z}(s),s\right)=\begin{bmatrix} \dfrac{1}{2Kr}\sin\left(2\theta^{+}\right)\dfrac{F(s)}{2\pi}+\cos\theta^{+}-\cos\theta \\ -\dfrac{\mu}{r}\left(\sin\theta^{+}-\sin\theta\right)+\dfrac{1}{R_1} \\ \dfrac{\mu}{r}\sin\theta^{+}\dfrac{F(s)}{2\pi}-q_r r \\ -\cos\theta^{+}\dfrac{F(s)}{2\pi}+\dfrac{D\left(1-\mu^2\right)}{2r}\left(\sin\left(2\theta^{+}\right)-2\cos\theta^{+}\sin\theta\right) \\ \dfrac{1}{Kr}\sin^2\theta^{+}\dfrac{F(s)}{2\pi}+\sin\theta^{+}-\sin\theta \end{bmatrix}. \quad (7)$$

The study of the convergence of the algorithm for solving such boundary value problems, which arise by transferring (linearizing) nonlinear components from the matrix $A\left(\overline{z}(s),s\right)$ to the matrix of free members $B\left(\overline{z}(s),s\right)$, was carried out based on the results of a computational experiment using various iterative processes for the variable $\theta^{+}(s)$. The solution of the problem in a linear formulation was chosen as the initial approximation. It turned out that the algorithms for solving the boundary value problem (1), (2) with unstructured (sparse) matrices of coefficients (6), (7) showed low and unstable convergence, convergence to different solutions was observed, and sometimes its absence [7].

In order to overcome these difficulties, in this work, it was proposed to present the matrices (6), (7) of the system of nonlinear equations at large displacements in a structured form, close to the matrices of the system of linear equations, the solution of which has been sufficiently tested [14 – 16]. For this purpose, two identical terms with different signs were added to the component matrices of the right-hand parts of each of the 5 equations of system (3), taking into account $\theta^{+}(s)=\theta(s)+\vartheta_r(s)$:

$$1)-\vartheta_r\sin\theta+\vartheta_r\sin\theta;\quad 2)-\vartheta_r\frac{\mu}{r}\cos\theta+\vartheta_r\frac{\mu}{r}\cos\theta;\quad 3)-0+0;$$

$$4)\vartheta_r\frac{D\left(1-\mu^2\right)}{r}\cos\left(\theta+\vartheta_r\right)\cos\theta-\vartheta_r\frac{D\left(1-\mu^2\right)}{r}\cos\left(\theta+\vartheta_r\right)\cos\theta;\quad (8)$$

$$5)\vartheta_r\cos\theta-\vartheta_r\cos\theta.$$

At the same time, the first components of the terms were assigned to matrix A, and the second to matrix B. Despite the fact that such an artificial technique increased the number of nonlinear components of these matrices, it allowed not only to present matrix A in the form of a corresponding matrix for a linear system [14], but also to increase its conditioning:

$$A\left(\bar{z}(s)\right)=\begin{bmatrix} -\frac{\mu}{r}\cos(\theta+\vartheta_r) & -\sin\theta & \frac{1}{Kr}\cos^2(\theta+\vartheta_r) & 0 & 0 \\ 0 & -\frac{\mu}{r}\cos\theta & 0 & \frac{1}{Dr} & 0 \\ \frac{K\left(1-\mu^2\right)}{r} & 0 & \frac{\mu}{r}\cos(\theta+\vartheta_r) & 0 & 0 \\ 0 & \frac{Eh^3}{12r}\cos(\theta+\vartheta_r)\cos\theta & \sin(\theta+\vartheta_r) & \frac{\mu}{r}\cos(\theta+\vartheta_r) & 0 \\ -\frac{\mu}{r}\sin(\theta+\vartheta_r) & \cos\theta & \frac{1}{2Kr}\sin(2(\theta+\vartheta_r)) & 0 & 0 \end{bmatrix}, \quad (9)$$

$$B\left(\bar{z}(s)\right)=\begin{bmatrix} \frac{1}{Kr}\sin(\theta+\vartheta_r)\cos(\theta+\vartheta_r)\frac{F(s)}{2\pi}+\cos(\theta+\vartheta_r)-\cos\theta+\vartheta_r\sin\theta \\ \frac{\mu}{r}\left(-\sin(\theta+\vartheta_r)+\sin\theta+\vartheta_r\cos\theta\right) \\ \frac{\mu}{r}\sin(\theta+\vartheta_r)\frac{F(s)}{2\pi}-q_r r \\ -\cos(\theta+\vartheta_r)\frac{F(s)}{2\pi}+\frac{D\left(1-\mu^2\right)}{r}\cos(\theta+\vartheta_r)\left(\sin(\theta+\vartheta_r)-\sin\theta-\vartheta_r\cos\theta\right) \\ \frac{1}{Kr}\sin^2(\theta+\vartheta_r)\frac{F(s)}{2\pi}+\sin(\theta+\vartheta_r)-\sin\theta-\vartheta_r\cos\theta \end{bmatrix}. \quad (10)$$

The results of the system numerical experiments showed that the presentation of matrices A and B of system (3) in the form (9), (10) made it possible to construct a stably convergent and positively sensitive to the application of convergence acceleration algorithms iterative process of solving the boundary value problem and to significantly reduce the total number of iterations.

In order to accelerate the convergence of the iterative process of solving boundary value problems of shell mechanics, it is proposed to determine the nonlinear component components of the next approximation vector $\bar{z}_i^{(n+1)}$ by extrapolation (imitation) of their values based on the results of calculations $\bar{z}_i^{n-2}$, $\bar{z}_i^{n-1}$, $\bar{z}_i^{n}$ at the previous steps for each of the nodal points $s_i\left(i=\overline{0,L}\right)$ of the interval $s_0 \le s_i \le s_L$ of solving the sequence of the corresponding linearized systems.

Three steps of the iterative process are carried out, in which the nonlinear components are refined using the method of upper relaxation (linear extrapolation)

$$\bar{z}_i^{(n+1)}=\bar{z}_i^{n}-\gamma\left(\bar{z}_i^{n}-\bar{z}_i^{(n-1)}\right), \quad (12)$$

where $0 \le \gamma \le 1$ is the relaxation factor.

For $n=0$, the linearized problem is solved at the initial value $\overline{z}^0$ corresponding to the solution of the linear problem. Next, the form of the forecast for each nonlinear *j*-th component of the vector $\overline{z}_{ji}$ $(j=\overline{1,5})$ is divided by the form of an iterative process [17 – 19]. In the case when the sequence $z_i^{n-2}, z_i^{n-1}, z_i^n$ is non-monotonic $\left(z_i^{n-1}-z_i^{n-2}\right)\left(z_i^n-z_i^{n-1}\right)<0$ for a certain *j* (in the future, the index *j* is omitted), the forecast is proposed to be carried out according to the Aitken – Steffensen formula

$$z_i^{(n+1)}=\frac{z_i^n z_i^{(n-2)}-\left(z_i^{(n-1)}\right)^2}{z_i^{(n-2)}-2z_i^{(n-1)}+z_i^n},\ n\geq 2,\ (i=\overline{0,L}), \tag{13}$$

and in the case when the process is monotonic $\left(z_i^{n-1}-z_i^{n-2}\right)\left(z_i^n-z_i^{n-1}\right)\geq 0$ – in the form of an analogue of the Adams method, which is based on the extrapolation dependences of Lagrange and Newton

$$z_i^{(n+1)}=\frac{23z_i^n-16z_i^{(n-1)}+5z_i^{(n-2)}}{12},\ \ n\geq 2,\ \ (i=\overline{0,L}), \tag{14}$$

Next, according to the main algorithm, the linearized system is solved again for the next two (using (12)) successive steps, and the process continues until the specified accuracy is reached, taking into account the obtained three new points, starting from the forecast point.

The estimation of the accuracy of the iterative process is calculated such

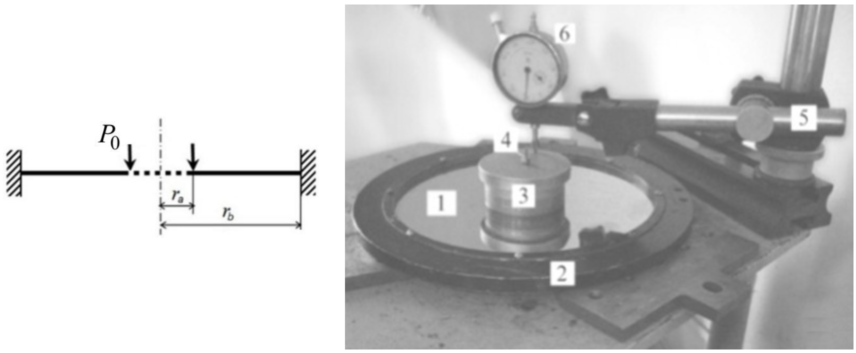
$$\sqrt{\frac{\sum_{j=1}^{m}\sum_{i=0}^{N}\left(z_{i(j)}^{n+1}(s_i)-z_{i(j)}^{n}(s_i)\right)^2}{\sum_{j=1}^{m}\sum_{i=0}^{N}\left(z_{i(j)}^{n}(s_i)\right)^2}}\leq\varepsilon\ , \tag{15}$$

where ε is the specified accuracy.

This approach made it possible to replace the solution of the linearized boundary value problem at every third step with predictive values calculated by simple formulas (13), (14), and which, in addition, turn out to be closer to the solution than those obtained by the main algorithm, and significantly (up to 2 times) speed up the computing process.

4. Numerical and experimental studies. The reliability of the results obtained with the help of the constructed algorithm was evaluated by comparing the data of numerical and experimental research.

In order to carry out experimental studies of the deformation of the annular plate under the action of an axisymmetric load distributed along the inner hole (Fig. 1, a), a special test setup was developed (Fig. 1, b). The load of the rigidly clamped – 2 plate – 1 was carried out through a thin–walled cylindrical plug washer – 3 with a micrometric thread, which made it possible to accurately screw the washer and, thus, to obtain a load evenly distributed over the inner hole of the plate. In the center of the puck, a hole was made for fixing a rack with a platform – 4, on which weights were installed for loading the plate.



(a) (b)

Fig. 1. Calculation scheme (a) and device (b) to determine the deflection of an annular plate

Measurements of plate deflections were made with a watch-type indicator – 6. The deflection value was chosen as the arithmetic mean of the measurement results at four points located at the ends of the mutually perpendicular diameters of the plate opening.

Calculations of the maximum deflection along the inner contour of the annular plate under the action of a transverse load uniformly distributed along the inner contour were carried out according to models of nonlinear deformation (3), (at) and (5) with boundary conditions (4, a), (4, c), the results of which are shown in Fig. 3. The signs «■», «□» indicate the data obtained according to models (3), (5), respectively. Line 1 corresponds to a linear solution. For comparison, Fig. 2 also shows the results of the numerical calculation of the plate in the Ansys R19.0 Academic, which are marked with «▲» in Fig. 3, and the «●» sign shows the data of the conducted experimental studies.

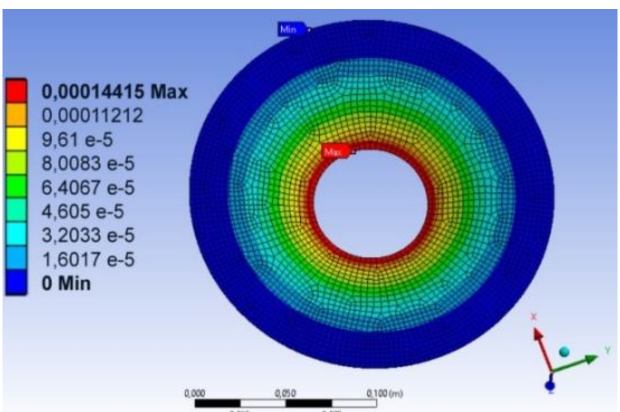


Fig. 2. Finite element calculation of the ring plate

The deviation of the calculation results obtained according to the model (3), (5) reaches 15%, and the finite element calculation according to the model (5) and the experimental data are within 5%. It should be noted that with an increase in the load and, as a consequence, the deflection, the experimental data approach the middle of the gap between the data of calculations according to models (3), (5).

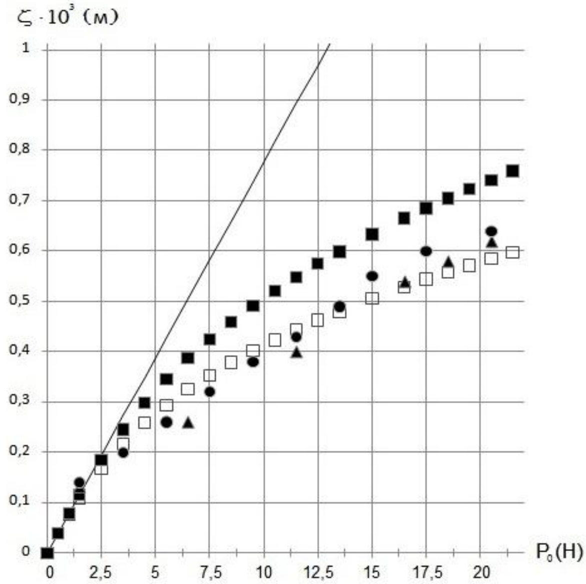


Fig. 3. Dependence of «load – deflection» for an annular plate
under the action of an axisymmetric load, distributed along the inner opening

The study of the deformation of flexible shells of rotation with a complex meridian shape is demonstrated on the example of the problem of numerical and experimental modeling of the behavior of a bellows under the action of longitudinal loading. Fig. 4, a. In some areas, the geometry of the middle surface (Fig. 4, b) can be presented as follows:

– **AB** (plate): $R_a \le s < AB,\ r(s) = s,\ \theta(s) = 0;$

– **BC** (torus): $AB \le s < ABC,\ r(s) = |r_b \sin\theta| + AB,\ \theta(s) = (s - AB)/r_b;$

– **CD** (plate): $ABC \le s < ABCD,\ r(s) = AB - (s - ABC),\ \theta(s) = \pi;$ (16)

– **DE** (torus): $ABCD \le s < ABCDE,\ r(s) = R_a + r_a - |r_a \sin\theta|,\ \theta(s) = \pi - (s - ABCD)/r_a;$

– **EF** (plate): $ABCDE \le s < ABCDEF,\ r(s) = s - ABCDE + R_a + r_a,\ \theta(s) = 0;$

– **FK** (torus): $ABCDEF \le s < ABCDEFK,\ r(s) = AB + |r_b \sin\theta|,\ \theta(s) = (s - ABCDEF)/r_b;$

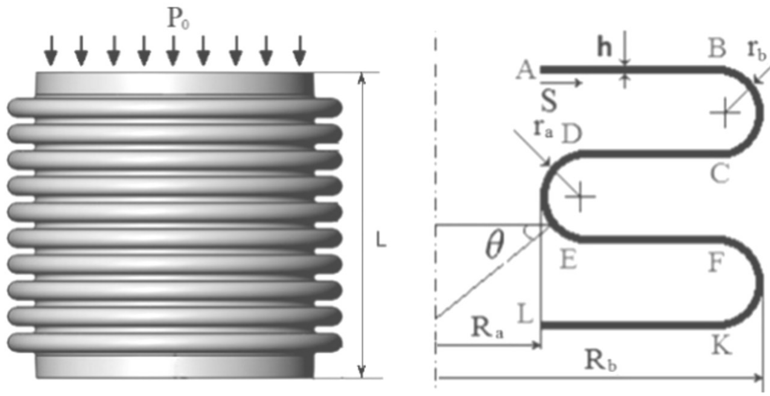
– **KL** (plate): $ABCDEFK \le s < ABCDEFKL,\ r(s) = AB - (s - ABCDEFK),\ \theta(s) = \pi,$

where s is the distance of the point along the meridian from point A; r_a, r_b are the inner and outer radii of tori, respectively; R_a, R_b are the inner and outer radii of the bellows; h is the thickness of its wall.

When integrating the boundary value problem (1), (2) taking into account (9), (10) and (16), which describes the behavior of the bellows, each of the sections (16) was divided in such a way that the nodal point was on the border of the transition from the section to sections, and the connection between conditional sections was considered mechanically ideal.

The boundary conditions were as follows: the lower end was considered rigidly clamped (4,a), and the upper end was considered to be loaded by the longitudinal compressive force P_0 (4, c) uniformly distributed along the upper edge.

A special test setup was developed for the experimental study of the axial movements of the bellows under the action of axial compression (Fig. 5).



(a) (b)

Fig. 4. Bellows geometry

Axial displacement was measured with a clock-type indicator with division price is 0.01 mm, and the load was carried out according to the «dead» load scheme. The physical parameters of the bellows were as follows: height, inner and outer radii of the bellows – $L = 0.23\times10^{-3}$m, $R_a = 37.5\times10^{-3}$m, $R_b = 45\times10^{-3}$ m, respectively; the diameter of the inner and outer torus – $r_a = 1\times10^{-3}$ m, $r_a = 2.5\times10^{-3}$ m, respectively; material – 12X18H9T (stainless steel) with appropriate physical characteristics.

Fig. 5. Experimental installation

The results of calculating the deformation parameters (axial displacements) of the bellows were obtained using the developed approach, which was implemented in the form of the author's software package in the PGI Visual Fortran language and is presented in the table. 1. The calculation results obtained using the ABAQUS 6.12-1 / CAE and the results of experimental tests are also given there.

Table 1

Dependence of the bellows length change on the axial load

Load P_0, N / Axial movement, $\zeta\times10^3$, m	**9,8**	**19,6**	**29,4**	**32,3**	**33,3**	**39.2**
Visual Fortran	2.10	4.10	6.40	7	7.20	10.2
ABAQUS / CAE	2.18	4.36	6.54	7.19	7.41	10.0
experiment	2.00	4.00	6.00	7.00	7.50	11.0

The obtained results of numerical calculations and experimental studies demonstrate the reliability and efficiency of the application of the developed algorithm for the calculation of envelopes of rotation with a complex meridian shape at large displacements. The application of the convergence acceleration algorithm (12) – (14) allows you to significantly reduce the number of direct calculations by up to 1.5÷1.8 times (and several times with certain input parameters of the problem) compared to known algorithms for direct sequential integration of linearized boundary value problems shell mechanics.

5. Corrugated membrane of the highest sensitivity. The sensitivity of the measuring device, which contains a membrane, in particular a sinusoidal profile (Fig. 6), is determined by the angle of inclination of the characteristic curve, which expresses the dependence between the deflection in the center and the pressure on the membrane. The elastic characteristic of the corrugated membrane, among other parameters, also depends on the shape of the middle surface of the shell [5], which forms the membrane. At the same time, the sensitivity of the membrane increases as the corrugation depth decreases, so that the thin smooth membrane (plate) has the highest sensitivity among round membranes, the behavior of which becomes significantly nonlinear even at low loads (Fig. 2). This is a disadvantage in the practical application of membranes as a sensitive element, as it reduces the range of measuring pressures on a scale with a uniform division price.

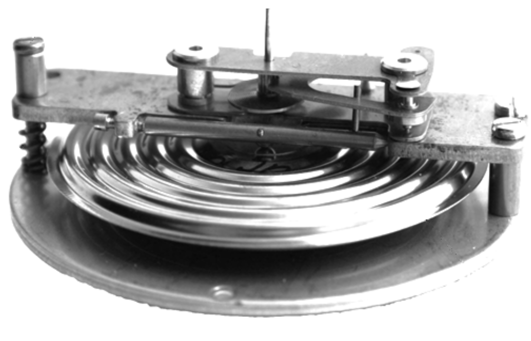


Fig. 6. Corrugated membrane with transmission mechanism

Creating a membrane of the desired sensitivity and with a wide range of measurable (within the linear scale) pressures is possible by choosing a rational depth of its corrugation. The variational problem of optimizing the membrane of the highest sensitivity consists in maximizing its deflection in the center according to the 5th equation in (3)

$$\varsigma(r_a)=-\int_{r_a}^{r_b}\Bigg(-\frac{\mu}{r}\sin\left(\theta+\vartheta_r\right)u+\vartheta_r\cos\theta+\frac{1}{2Kr}\sin\left(2\left(\theta+\vartheta_r\right)\right)\left(N_r r\right)+$$

$$+\frac{1}{Kr}\sin^2\left(\theta+\vartheta_r\right)\frac{F(s)}{2\pi}+\sin\left(\theta+\vartheta_r\right)-\sin\theta-\vartheta_r\cos\theta\Bigg)\cos\theta dr, \quad (17)$$

when limiting the length of the meridian and, thus, the depth of corrugation of the membrane, in the form

$$L=\int_{r_a}^{r_b}\sqrt{1+y'^2}\,dr=\int_{r_a}^{r_b}\sqrt{1+tg^2\theta}\,dr \quad (18)$$

and availability of strength requirements

$$\max_{z}\sigma_i\le[\sigma], \quad (19)$$

where $\sigma_i=\sqrt{\sigma_r^2-\sigma_r\sigma_\varphi+\sigma_\varphi^2+3\tau_{rz}^2}$.

In the paper, the profile of the membrane is given in the form $y(r)=A(r)\sin\left(\omega(r-r_a)\right)$; $A(r),\omega$ is depth and frequency of corrugation, respectively; r_a is the radius of the rigid center, r_b is the outer radius of the membrane; h is membrane wall thickness; $\sigma_r,\sigma_\varphi,\tau_{rz}$ are normal (radial and annular) and tangential stresses. It should also be noted that in equations (3) it is convenient to switch to integration over the variable r (radius of the membrane) taking $ds=dr\cos\varphi$. It is also assumed that the greatest stresses occur on the surface of the shell $z = h/2$.

The task of designing the membrane of the greatest sensitivity consists in finding the optimal control from the condition of maximum deflection of the rigid center (17) in the presence of restrictions (18), (19). The solution of the problem is carried out on the basis of the necessary conditions of optimality in the form of L. S. Pontryagin’s maximum principle. As is known [20, 21], the task in this case is to find the optimal control $A(r)$ from the conditions of the maximum of the Hamiltonian

$$H\left(\bar{z}(s),\bar{\lambda}(s),A(s),s\right)=\sum_{j=0}^{5}\lambda_j\psi_j+\delta(x)\left(\sigma(x)-[\sigma]\right)+c\sqrt{1+tg^2\theta}, \quad (20)$$

where $\psi_j\ (j=\overline{1,5})$ are the right-hand parts of the equations of the mathematical model (3) of the deformation of the corrugated sinusoidal membrane; ψ_0 is integral expression of the objective function (17); $\delta(s)$ is Lagrange function, ($\delta(s)=0$ *when* $\sigma(s)<[\sigma]$; $\delta(s)\ne 0$, *when* $\sigma(s)=[\sigma]$); c is the Lagrange multiplier, which is found from the conditions of execution (18); $\lambda_0=-1$; $\lambda_i(s)(i=\overline{1,5})$ are conjugate functions found by solving the boundary value problem

$$\frac{d\lambda_j}{ds} = -\frac{\partial H}{\partial z_j} \quad (21)$$

with boundary conditions of transversality (2)

$$\lambda_j\left(\bar{z}(s_p)\right) = \sum_j c_j qrad\, f_j\left(\bar{z}(s_p)\right), \quad (22)$$

where $s_p = s_{r_a} \vee s_{r_b}$.

The algorithm for satisfying the necessary optimality conditions of the maximum principle in the presence of constraints (19), containing control and phase variables, and integral conditions (18) is based on the scheme of successive approximations $A(r)^{k-1} \to \bar{z}^k \to \bar{\lambda}^k \to \sup_{A(r)} H \to A(r)^k$ [22].

Here, the calculation $\bar{z}^k$, $\bar{\lambda}^k$ with a given initial approximation $A(r)^0$ is carried out by successive integration of the corresponding boundary value problems (3), (4) and (21), (22). Attention should be paid to the peculiarities of solving the conjugate system (21), the right-hand parts of which may differ in some sections $r_a \le r \le r_b$ by the presence or absence of the Lagrange multiplier $\delta(s)$ associated with the fulfillment of constraint (19). The optimal control of the next step is sought from the maximum condition (20) by solving a number of auxiliary nonlinear programming problems for fixed (nodal) points r_i of the integration interval $[r_a, r_b]$.

To determine the control of the next step of approximations from the point of view of acceleration of convergence (as shown by the results of numerical simulations obtained when solving specific problems), it is advisable to use the relations

$$A^{k+1}(r_i) = A^k(r_i) + \left(F\left(\bar{z}^k, \bar{\lambda}^k, A^k, r_i\right) - A^k(r_i)\right) \cdot \bar{\gamma}^T, \quad (23)$$

where $F\left(\bar{z}^k, \bar{\lambda}^k, A^k, r_i\right)$ is the operator matching the control A^k with a new control A^* value that satisfies the conditions of the (k+1)-th step maximum, and the relaxing components of the vector $\bar{\gamma}$ ($0 < \gamma_j \le 1$) are taken from the requirements of the best convergence of the iterative process. An analysis of the algorithmic processes of satisfying the necessary optimality conditions of the Pontryagin's maximum principle is given in [22].

Numerical results were obtained for a membrane with seven corrugations under the action of uniform transverse pressure and the following parameters: $l = 1.8 \cdot 10^{-3}$ m; $h = 0.2 \cdot 10^{-3}$ m; $R = 25 \cdot 10^{-3}$ m; $E = 100$ MPa/m^2.

The application of the proposed approaches to the linearization of the nonlinear boundary value problem and the convergence acceleration algorithm made it possible to obtain a convergent process of solving the nonlinear problem of membrane deformation in the form of a sequence of linearized boundary value problems in 8 iterations (Fig. 7), where the results of the calculation of the membrane deflection of a sinusoidal profile under the action of uniform pressure are presented in the form of an undeformed profile (line 1) and membrane deflections (lines 2 – 4, 8) at individual steps of the iterative process. For comparison, it should be noted that the simple iteration method converged in 91 iterations.

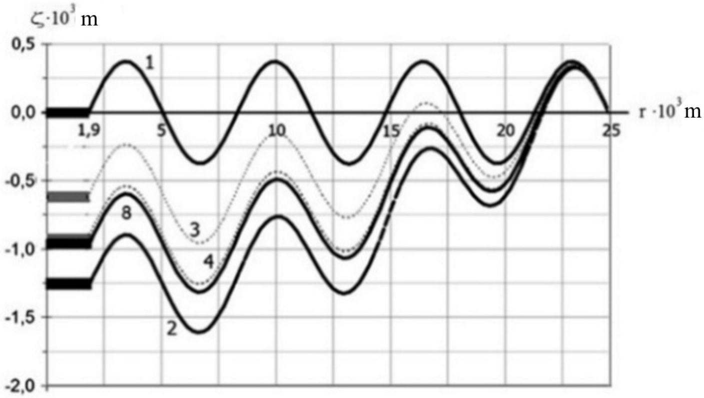


Fig. 7. Graph of membrane deflection at individual steps of the iterative process

The membrane profile of the optimal shape is shown in Fig. 8, where $H_1 = 0.47{\cdot}10^{-3}$ m; $H_2 = 1.1{\cdot}10^{-3}$ m. (In the general case, for the membrane of the highest sensitivity, H_1/H_2 varies in the range of 0.44÷0.75).

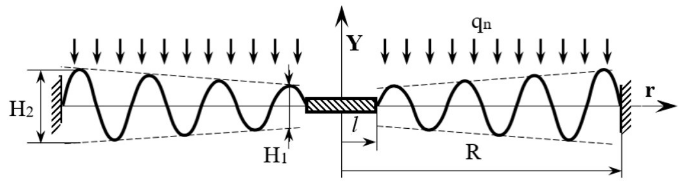


Fig. 8. The optimal shape of the corrugated membrane of the greatest sensitivity

The elastic characteristics of such a membrane are shown in Fig. 9. Here, for comparison, the characteristics of the designed membrane (line 1) and two membranes of permanent corrugation are shown, one of which has the same length ($OA = OA'$) of the linear section of the «load – deflection» characteristic as for the optimal one (line 2), and the second (line 3) has the same sensitivity as the optimal profile membrane.

The obtained results show that the sensitivity of the proposed membrane is 2.1 times greater than the sensitivity of the membrane $t_2 = \tan\alpha_2$ with a constant depth of corrugation, and in the case of the same sensitivity $t_1 = t_2$, the length OA of the linear section of the elastic characteristic of the proposed membrane is 12% longer than the corresponding section OA'' for the membrane with a constant depth of corrugation.

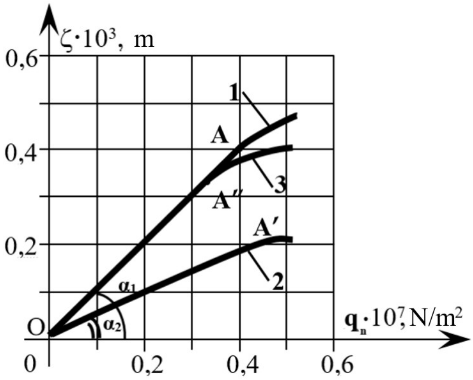


Fig. 9. Elastic characteristics of membranes

Conclusions

Techniques for effective linearization of nonlinear boundary-value problems of the mechanics of flexible shells of rotation with a complex meridian shape and algorithms for accelerating the convergence of iterative processes for their solution based on extrapolation (forecasting) of the solutions obtained in the previous approximation steps and their application to the study of the behavior of flexible ring plates, bellows and corrugated membranes, demonstrating the effectiveness of the proposed approach.

The results of the numerical analysis obtained from the application of the described approach for two mathematical models of deformation of flexible shell elements are presented. Their comparative analysis with the data of the conducted experimental tests is presented, indicating the sufficient adequacy of the selected models. The results of the calculations according to which there are deviations from the experimental data of up to 5% in both directions.

The results of the optimization of the corrugated membrane of the sinusoidal profile were obtained. They demonstrate advantages in its sensitivity more than 2 times, in the range of the linear scale of measurements.

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Дзюба А.П., Сафронова І.А., Левитіна Л.Д.

ЧИСЛОВЕ ТА ЕКСПЕРИМЕНТАЛЬНЕ МОДЕЛЮВАННЯ ПОВЕДІНКИ ГНУЧКИХ ОБОЛОНКОВИХ ЕЛЕМЕНТІВ КОНСТРУКЦІЙ

Для задач осесиметричного нелінійного згинання тонких оболонок обертання зі складною формою меридіану (тонких кільцевих пластин, гофрованих мембран синусоїдального профілю та сильфонів) проведено порівняльний аналіз застосування двох математичних моделей деформування гнучких оболонкових елементів. Числові результати отримані шляхом безпосереднього інтегрування крайових задач механіки оболонок, методом скінчених елементів та експериментальних досліджень. Результати оптимального проектування гнучкої гофрованої мембрани синусоїдального профілю найбільшої чутливості отримані з використанням необхідних умов оптимальності принципу максимуму Л. С. Понтрягіна. Результати подані у вигляді таблиць, фотографій та графіків.

Ключові слова: гнучкі оболонки обертання, гофровані мембрани, сильфони, числовий аналіз, експериментальні дослідження.

Dzyuba A.P., Safronova I.A., Levitina L.D.

NUMERICAL AND EXPERIMENTAL MODELING OF THE BEHAVIOR OF FLEXIBLE SHELL ELEMENTS OF STRUCTURES

This article is devoted to computer and experimental modeling of the behavior of flexible elements of shell structures, and the development of effective algorithms for solving emerging nonlinear boundary value problems of their calculation and optimization of parameters. At the same time, the probability of the results obtained using different approaches to the construction of a nonlinear theory is established. Their comparative analysis, error estimation, which in this case is given by calculation according to linear and corresponding nonlinear theories, is carried out. The results of calculated data and experimental studies of the behavior of real structural elements are compared.

The results of a comparative analysis of the application of two mathematical models of deformation of flexible shell elements, obtained by direct integration of boundary value problems of shell mechanics, by the finite element method and experimental research, are presented.

The problems of axisymmetric nonlinear bending of thin ring plates, corrugated membranes of a sinusoidal profile and bellows as a shell of rotation with a complex meridian shape are considered.

Using the necessary optimality conditions of the principle of maximum L. S. Pontryagin obtained the results of the optimal design of a flexible corrugated membrane with a sinusoidal profile of the highest sensitivity. The results are presented in the form of tables, photos and graphs.

Keywords: flexible shells of rotation, corrugated membranes, bellows, numerical analysis, experimental studies.

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Дзюба А.П., Сафронова І.А., Левитіна Л.Д. **Числове та експериментальне моделювання поведінки гнучких оболонкових елементів конструкцій** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2023. – Вип. 110. – С. 3-20. – Англ.
Подано результати експериментальних випробувань та числового порівняльного аналізу поведінки гнучких елементів оболонкових конструкцій (сильфонів, гофрованих мембран синусоїдального профілю та ін.) з оптимальними параметрами.
Табл. 1, Іл. 9. Бібліогр. 22 назв.

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Dzyuba A.P., Safronova I.A., Levitina L.D. **Numerical and experimental modeling of the behavior of flexible shell elements of structures** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUCA, 2023. – Issue 110. – P. 3-20.
The results of experimental tests and numerical comparative analysis of the behavior of flexible elements of shell structures (bellows, corrugated membranes of a sinusoidal profile, etc.) with optimal parameters are presented.
Tabl. 1. Fig. 9. Ref. 22.

Автор (вчена ступень, вчене звання, посада): доктор технічних наук, професор, професор кафедри теоретичної та комп'ютерної механіки ДЗЮБА Анатолій Петрович
Адреса робоча: 49010 Україна, м. Дніпро, проспект Гагаріна 72, Дніпровський Національний університет імені Олеся Гончара.
Робочий тел.: +38(056) 745-00-85.
Імейл: dzb@ua.fm
ORCID ID: 0000-0001-6331-7783

Автор (вчена ступень, вчене звання, посада): кандидат фізико-математичних наук, доцент кафедри комп'ютерних технологій САФРОНОВА Інга Анатоліївна
Адреса робоча: 49010 Україна, м. Дніпро, проспект Гагаріна 72, Дніпровський Національний університет імені Олеся Гончара.
Робочий тел.: +38(0562) 46-10-47.
Імейл: safronovainga12@gmail.com
ORCID ID: 0000-0003-0242-3798

Автор (вчена ступень, вчене звання, посада): завідувач навчальної лабораторії обчислювальної механіки і міцності конструкцій кафедри теоретичної та комп'ютерної механіки ЛЕВИТІНА Лариса Данилівна
Адреса робоча: 49010 Україна, м. Дніпро, проспект Гагаріна 72, Дніпровський Національний університет імені Олеся Гончара.
Робочий тел.: +38(056) 745-00-85.
Імейл: ldlora@i.ua