UDC 539.3

A METHOD FOR ANALYSIS OF NONLINEAR DEFORMATION, BUCKLING, AND VIBRATIONS OF THIN ELASTIC SHELLS WITH INHOMOGENEOUS STRUCTURES

O.P. Krivenko,
Candidate of Science (Engineering), Senior Researcher

P.P. Lizunov,
Doctor of Science (Engineering), Professor

Yu.V. Vorona,
Candidate of Science (Engineering), Associate Professor

O.B. Kalashnikov,
Applicant for Candidate of Science (Engineering)

Kyiv National University of Construction and Architecture
31, Povitroflotsky ave., Kyiv, Ukraine, 03037

DOI: 10.32347/2410-2547.2023.110.131-149

The formulation of the problem and the method of analysis of the stress-strain state, buckling and vibrations of elastic shells with inhomogeneous structure are considered. The modal analysis of the shells is performed at each stage of loading. The method allows one to study the behavior of shells with a complex shape of the middle surface, geometric features throughout the thickness, and a multilayer material structure under thermomechanical loading. We approximate a thin shell with one finite element (FE) over the entire thickness. At the same time, we use spatial FEs of the same type to model shell portions with stepwise-varying thickness. So we apply the universal finite element. It is based on an isoparametric 3D element with polylinear shape functions for coordinates and displacements and has additional parameters. The universal finite element can be transformed (modified) to accurately describe portions of the shell with stepped-variable thickness. This element can be eccentrically displaced relative to the average surface of the shell and change its own thickness. The side edges of neighboring FEs are in continuous contact, and the FE allows simulating sharp bends of the shell. The approach is modern and easy to implement, since it is based on the use of the relations of the three-dimensional geometrically nonlinear theory of thermoelasticity and the application of the moment finite-element scheme. The effectiveness of the method is demonstrated on classical test examples. The convergence, accuracy and reliability of the obtained solutions are investigated. Comparison of the results of calculations obtained by the moment finite-element scheme with the data of other authors shows a good agreement between the solutions.

Keywords: inhomogeneous shell, geometrically nonlinear deformation, buckling, modal analysis, thermomechanical loading, universal 3D finite element, moment finite-element scheme.

Introduction. The analysis of the processes of nonlinear deformation, buckling, post-buckling behavior, and vibrations are the most important part in a research of the behavior of a thin elastic shell under the action of a static thermo-mechanical load. Particular attention to these problems is explained by the fact that the process of buckling and further deformation of the shell usually exhausts its load-bearing capacity. And this, in turn, can lead to catastrophic consequences. In addition, it is necessary to know the conditions of strength and reliability of a thin-walled structure for the safe operation. In this regard, methods for analyzing their dynamic characteristics are of particular importance, since the natural frequencies and vibration modes of the shell are one of the main dynamic characteristics of any elastic system.

Usually, the elements of real thin-walled structures are shells, which geometry has a non-canonical shape. As a rule, real shells are characterized by a complex shape outline. They are made inhomogeneous throughout the thickness in order to enhance reliability and reduce materials consumption. They can be constant, linear-varying or piecewise-varying thickness, have facets, ribs, cover plates, holes, cavities, channels, layers. Such shells are usually called: inhomogeneous, shells of inhomogeneous structure, shells of inhomogeneous rigidity, and shells with variable parameters in thickness [1]. During operation, the considered shells can be under the action of a static thermo-mechanical load.

Various methods of numerical analysis are used to investigate the stress-strain state (SSS), buckling and vibrations of shell structures [1-12]. Over the last decades, the finite element method has been especially widely used in solving the problems under consideration and is, in fact, the dominant method among the methods used. However, most studies are devoted to the analysis of the behavior of shells of a certain class under simple types of loading, mainly mechanical.

This work is devoted to expanding the capabilities of the existing method for studying the stress-strain state and stability of shells to the problems of modal analysis, taking into account static loading. The method of modal analysis of the structure at each step of thermomechanical loading, on the one hand, is focused on obtaining deeper knowledge about the ongoing processes and, on the other hand, makes it possible to determine the moment of buckling of the shell simultaneously by two criteria: static and dynamic.

The method allows considering a wide class of shells both in terms of geometric characteristics and types of load. An approach is used that makes it possible, within the framework of one finite element, to simulate the behavior of shells of a complex inhomogeneous structure, which are under the action of mechanical, thermal and thermomechanical loads. The approach is modern and easy to implement, since it is based on the use of the relations of the three-dimensional geometrically nonlinear theory of thermoelasticity [13] and the application of the moment finite-element scheme [10, 14].

1. A problem statement. An integrated approach is used to solve the problem. It consists of two stages for each step of thermomechanical loading. At the first stage, the stress-strain state of the shell is determined [14]. At the second stage, a modal analysis of the shell is performed taking into account the deformed and prestressed state [1, 15, 16]. This approach allows to trace how the modal characteristics of the shell change during its loading. In addition, it becomes possible to determine the moment of buckling of the shell simultaneously by two criteria: static and dynamic.

In general, the solution of nonlinear shell stability problems is often a process of obtaining results that are difficult to predict in advance. They depend on the influence of various system parameters, such as geometry, boundary conditions, type of load, material, presence of various structural elements, etc. These factors require a more thorough construction of universal algorithms and a more detailed analysis of the process of obtaining solutions for each problem. Therefore, the analysis of the bifurcation points of the "load-deflection" curves and the modal analysis of the shell during its loading come to the fore.

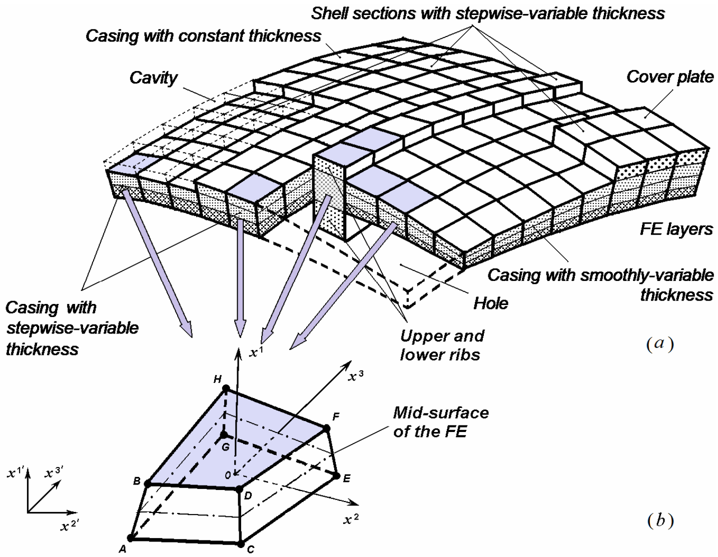


Fig. 1. Finite element fragment of an inhomogeneous shell

Elastic shells of thin and medium thickness are considered. In general, the shell has a complex shape of the middle surface, smooth and stepped-variable thickness, holes, faceting, multilayer material structure and other features in thickness, as well as inhomogeneous conditions for fixing the contour. The shell is under the action of thermomechanical load. An example of a fragment of such a shell is shown in Fig. 1.

The finite element method for solving static problems of nonlinear deformation, buckling and postbuckling behavior of shells under the action of thermomechanical loads has been created on the basis of the unified methodological positions of the three-dimensional geometrically nonlinear theory of thermoelasticity. A detailed description of the method, substantiation of its reliability, solution of a number of problems are presented in [1, 14, 17-22].

The casing of the shell and the ribs reinforcing it can consist of an arbitrary number of layers of varying thickness bonded into a single piece. The layers of the shell are considered linear elastic and described by the generalized Duhamel–Neumann law. The shell is modeled by a nonlinear elastic continuum subject to large displacements and small strains whose components are linear functions of stresses. We consider a steady-state thermal process in which the temperature field in the shell is a known function of coordinates, $T = T\left(x^{i'}\right)$, independent of the stress-strain state.

Thin inhomogeneous shells of variable thickness and complex geometry (Fig. 1 (a)) are considered as a three-dimensional bodies. To develop a finite element shell model (FESM), we approximate a thin shell by one spatial finite element (FE) throughout the thickness, which is an efficient approach [14].

Two hypotheses are used to describe the SSS of the shell: (i) the non-classical kinematic hypothesis of a deformed straight line, which makes it possible to join spatial FEs keeping compatibility of the coordinates and displacements and to naturally model sharp bends; (ii) the static hypothesis, the use of which the stress state of the shell does not deprive the three-dimensional properties. According to the kinematic hypothesis, a straight segment along the thickness remains straight even after deformation. This segment is not necessarily normal to the mid-surface of the shell. The static hypothesis assumes that the compressive stresses σ_n^{11} in the fibers of the n th layer are constant throughout the thickness (along the x^1 -axis).

Spatial discretization of an inhomogeneous shell is implemented using a universal spatial finite element (Fig. 1 (b)), which models shell portions with different types of structural features. The universal FE is based on the well-known ("standard") 8-node isoparametric spatial FE with polylinear shape functions for coordinates and displacements [14]. We will call it the casing finite element (CFE) (Fig. 2 (a)). The casing of the shell is understood as a shell without geometric features in thickness. The spatial CFE can be transformed into a modified finite element (MFE) by introducing additional variable parameters. These parameters expand the capabilities of the universal FE. The element can be eccentrically displaced relative to the mid-surface of the casing and can change its dimensions in the thickness direction to model ribs (MFE^+; Fig. 2 (c)) and cavities (MFE^-; Fig. 2 (d)).

A unified computational model has been developed that takes into account various features of the structural elements of an inhomogeneous shell based on the universal 3D FE. According to the adopted approach, the investigation of the behavior of shell structures of various types is implemented within the framework of a unified methodology [14]. The moment finite-element scheme (MFES) is used to derive the governing finite-element equations for displacements [10, 14].

The geometrically nonlinear deformation of shells is analyzed using the incremental method based on the general Lagrangian formulation. According to this approach, the frequencies and forms of natural vibrations of the shell are determined at the moments of stepped thermomechanical loading. At each step, the new deformed state of the shell and the stresses accumulated in the previous steps are taken into account. Thus, in the problems of the shell vibrations, the presence of prestressing from the action of various static thermomechanical loads is taken into account.

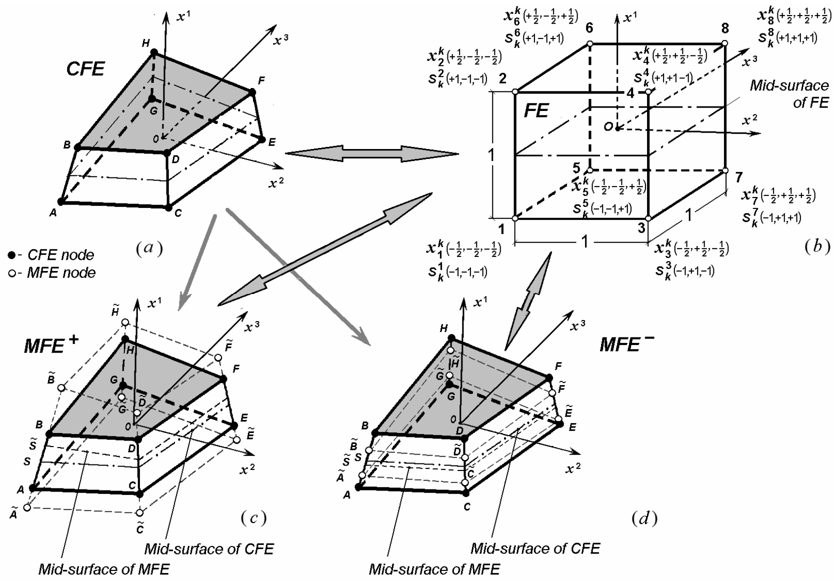


Fig. 2. Universal 3D finite element

The problem of nonlinear deformation, buckling, and postbuckling behavior of inhomogeneous shells is solved by a combined algorithm that employs the parameter continuation method, a modified Newton–Kantorovich method, and a procedure for automatic correction of algorithm parameters. An algorithm based on these principles makes it possible to obtain the dependence represented by a load-deflection curve, regardless of its shape and complexity. The subspace iteration method is used to determine the low-frequency spectrum and eigenvectors of shells of an inhomogeneous structure.

2. Algorithm for a comprehensive investigation of the buckling and vibrations of the shells. The algorithm of the incremental (step-by-step) method for solving problems of buckling and vibrations of the shells is shown in Fig. 3. According to this approach, each step corresponds to an increment in the parameter of external loads P. The solution of the static problem of nonlinear stability is the found relationship between the parameter of external load P and the displacement field U of the FESM. This relationship is determined at each step of increasing the generalized load ΔP and is usually represented by a "load-deflection" ($P-U$) curve at characteristic points of the shell. The effect of the mechanical Q and thermal T fields on the shell is considered as a single process of loading described by a relationship between the general load parameter $P = P(Q,T)$ and the parameters of mechanical and temperature fields.

The technique for solving the problems of nonlinear deformation, buckling and vibration of elastic shells is a two-stage algorithm implemented at each step k of the load increment. At the first stage, the static problem of nonlinear deformation of the shell is solved. The SSS of the shell is determined for the corresponding load increments: its deformed shape (new coordinates) and increments of displacement and stress fields.

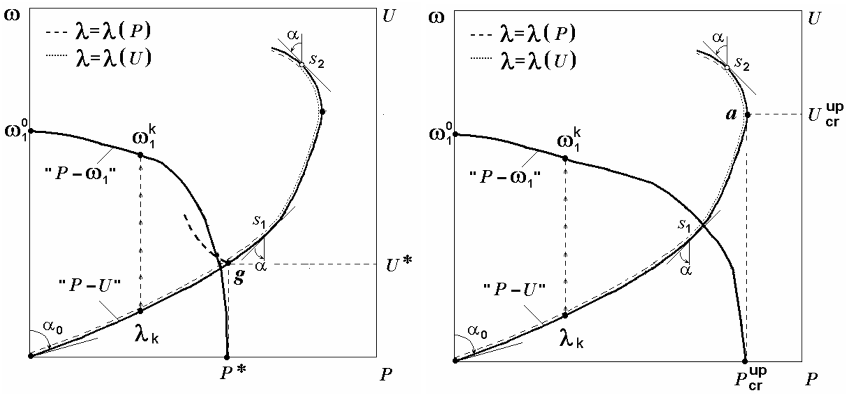
To perform modal analysis, files are generated with the necessary information: the geometry (field of coordinates) of the shell at the beginning of the step $\{x_{k-1}^{i'}\}$; new coordinate field $\{x_k^{i'}\} = \{x_{k-1}^{i'}\} + \{u_k^{i'}\}$ after deformation from the applied load ΔP_k; stresses $\{\sigma_k^{ij}\} = \{\sigma_{k-1}^{ij}\} + \{\Delta\sigma_k^{ij}\}$ in the elements of the finite element shell model.

The second step of the current stage uses the new shell shape and its prestressed state, which have been determined in the first step. The load is assumed to be zero (that is, it is removed) and a modal analysis is carried out [1, 15, 16]. Small vibrations of the shell relative to the static equilibrium position are considered. In accordance with this, the natural frequencies and vibration modes of the shell are determined for each moment of the load increment. The input is the number of frequencies to be determined. The modal analysis of the shell is performed down to zero (or negative) value of the lowest frequency ω_1. The equality of the eigenfrequency to zero is a dynamic criterion for the loss of stability of the structure. As is known, under loads reaching critical values, the deformed system loses its ability to perform oscillatory motions [23]. The resulting load value can be taken as critical. At subsequent stages of the load increment, the modal analysis of the shell is not carried out, and only its postbuckling behavior is investigated.

The problem of nonlinear deformation, buckling, and postbuckling behavior of inhomogeneous shells is solved by a combined algorithm that employs the parameter continuation method, a modified Newton–Kantorovich method, and a procedure for automatic correction of algorithm parameters. Either the parameter of the field of external nodal loads $P = P(Q,T)$, or the displacement U of the characteristic node selected by the algorithm can be used as the continuation parameter $\lambda_k = \lambda_k(P,U)$ at each moment k of load increment. The node of the entire finite element shell model for which the increment of the modulus of the nodal displacement vector has been the largest in the previous step is characteristic [14].

The natural frequencies ω_i^k (it is ω_1^k in Fig. 3) and the corresponding vibration modes are obtained in the module of the algorithm that implements the modal analysis of the shell. The result of the modal analysis is presented as a "load P – lower frequency ω_1" ($P-\omega_1$) curve. If there is the branching point g on the $P-U$ curve (Fig. 3 (a)), then the value P^* with $\omega_1 = 0$ can be taken as the upper critical load according to the dynamic criterion. If there is no branching point, the point a of the maximum of the $P-U$ curve

(Fig. 3 (b)) corresponds to the upper critical load P_{cr}^{up} both according to the static criterion and to the dynamic one ($\omega_1 = 0$).



(a) (b)

Fig. 3. Combined algorithm for solving the problem of geometrically nonlinear deformation, stability and vibrations of flexible shells

3. Numerical analysis. The effectiveness of the method is demonstrated by solving test problems. The convergence and accuracy of the obtained solutions are investigated.

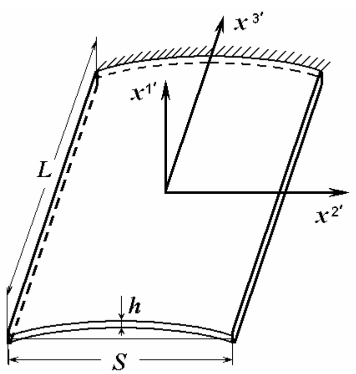


Fig. 4

3.1. The test problem of natural vibrations of a cantilevered cylindrical panel is considered (Fig. 4). The shell is rigidly fixed along a curvilinear contour.

The input data: $E = 0{,}2 \cdot 10^{12}$ N/m^2, $\nu = 0.3$, $\rho = 0{,}704 \cdot 10^4$ kg/m^3; size in plan $L = S = 0.3048$ m, radius of mid-surface $R = 0.6096$ m, thickness $h =$ 0.003048 m.

The analysis (Tab. 1) shows that the solutions converge on the 15×15 FE mesh, and the error with respect to the solution with the 30×30 FE mesh is less than 2%.

Comparing the solution obtained by the MFES with those obtained by using well-known software (SW): SCAD [24], LIRA [25], NASTRAN [26], and by other authors, we conclude that the MFES solutions are in good agreement for all considered frequencies (Tab. 2).

The obtained oscillation modes (Fig. 5) agreement with those obtained using the SCAD and LIRA software. The lower right edge of the forms shown in Fig. 5 is rigidly fixed. The mode shapes are given for the finite element shell model with the 20×20 FE mesh.

Table 1

№ ω_i	FEs						
	10×10, Hz	Δ, %	15×15, Hz	Δ, %	20×20, Hz	Δ, %	30×30, Hz
1	90,229	0,82	89,839	0,38	89,658	0,005	89,494
2	146,08	0,75	145,50	0,35	145,23	0,16	144,99
3	260,86	1,85	257,96	0,72	256,91	0,31	256,11
4	365,73	2,21	361,13	0,92	359,28	0,41	357,82
5	407,09	1,66	403,32	0,72	401,74	0,32	400,45
6	570,28	5,23	552,05	1,86	546,17	0,78	541,92
7	790,99	4,28	772,56	1,83	764,56	0,78	758,65
8	794,19	4,25	773,31	1,51	766,68	0,64	761,78

Table 2

Calculation method	ω_1, Hz	ω_2, Hz	ω_3, Hz	ω_4, Hz	ω_5, Hz
20×20 FE MFES	89,658	145,23	256,91	359,28	401,74
Experiment [27, 28]	85,60	134,50	258,90	350,60	395,20
20×20 FE SCAD	91,0345	146,94	256,02	364,03	404,81
20×20 FE LIRA	89.4434	145.02	256.53	357.67	400.39
20×20 FE NASTRAN	89,2368	144,65	253,44	355,32	395,39
M.A. Bossak, O.C. Zienkiewicz [29]	88,30	142,80	257,60	369,20	441,80

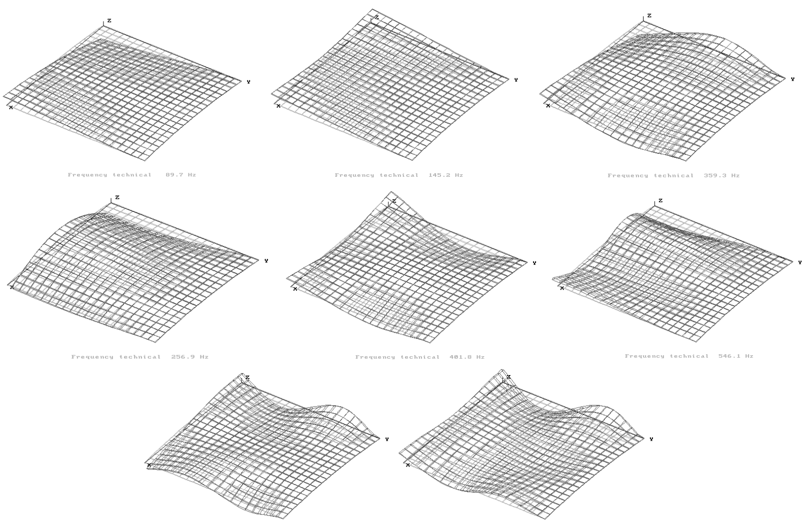


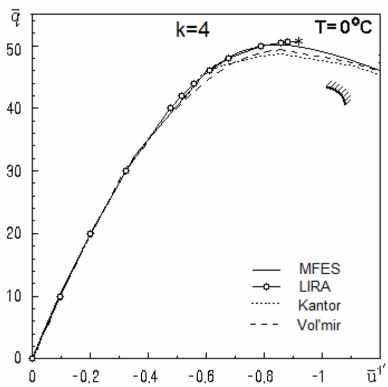
Fig. 5

3.2. A shallow axisymmetric spherical panel of constant thickness is considered. The input data: $E = 19.6 \cdot 10^4$ MPa, $\nu = 0.3$, linear expansion coefficient $\alpha = 0.125 \cdot 10^{-4} \deg^{-1}$; thickness $h = 0.01$ m, radius of support contour $a = 100h$, radius of mid-surface $R = 1252h$, rise $H = 4h$. The results are presented in dimensionless form [7, 14, 23]: $\bar{q} = q/E \cdot (a/h)^4$, $\bar{u}^{1'} = u^{1'}/h$, $\bar{t} = t\alpha/2 \cdot (a/h)^2$, where $u^{1'}$ is the deflection of the panel (along the $x^{1'}$-axis); $k = H/h$ is the panel shallowness parameter. The FESM is a half of the panel uniformly partitioned along radius and circumference.

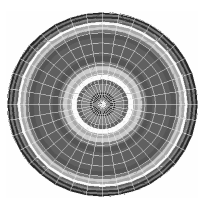
The shell is subjected to the thermomechanical load. The impact of the thermomechanical load on the panel consists of two stages. First, the SSS of the shell is perturbed by the temperature field, and then the panel is subjected to external pressure q, the temperature field remaining constant.

The effect of two factors on the stability of the panel is examined: (i) the type of the contour fastening; (ii) value of preheating. Three cases of uniform heating by T°C ($T = 0, 20, 40^\circ$C) is considered.

The panel is clamped at the edges. This type has an icon () in Figures. A numerical analysis for the nonheated panel ($T = 0^\circ$C) of the convergence of solutions shows that the upper and lower critical loads converge with an error of 3% even on a 10×10 FE mesh [14, 17, 19, 21]. The results agree well with those from [7, 23] and with obtained by the LIRA SW on the entire "load-deflection" ($\bar{q} - \bar{u}$) curve, with insignificant discrepancy for the upper critical load $\bar{q}_{cr}^{up}$ (Fig. 6 (a), Tab. 3). The buckling mode of the panel is characterized by snap-through in the middle (Fig. 6 (b)).



(a)



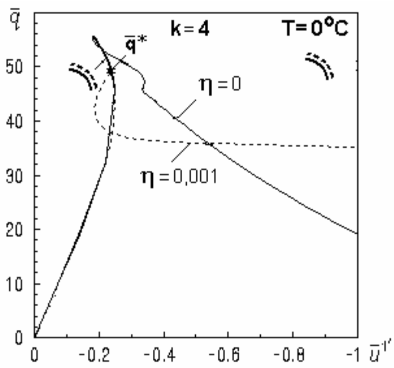
(b)

Fig, 6

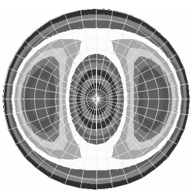
Table 3

Calculation method	$\bar{q}_{cr}^{up}$	Δ, %	$\bar{u}_{cr}^{1'up}$	Δ, %
MFES	50.12	1.46	-0.882	2.56
LIRA	50.70	2.63	-0.920	6.98
Kantor [7]	48.80	-1.21	0.859	-0.12
Vol'mir [23]	49.40	0	-0.860	0

The panel is hinged at the edges ( ). *The nonheated panel* ($T = 0^\circ$C). Comparison of the MFES results with the results obtained by the LIRA SW shows that the $\bar{q} - \bar{u}$ curves (at the center of the panel) are in good agreement in the prebuckling domain (Fig. 7 (a)). The MFES algorithm has allowed to find a branching point ($*$) on the $\bar{q} - \bar{u}$ curve in the prebuckling domain. The solution obtained by the LIRA SW determines the same point ($*$) with insignificant discrepancy (5%). The LIRA SW identifies this point as the upper critical load.



(a)



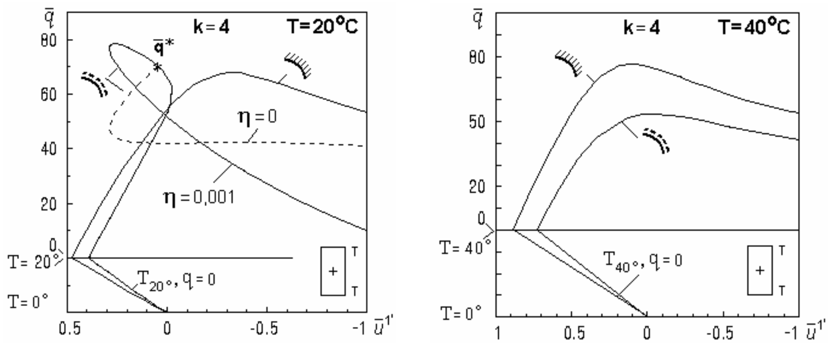
(b)

Fig. 7

The MFES algorithm can find the branching points and allows determining adjacent deformation modes in their neighborhood. To identify a branching point, a qualitative theory is used. It states that at least one negative eigenvalue of the linearized stiffness matrix represents a new equilibrium configuration of the shell. To identify the adjacent deformation mode, the perfect initial mid-surface was perturbed according to the formula $\eta \sin(\pi r/a) \cos(\varphi)$, where $\eta = 0.001$ is a small perturbing parameter, r and φ are polar coordinates. The initial imperfection allows to identify accurately the point ($*$) as a bifurcation point for a perfect panel whose solution has branches. This point ($*$) is considered as critical. The axisymmetric deformation mode transforms into an adjacent nonaxisymmetric buckling mode (Fig. 7 (b)). Fig. 7 (a) shows, by the dotted line, the obtained branch of the solution.

The value of preheating of the shell significantly affects the shape of the $\bar{q} - \bar{u}$ curves, and the type of the contour fastening also affects their character (Figs. 6, 7, 8; Tab. 4). The solution for the panel hinged at the edges when $T = 20^\circ$C (Fig. 7 (a)) also has a branch point in the prebuckling domain, as in the case of a non-heated panel ($T = 0^\circ$C, Fig. 7).

3.3. Consider a shallow spherical panel of square planform with curvature parameter $K = 32$ ($K = 2a^2/(Rh)$; h is the thickness; $R = 225h$ is the radius; $a = 60h$ is the side length of the panel). The shell has a central square hole of width $b = 12h$. The panel is hinged at the edges and subjected to heating and pressure. The input data: $h = 0.01$ m, $E = 20.59 \cdot 10^4$ MPa, $\nu = 0.3$, $\alpha = 0.12 \cdot 10^{-4} \deg^{-1}$.



(a) (b)

Fig. 8

Table 4

Effect of the type of the contour fastening and the value of heating on the $\overline{q}_{cr}^{up}$

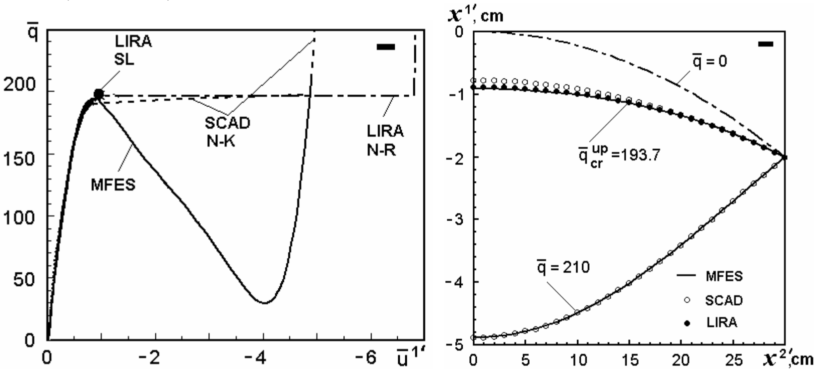
Type of fastening	$T=0^{\circ}$C		$T=20^{\circ}$C		$T=40^{\circ}$C	
	$\overline{q}_{cr}^{up}$	$\Delta(\overline{q}_{cr}^{up})$, %	$\overline{q}_{cr}^{up}$	$\Delta(\overline{q}_{cr}^{up})$, %	$\overline{q}_{cr}^{up}$	$\Delta(\overline{q}_{cr}^{up})$, %
	50,40	–	67,82	–	76,40	–
	55,48 $\overline{q}^{*}$=50,05	10,08 -0,69	78,35 $\overline{q}^{*}$=69,82	15,53 2,95	53,50	-29,97

The effect of three cases preheating by $T=-20^{\circ}$, 0°, 20°C on the stability and vibration of the shell is investigated [14, 17, 19, 20]. The design model has the mesh 40×40 FEs. The results of the investigation of a smooth panel behavior are basic for analyzing the influence of a such geometric feature as a hole on the buckling and natural vibrations of the shell. The results for a panel with the hole are marked by an icon (■::■), and the solutions for a smooth panel are marked by an icon (▬). Deflection is considered at a center of the smooth panel, and at point A for the panel with a hole.

For the smooth shell (▬) the upper critical load $\overline{q}_{cr}^{up}$ obtained using the LIRA SW is in good agreement with that obtained using the MFES (Fig. 9 (a)): for two variants of the method of successive loadings (SL) discrepancy is less than 3%, the Newton–Raphson (N–R) method gives error -1.8% [22]. This problem with the SCAD SW has been solved by the Newton–Kantorovich (N–K) method and by the Newton–Raphson method (errors -4.9%) [22]. Disagreement with the MFES solution is -3.15%. Shapes of the deformed panel for all solutions have a simple form in the prebuckling and postbuckling domains and are in good agreement with each other (Fig. 9 (b)).

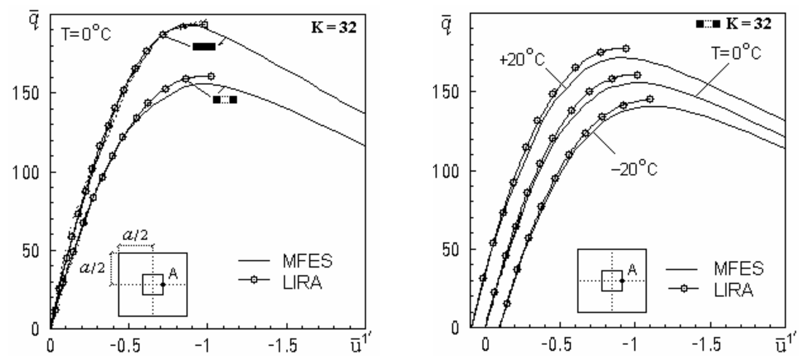
For the panel with the hole (■::■) the MFES results agree well with those obtained by the LIRA SW on the entire $\overline{q}-\overline{u}$ curve, with insignificant discrepancy for the upper critical load $\overline{q}_{cr}^{up}$ (Fig. 10, Tab. 5). The critical load

$\bar{q}_{cr}^{up}$ for a panel with a hole (■⁚⁚■) differs from that for a smooth panel (▬) by 19.2% ($T = 0^{\circ}$C).



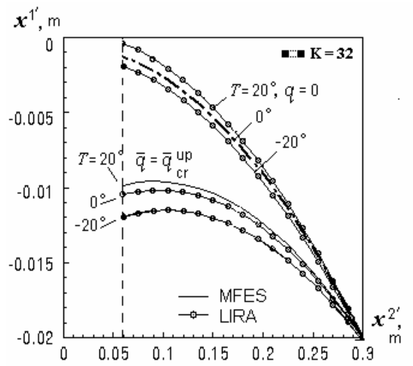
(a) (b)

Fig. 9

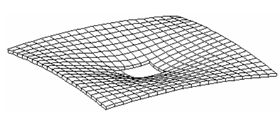


(a) (b)

Fig. 10



(a)



(b)

Fig. 11. Deformation and buckling shapes of the panel

For both methods, we have a complete agreement of the deformation shapes obtained after heating by $T = +20^\circ\text{C}$ and cooling by $T = -20^\circ\text{C}$ (Fig. 11 (a)). The shapes differ little from the initial panel configuration. Buckling occurs through the snapping at the center of the shell (Fig. 11 (b)).

An analysis of the calculations of natural vibrations of smooth (▬) and weakened by a hole (■::■) panels shows that for unloaded shells ($T = 0^\circ\text{C}$, $\bar{q} = 0$) the presence of a hole reduces the natural frequency by 3.3% (Tab. 6). There is a good agreement between the frequencies obtained by the MFES and using the LIRA SW. In this case, if the frequencies ω_1 and ω_2 are multiples for the smooth shell, then the frequencies ω_2 and ω_3 are multiples for the panel with the hole. Therefore, the vibration modes for the corresponding shells are different (Figs. 12, 13). The mode shapes obtained using the LIRA SW are shown in the figures in the form of moiré fringes for clarity.

Table 5

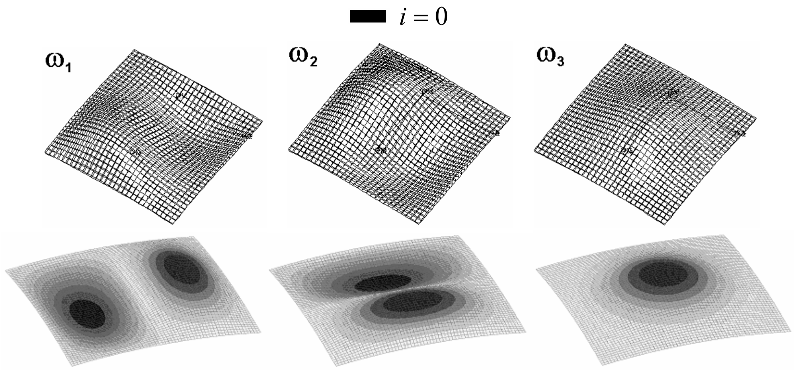
Calculation method	$T = -20^\circ$ C		$T = 0^\circ$ C		$T = +20^\circ$ C	
	$\bar{q}_{cr}^{up}$	$\bar{u}_{cr}^{1'up}$	$\bar{q}_{cr}^{up}$	$\bar{u}_{cr}^{1'up}$	$\bar{q}_{cr}^{up}$	$\bar{u}_{cr}^{1'up}$
MFES	140.83	-1.095	156.41	-0.950	171.55	-0.904
LIRA	145.00	-1.098	161.00	-1.018	177.56	-0.941
Δ, %	3.0	0.3	2.9	7.2	3.5	4.1

Table 6

Panel vibration frequencies ω_i at different load values $\bar{q}^i$ ($T = 0^\circ\text{C}$)

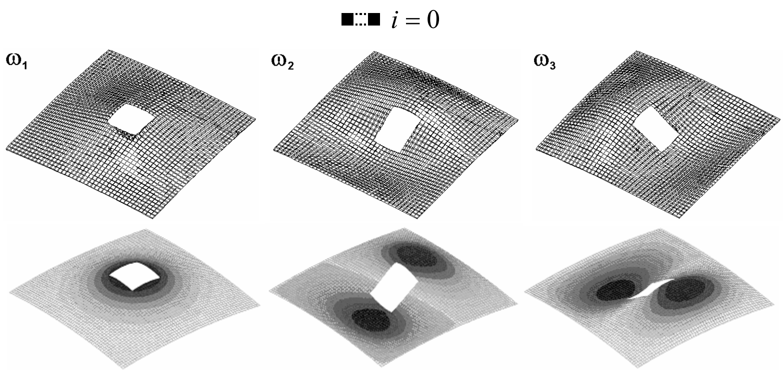
№ $\bar{q}^i$	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
▬ $i = 0$						
MFES	533.78	533.78	547.40	691.24	796.09	816.64
LIRA	529.29	529.29	545.25	682.01	774.27	796.41
■::■ $i = 0$						
MFES	516.04	519.38	519.38	609.26	714.34	819.60
LIRA	515.17	519.63	519.63	610.02	713.51	818.96
1	512.13	512.51	512.51	599.17	703.29	810.49
2	498.05	498.05	500.57	581.03	685.30	793.48
3	474.53	474.53	481.76	551.37	656.03	766.20
4	433.68	433.68	449.05	499.43	605.37	720.19
5	348.14	348.14	379.47	387.73	499.78	631.41
6	275.18	275.18	292.54	332.49	455.78	584.81
7	245.66	245.66	250.91	319.14	443.42	574.59
8	181.04	191.33	191.33	285.32	410.71	555.81
9	144.46	165.03	165.03	271.48	396.84	549.46
10	85.034	132.49	132.51	264.27	387.24	548.44
11	-31833.	93.008	93.082	260.60	379.89	550.59

Preheating and pre-cooling of a shell with a hole ($\overline{q}=0$) lead to insignificant changes in frequencies (Tab. 7). In the case of pre-cooling ($T=-20^{\circ}\mathrm{C}$), the first three vibration modes are similar to those shown in Fig. 13. In the case of preheating ($T=+20^{\circ}\mathrm{C}$), the shape modes are similar to the corresponding shapes (b), (c), (a). The obtained results are in good agreement with the data obtained with the help of LIRA SW.



(a) (b) (c)

Fig. 12



(a) (b) (c)

Fig. 13

For smooth and perforated panels, the "$\overline{q}-\omega_1$" curves have the same shape when only pressure ($T=0^{\circ}\mathrm{C}$) is applied (Fig. 14). Loading moments $\overline{q}^{\,i}$ ($i=\overline{0,11}$) (Tab. 6), at which the modal analysis has been carried out, are marked in the figure by circles. The applied pressure causes both the restructuring of the frequency multiplicity and the corresponding transformation of the mode shapes. Vibrations for ω_1 of both shells (smooth and with a hole) occur in their central part (Figs. 12,c; 13,a) when a load approaching the critical one $\bar{q}_{cr}^{up}$.

Table 7

Vibration frequencies for a heated shell ("■::■", $\bar{q}$ =0)

T°C	ω_1	ω_2	ω_3	ω_4	ω_5
0	**516.04**	**519.38**	**519.38**	**609.26**	**714.34**
+20	530.17	530.28	531.51	616.65	716.62
Δ_0^{+20}	+2.37	+2.10	+2.33	+1.21	+0.32
-20	499.77	508.29	508.38	602.11	712.05
Δ_0^{-20}	-3.15	-2.13	-2.12	-1.17	-0.32

The "$\bar{q}-\omega_1$" curves are similar for all cases of shell preheating (Fig. 15).

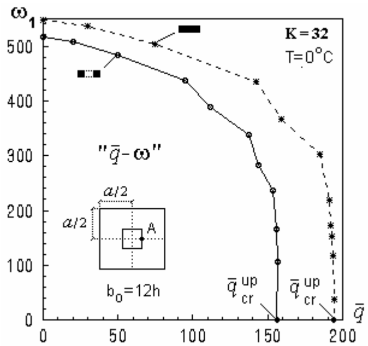


Fig. 14

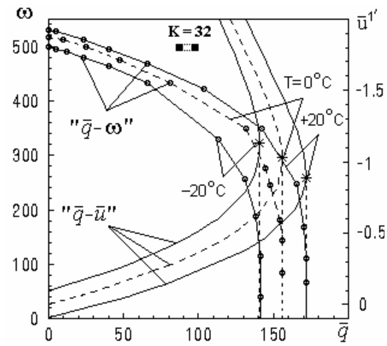


Fig. 15

Conclusions. A complex method for investigating geometrically nonlinear deformation, buckling, and vibrations of thin inhomogeneous shells under the action of static thermomechanical loads is considered. The method is based on geometrically nonlinear relations of the three-dimensional theory of thermoelasticity, the use of the finite element moment scheme, and the application of the universal 3D isoparametric finite element with multilinear shape functions. The modal analysis of the shell is performed at each step of thermomechanical loading, taking into account the prestressed state of the deformed shell.

The convergence and accuracy of solutions are investigated. Comparison of the calculation results obtained by the moment finite elements scheme with the data of other authors shows their good agreement.

REFERENCES

1. *Bazhenov V. A., Krivenko O. P.* Buckling and vibrations of elastic inhomogeneous shells under thermo-mechanical loads (Stiikist i kolyvannia pruzhnykh neodnoridnykh obolonok pry termosylovykh navantazhenniakh). – Kyiv: Karavella, 2020. – 187 p. ISBN: 978-966-8019-85-2 (in Ukrainian).
2. *Grigorenko Y.M., Gulyaev V.I.* Nonlinear problems of shell theory and their solution methods (review) // Appl. Mech 27, 929–947 (1991)
3. *Grigolyuk E. I., Kabanov V. V.* Shell stability. (Ustojchivost obolochek). – M.: Nauka, 1978. – 360 p. (in Russian)
4. *Zarutskii V.A., Prokopenko N.Y.* Vibrations and Stability of Shallow Ribbed Shells with a Rectangular Planform // Int. Appl. Mech. 38, 710–715 (2002).
5. *Gavrilenko G.D., Matsner V.I., Kutenkova O.A.* Dent and thickness effects on the critical loads of stiffened shells // Strength of Materials, 2011. – Vol. 43, No. 3. – Pp. 347-351
6. *Podvornyi A.V., Semenyuk N.P., Trach V.M.* Stability of Inhomogeneous Cylindrical Shells Under Distributed External Pressure in a Three-Dimensional Statement // Int. Appl. Mech. 53, 623–638 (2017).
7. *Kantor B. Ya.* Nonlinear Problems in the Theory of Inhomogeneous Shallow Shells (Nelineynyye zadachi teorii neodnorodnykh pologikh obolochek). – Kyiv: Naukova Dumka, 1974. – 136 p. (in Russian)
8. *Hutchinson J. W., Thompson J. M. T.* Nonlinear Buckling Interaction for Spherical Shells Subject to Pressure and Probing Forces // J. Appl. Mech 84(6), 061001 (2017).
9. *Gavrilenko G. D., Trubitsina O. A.* Oscillations and stability of ribbed shells of revolution (Kolebaniya i ustoychivost' rebristykh obolochek vrashcheniya). - Dnipropetrovsk: TOV "Barviks", 2008. - 155 p. (in Russian).
10. Finite element method in solid mechanics (Metod konechnykh elementov v mekhanike tverdykh tel) / *A. S. Sakharov, V. N. Kislooky, V. V. Kirichevsky [et al.].*– Kyiv: Vishcha shk. Golov. izd-vo, 1982. - 480 p. (in Russian).
11. *Cinefra M.* Formulation of 3D finite elements using curvilinear coordinates // Mechanics of Advanced Materials and Structures, pp, 1-10 (2020).
12. *Thornton E. A.* Thermal Buckling of Plates and Shells // Applied Mechanics Review. 1993. Vol. 46, Iss. 10. P. 485–506.
13. *Bloch V.I.* Theory of elasticity (Teoriya uprugosti). – Kharkov: Publishing House of KhGU, 1964. – 483 p. (in Russian).
14. *Bazhenov V. A., Krivenko O. P., Solovei M. O.* Nonlinear deformation and buckling of elastic shells with inhomogeneous structure (Neliniine deformuvannia ta stiikist pruzhnykh obolonok neodnoridnoi struktury). – Kyiv: ZAT Vipol, 2010. – 316 p. ISBN: 978-966-646-097-7 (in Ukrainian).
15. *Bazhenov V. A., Krivenko O. P., Legostaev A. D.* Stability and natural vibrations of inhomogeneous shells taking into account the stress state (Stiykist□ i vlasni kolyvannya neodnoridnykh obolonok z urakhuvannyam napruzhenoho stanu) // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2020. *2015.* - Issue 95. - C. 96-113. (in Ukrainian).
16. *Bazhenov V., Krivenko O.* Buckling and Natural Vibrations of Thin Elastic Inhomogeneous Shells. – LAP LAMBERT Academic Publishing. Saarbruken, Deutscland, 2018. – 97 p. ISBN: 978-613-9-85790-6
17. *Solovei M. O., Krivenko O. P., Kalashnikov O. B.* Comparative analysis of the results of stability calculations of thin elastic shells (Porivnyal□nyy analiz rezul□tativ rozrakhunkiv stiykosti tonkykh pruzhnykh obolonok) // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2009. - Issue 83. - P. 63-73. (in Ukrainian).
18. *Solovei M. O., Krivenko O. P., Kalashnikov O. B., Tamilko O. A.* Comparative analysis of the stability of axisymmetric shells of linearly variable thickness (Porivnyal□nyy analiz stiykosti osesymetrychnykh obolonok liniyno-zminnoyi tovshchyny) // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2009. - Issue 84. - P. 89-96. (in Ukrainian).
19. *Bazhenov V. A., Solovei N. A., Krivenko O. P., Gusar P. P., Dubina A. S., Kalashnikov A. B., Mishchenko O. A.* Study of nonlinear deformation and stability of elastic inhomogeneous

shells based on shell and spatial finite element models (Issledovaniye nelineynogo deformirovaniya i ustoychivosti uprugikh neodnorodnykh obolochek na osnove obolochechnoy i prostranstvennoy konechno-elementnykh modeley) // Proceedings of the V International Scientific and Practical Conference "Engineering Systems - 2012". April 16-18, 2012. - M.: RUDN University, 2012. - P. 162-167. (in Russian).

20. *Solovei M. O., Krivenko O. P., Kalashnikov O. B.* The influence of preheating on the loss of stability of hollow shells under the action of pressure // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2012. - Issue 90. - P. 143-157. (in Ukrainian).
21. *Bazhenov V. A., Solovei M. O., Krivenko O. P., Mishchenko O. O., Kalashnikov O. B.* Study of the stability of thin elastic shells of constant and linearly variable thickness (Doslidzhennya stiykosti tonkykh pruzhnykh obolonok staloyi ta liniyno-zminnoyi tovshchyny) // Building mechanics and building structures: Collection of articles. - M.: Izdatelsto SKAD SOFT, 2013. - P. 27-33. (in Ukrainian).
22. *Bazhenov V. A., Solovey N. A., Krivenko O. P., Kalashnikov A. B., Mishchenko O. A.* Analysis of modeling of nonlinear deformation and stability of elastic inhomogeneous shells in some software systems (Analiz modelirovaniya nelineynogo deformirovaniya i ustoychivosti uprugikh neodnorodnykh obolochek v nekotorykh programmnykh kompleksakh) // Proceedings of the VI International Scientific and Practical Conference "Engineering Systems - 2013", April 24–26, 2013. - M.: RUDN University, 2013. - P. 96-102. (in Russian).
23. *Vol'mir A. S.* Stability of Deformable Systems (Ustoychivost' deformiruyemykh system) - M.: Nauka, 1967. – 984 p.
24. *Karpilovsky V. S., Kriksunov E. Z., Perel'muter A. V., Perel'muter M. A.* Software SCAD (Vyichislitelnyiy kompleks SCAD). – M.: SCAD SOFT, 2009. – 656 p. (in Russian).
25. *Strelets-Streletskiy E.B., Bogovis V.E., Genzersky Y.V., Geraymovich Y.D. [et al.].* LIRA 9.4. User Guide. Basics. Textbook (LIRA 9.4. Rukovodstvo polzovatelya. Osnovy. Uchebnoe posobie). – Kyiv: Fact, 2008. 164 p. (in Russian).
26. Rychkov S. MSC.visualNASTRAN for Windows. – M.: NT Press, 2004. – 552 p. (in Russian).
27. *Cowep G.R., Lindberg G.M., Olson M.D.* A shallow shell finite element of triangular shape // Int. J. Solids and Structures, 1970. – V.6, №8. – Pp. 1133-1156.
28. *Olson M.D., Linberg G.M.* Vibration analysis of cantilever plates, using a new cylindrical shell finite elements // Pros of 2nd Conf. math. struct. mech., AF base Wright. Peterson. Otto, 1968. – P. 247-269.
29. *Bossak M.A., Zienkiewicz O.C.* Free vibration of initially stressed solids with particular referents to centrifugal force in rotation machinery // J.Strain Anal. – 1973. – V.8, №4. – P. 245-252.

Стаття надійшла 28.04.2023

Кривенко О.П., Лізунов П.П., Ворона Ю.В., Калашніков О.Б.

МЕТОДИКА ДОСЛІДЖЕННЯ НЕЛІНІЙНОГО ДЕФОРМУВАННЯ, СТІЙКОСТІ ТА КОЛИВАНЬ ТОНКИХ ПРУЖНИХ ОБОЛОНОК НЕОДНОРІДНОЇ СТРУКТУРИ

Розглянуто постановку задачі та методику аналізу напружено-деформованого стану, втрати стійкості та коливань пружних оболонок з неоднорідною структурою. Модальний аналіз оболонок виконується на кожному кроці навантаження. Метод дає змогу досліджувати поведінку оболонок зі складною формою серединної поверхні, геометричними особливостями за товщиною та багатошаровою структурою матеріалу при термосиловому навантаженні. Тонка оболонка апроксимується одним скінченним елементом (СЕ) за товщиною. При цьому використовуються просторові СЕ одного типу для моделювання ділянок оболонки зі ступінчасто-зміною товщиною. Тому застосовується універсальний скінченний елемент. Він побудований на базі ізопараметричного просторового елемента з полілінійними функціями форми для координат і переміщень і має додаткові параметри. Універсальний скінченний елемент може трансформуватися (модифікуватися) для точного опису ділянок оболонки зі ступінчастою зміною товщини.

Цей елемент може ексцентрично зміщуватися щодо середньої поверхні оболонки і змінювати товщину. Бічні грані сусідніх СЕ знаходяться в безперервному контакті, а СЕ дозволяє моделювати різкі зломи оболонки. Підхід є сучасним і простим у реалізації, оскільки базується на використанні співвідношень тривимірної геометрично нелінійної теорії термопружності та застосуванні моментної схеми скінченних елементів. Ефективність методу продемонстровано на класичних тестових прикладах. Досліджено збіжність, точність і надійність отриманих розв’язків. Порівняння результатів розрахунків, отриманих за моментною схемою скінченних елементів, з даними інших авторів показує хороший збіг розв’язків.

Ключові слова: неоднорідна оболонка, геометрично нелінійне деформування, стійкість, модальний аналіз, термосилове навантаження, універсальний просторовий скінченний елемент, моментна схема скінченних елементів.

УДК 539.3
Кривенко О.П., Лізунов П.П., Ворона Ю.В., Калашніков О.Б. **Методика дослідження нелінійного деформування, стійкості та коливань тонких пружних оболонок неоднорідної структури** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2023. – Вип. 110. – С. 131-149.
Розглядається постановка задачі щодо методу аналізу напружено-деформованого стану, стійкості, закритичної поведінки та власних коливань неоднорідних оболонок при дії термосилових навантажень. Чисельні результати підтверджують точність та ефективність розробленого методу.
Табл. 7. Іл. 15. Бібліогр. 26 назв.

UDC 539.3
Krivenko O.P., Lizunov P.P., Vorona Yu.V., Kalashnikov O.B. **A Method for Analysis of Nonlinear Deformation, Buckling, and Vibrations of Thin Elastic Shells with Inhomogeneous Structures** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2023. – Issue 110. – P. 131-149.
The formulation of the problem of the method of analysis of the stress-strain state, buckling, post-buckling behavior and vibrations of inhomogeneous shells under the action of thermo-mechanical loads is considered. Numerical results confirm the accuracy and effectiveness of the developed method.
Tabl. 7. Fig. 15. Ref. 26

УДК 539.3
Кривенко А.П., Лизунов П.П., Ворона Ю.В., Калашников А.Б. **Методика исследования нелинейного деформирования, устойчивости и колебаний тонких упругих оболочек неоднородной структуры** // Сопротивление материалов и теория сооружений. – 2023. – Вып. 110. – С. 131-149.
Рассматривается постановка задачи метода анализа напряженно-деформированного состояния, устойчивости, закритического поведения и колебаний неоднородных оболочек при действии термосиловых нагрузок. Численные результаты подтверждают точность и эффективность разработанного метода.
Табл. 7. Ил. 15. Библиогр. 26 назв.

Автор (вчена ступень, вчене звання, посада): *кандидат технічних наук, старший науковий співробітник, провідний науковий співробітник НДІ будівельної механіки КРИВЕНКО Ольга Петрівна*
Адреса робоча: *03037 Україна, м. Київ, Повітрофлотський проспект 31, Київський національний університет будівництва і архітектури, НДІ будівельної механіки*
Робочий тел.: *+38(044) 245-48-29.*
мобільний тел.: *+38(066) 048-32-77*
E-mail: olakop@ukr.net
ORCID ID: https://orcid.org/0000-0002-1623-9679

Автор (вчена ступень, вчене звання, посада): *доктор технічних наук, професор, завідувач кафедри будівельної механіки ЛІЗУНОВ Петро Петрович*
Адреса робоча: *03035 Україна, м. Київ, Повітрофлотський проспект 31, Київський національний університет будівництва і архітектури, кафедра будівельної механіки*
Робочий тел.: +38(044) 245-48-29.
мобільний тел.: +38(067) 921-70-05
E-mail: lizunov@knuba.edu.ua
ORCID ID: https://orcid.org/0000-0003-2924-3025

Автор (вчена ступень, вчене звання, посада): *кандидат технічних наук, доцент, завідувач відділу НДІ будівельної механіки ВОРОНА Юрій Володимирович*
Адреса робоча: *03035 Україна, м. Київ, Повітрофлотський проспект 31, Київський національний університет будівництва і архітектури, НДІ будівельної механіки*
Робочий тел.: +38(044) 245-48-29.
мобільний тел.: +38(050) 750-13-61
E-mail: yuvv@ukr.net
ORCID ID: https://orcid.org/0000-0001-8130-7204

Автор (вчена ступень, вчене звання, посада): *здобувач ступеня кандидата технічних наук, кафедра будівельної механіки Калашніков Олександр Борисович*
Адреса робоча: *03035 Україна, м. Київ, Повітрофлотський проспект 31, Київський національний університет будівництва і архітектури, НДІ будівельної механіки*
Робочий тел.: +38(044) 245-48-29
мобільний тел.: +38(066) 71-88-099
E-mail: kalash2d@gmail.com
ORCID ID: https://orcid.org/0009-0009-7825-9809