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# DEVELOPMENT OF MATHEMATICAL MODELS OF PROCESS OF CHANGE OF THE TECHNICAL CONDITION OF SAMPLES OF BUILDING TECHNIQUE IS DURING REALIZATION OF STRATEGY OF TECHNICAL EXPLOITATION ACCORDING TO CONDITION

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In the article the brought procedure over of development of mathematical model of process of change of the technical state of standards of building technique during exploitation according to the technical condition. The brought procedure over is base on the use of method of the group taking into account of arguments and takes into account possibility of using as these initial estimations of indexes of faultlessness, operating supervisions got on non steady-precise results. The brought procedure over allows more fully to take into account the specific features of the use of samples of the special technique.

**Keywords:** Group Method of Data Handling, construction machinery, operation on a technical condition.

**Introduction.** Existing on this time plan-preventive (regulated) strategy of technical exploitation and repair (TE and R) of building technique in different industries, in particular and in military, oriented to the expensive periodic technical servicing of the samples (PTS) and major repairs (MR). Thus efficiency of the regulated strategy TE and R (according to operating time or on calendar duration), that envisages implementation of prophylactic works of certain volumes through before the pre-arranged time domains or operating time on all of the same type park of building machines regardless of the conditions of their systems and wares, depends on that, in that degrees she takes into account the level of reliability, stopped up in these samples, during their production, physical-geographical terms and intensity of their exploitation.

As a result of the review of global trends in the development of TE and R strategies, it was established that a closer connection between the condition of the object and the condition of its operation process is ensured by the TE and R strategy by state. A characteristic feature of the strategy of TE and R by condition is that during operation the condition of a specific object is monitored to make timely decisions about the need for repair and its scope. At the same time, the equipment can be operated without establishing inter-repair resources (service terms) and are subject to repair when they reach the appropriate type of limit condition.

The effectiveness of the implementation of the TE and R strategy by state is largely determined by the timely identification of moments of transition of technical samples to the limit state. In the process of operation, the reliability of specific samples of construction equipment (SCE) deteriorates, because by the nature of the main degradation processes that bring the sample to the limit condition, they belong to aging products. Therefore, the deviation of the value of the controlled reliability indicator beyond the established values is one of the signs of the limit condition of the safety equipment.

In the absence of changes in the design of the SCE, the maintainability indicators can be considered independent of the duration of operation. Therefore, when predicting the limit condition of the SCE, it is necessary to study the change in failure rate, and to determine the durability indicators of a specific SCE using the established patterns of individual changes in the values of the non-failure indicators (NFI).

In the scientific and technical literature, the set of methods for estimating durability indicators is divided into parametric and non-parametric [1-7]. At the same time, parametric methods are used under the assumption that the type of function of distribution of time of operation (duration of operation) of objects to resource failure (limit state) is known in advance, and its parameters are set based on a sufficiently large scope of results of operational observations of homogeneous objects. On the contrary, non-parametric methods can be applied assuming an unknown form of the earnings distribution function before resource failure.

Such methods of assessing durability indicators were developed in relation to the regulated strategy of TE and R and can be applied to a fleet of the same type of SCE; do not fully take into account the individual characteristics of the conditions and modes of operation of the SCE. In addition, in practice, the assessment of the durability indicators of SCE based on the results of the organization and performance of resource tests or operational observations is complicated due to the fact that there are no statistics of moments of transitions of SCE to the limit condition and, as a result, there is no sample of resource scope and lifetimes of SCE, which does not allow establish of distribution laws of these random variables; establishing the distribution laws of resources and life spans of the safety equipment by the calculation-experimental method according to the known laws of distribution of resources and service life of component products, determined by the results of operational observations, leads to large errors due to the impossibility of accounting for the conditions and modes of operation of specific component products in the composition of safety equipment and their constituent parts.

**Main part.** In order to individually solve the task of assessing the durability indicators of a sample of construction equipment when put into operation according to the technical condition (OATC) taking into account their actual technical condition and reliability, it is necessary to evaluate their durability indicators taking into account the specific modes and conditions of their operation, the degree of influence of these modes and operating conditions on the state of the sample. This evaluation is carried out, as a rule, using the dependencies of the change in the fail-safe rate (FSR) on the parameters characterizing the operating modes, such as the duration of operation, total mileage, total mileage of the base chassis, etc. At the same time, the results of operational observations, accumulated over a set of operation intervals of a fixed duration, in the form of a set of FSR estimates, are considered as initial data for constructing dependencies of changes in FSR values during operation for forecasting values for future operation intervals and, accordingly, evaluating the durability indicators of the SCE.

The mathematical apparatus that can be used to solve this task in the conditions of measurement instability, with the aim of taking into account the influence of factors on the change of FSR in the most complete way, is an inductive approach as a method of direct construction of models based on observational data [8-10].

Inductive methods make it possible to automatically find dependencies that are hidden in a sample of initial data.

Inductive modeling is based on sorting through a set of candidate models according to external selection criteria and solves the problem of structural-parametric identification. One of the most effective methods of modeling based on experimental data in conditions of incomplete information is the Group method of data handling (GMDH) [8-13]. In contrast to regression analysis, where the structure of the model is specified, in GMDH the structure of the optimal model and its parameters are found with the help of self-organization of models, that is, testing of many candidate models according to external selection criteria.

In work [14], a comparison of the effectiveness of regression analysis and GMDH for forecasting economic processes was carried out. A comparison of the predictive properties of models built on the basis of the least squares method (LSM) and GMDH was carried out on the examples of building models of the processes of changes in the volume of light industry production and inflation.

The work shows that the predictive properties of the GMDH model are significantly higher, and the model of optimal complexity is much simpler than the model built according to the LSM. That is, the factors that were not included in the GMDH model are not just redundant and uninformative, but even “harmful” in the conditions of the available short sample of data.

The obtained results show that regression models, even if they are statistically significant, are not suitable for forecasting purposes. Models built according to GMDH algorithms automatically, based on data sampling information, find significant internal regularities of the modeling object, so their predictive efficiency is much higher.

GMDH is built according to the principles of self-organization, which are based on two main principles: with the gradual complexity of the structure, the values of external criteria first decrease, and then increase, that is, there is a minimum; only external criteria calculated on data that were not used to build the models pass through the minimum.

That is why the separation of not less than two independent data samples is used in GMDH [8]. One of them is used to estimate the parameters, and the second one is used to select the optimal model. It is fundamentally impossible to apply only one criterion, because the situation may arise: “the more complex the model, the more accurate it is.”

All criteria used in the GMDH are based on the breakdown of the sample.

A feature of GMDH is the provision of “freedom of choice”. With the gradual complexity of the structure of the models, not one, but several of the best models are selected from the best models according to the external criterion on each row [8].

The most famous among the methods of inductive modeling is the combinatorial algorithm GMDH [8, 14-17]. This is a method that does not require proof of convergence, as it performs a complete search of all possible candidate models.

GMDH is designed to solve the problems of modeling complex systems, forecasting, identification and approximation of multifactor systems, diagnostics, pattern recognition and data clustering.

At the same time, to use this mathematical apparatus, it is advisable to accept the following assumptions:

1. The change in the value of the controlled (estimated) FSR for a fixed duration of the operation interval can be neglected, since this duration is disproportionate in comparison with the value of the assigned resource (service life) of the object. At the same time, their failure-free operation as a result of restoration of operational capacity within the operating interval of a fixed duration practically does not change (that is, the restoration of the failure-free operation of safety equipment after failures is assumed to be minimal).

2. On a set of operation intervals of a fixed duration, the values of controlled (estimated) FSRs change significantly, and the nature of this change is unknown in advance and must be established in the form of models of their change depending on the duration of operation and other factors based on the accumulated values of FSR assessments.

We select the set of parameters characterizing the operation modes of the SCE, taking into account the forms and mechanisms of registration of the results of operational observations, as follows: duration of operation $T_e$, total mileage $T$, total mileage of the base chassis S, total number of activations $N$. The dependence of the value of SFR on the values of the parameters characterizing the operating modes, in relation to a specific SCE, can be determined by a mathematical model that describes the process of changing the fail-safe of the SCE.

In general, the process of changing the technical condition (changes in the SFR) of the SCE can be represented in the form of a time series ($Y_t$), consisting of time-ordered sets of measurements of certain characteristics of the process being studied

$$y_t = x_t + \xi_t, \tag{1}$$

where $x_t$ is a deterministic component of the process; $\xi_t$ - stochastic process component.

At the same time, the deterministic component (trend) characterizes the existing dynamics of the development of the process as a whole, and the stochastic component reflects random fluctuations (noise) of the process.

The presence of a deterministic component is explained by the processes of degradation, wear, and aging of the SCE at the OATC, caused by the consumption of resources provided for in the regulatory operational documentation, the influence of climatic and other factors, as well as the performed maintenance and repair measures. The stochastic component is due to errors that occur during the measurement of parameters, deviations in the choice of the start of the countdown, and other reasons.

The basis of any forecast is a certain volume of initial information, for example, for time $T_1$, during which at discrete moments of time $t_i = t_0, t_n$ the studied indicator takes random values $Y(t_i) = Y_0, Y_n$.

In the assumption regarding the distribution according to the normal law of the deviation of the controlled indicator from the general trend of its change, the analytical expression of the mathematical expectation function M[Y(t)] of the controlled indicator must satisfy the following condition in the area $T_1$:

$$\begin{cases} M[Y(t_0)] = Y(t_0); \\ M[Y(t_1)] = Y(t_1); \\ M[Y(t_2)] = Y(t_2); \\ \ldots\ldots \\ M[Y(t_n)] = Y(t_n). \end{cases} \tag{2}$$

At the same time, in the future, through a formal and logical analysis of the process, we can claim that the tendency of the change of the indicator Y(t) is preserved in the $T_2$ region with a certain error ε(t), that is, the indicator acquires a value $M[Y(t_{n+1})] \pm \varepsilon(t_{n+1}), M[Y(t_{n+2})] \pm \varepsilon(t_{n+2})$ and so on until the moment when the function Y(t) reaches permissible (limit) value:

$$M[Y(t_{n+j})] \pm \varepsilon(t_{n+j}) \geq Y_{доп}. \tag{3}$$

At the same time, determining the permissible value of $Y_{доп}$ is considered a secondary task, since it makes sense to talk about finding its solution in the case of solving the task of determining the regular component of the component and, on its basis, obtaining a correct representation of the process of changing the level of reliability over time.

It should be noted that if the law of the distribution of the controlled indicator is unknown, then the task of describing the random process and forecasting its values at the $T_2$ interval arises, which can be successfully solved by applying GMDH.

According to GMDH, such a task can be reduced to finding the extremum of some CR criterion on many different F models:

$$f^* = \arg\min_{f\in F} CR(f). \tag{4}$$

It is obvious that (4) does not contain an exhaustive formulation of the task, therefore it is additionally necessary:

set the form and volume of the initial information;

specify the class of basis functions from which the set F is formed;

determine the method of generation of f models together with the method of parameter estimation;

choose a model comparison criterion;

specify the CR minimization method.

In order to clarify the formulation of the task, we introduce the following assumptions:

1. The process is characterized by m input (independent) variables $x_{i1}, x_{i2}, ..., x_{im}$ and one output (dependent) variable $y_i$, which are related by the ratio:

$$y_i = x_i\Theta + \xi_i, \tag{5}$$

where $\Theta$ is a vector of m independent parameters; i - observation number; $\xi_i$ - the vector of realization of a random variable (noise).

2. A random variable has the following properties:

$$E\xi_i = 0 \ , E\xi_i\xi_j = \sigma^2 \ , E\xi_i\xi_j = 0, \ i \neq j \ ,$$

where $\sigma^2$ is the final unknown variance with an unknown distribution function $\xi_i$; E is the mathematical expectation operator.

3. A given sample W is considered, which contains n observation points that create a matrix of arguments $X = \left\{x_{ij}, i = 1,n, j = 1,m\right\}$ and a vector of the noisy output variable $y = (y_1, ..., y_n)^T$, and $n \geq m$.

Taking into account the introduced assumptions, the task of modeling the process consists in the formation of a number of models of different structures:

$$y_f = f(x, q_f) \tag{6}$$

and finding the best model provided:

$$f^* = \arg\min_{f\in F} CR(y, f(x, \theta_f)). \tag{7}$$

The search for predictive values should be conducted in the area of unbiased and effective estimates, which is successfully solved by the method of group consideration of arguments.

Calculation of models of the process of changing the level of reliability of SCE, which characterize the process of changing its technical condition, is carried out by solving the problems of expression (4). To do this, it is necessary to analyze the modeling process based on experimental data in order to determine its main stages in relation to the methodology of forecasting the intended service life of the developed SCE.

As it was established, the task of modeling consists in choosing the best model from a certain set of models according to a given criterion. At the same time, it is necessary: to specify the form and volume of the initial information; specify the class of basic functions (operators) from which many models are formed; determine the method of formation (generation) of model structures together with the method of evaluating their parameters; choose an external criterion for comparing models; specify the method of minimization of the model selection criterion and the adequacy assessment procedure. Thus, the task of assessing the level of reliability of SCE complexes is reduced to finding a function describing a random process with unshifted, reliable parameters and determining its predictive values. The following work materials are devoted to solving this problem.

Based on a sample of observations of the variables Y and X, it is necessary to select a structure f from some class of structures F, assuming that $f \in F$, and estimate the value of the parameter Θ. The model of the researched process is defined as

$$y = f(x, \Theta) \tag{8}$$

and y is a scalar output variable, x is a vector input variable. All the a priori information is that for some values $x \in X$ we observe the value y distorted by noise.

There is a function for describing the process of changing the technical condition (reliability level) of equipment samples

$$y_i = f(x_{ij}) + \xi_i,\ i = 1,2,...,n,\ j = 1,2,...,m, \tag{9}$$

where x, y are elements of matrices X and Y, respectively; $\xi_i$ - a random error in the measurement of quantities. At the same time, in y - the researched SFR of SCE, $x_{i1}, x_{i2}, ..., x_{i4}$ - the results of operational observations: $T_e$ duration of operation, $T$ total mileage, total mileage of the base chassis S, total number of activations $N$ .

The initial data obtained from the consequences of controlled operation may not have additional information about the characteristics of uncontrolled disturbances.

The developed GMDH algorithms use the same selection criteria and the same support function systems. The difference between the modeling algorithms is determined by the structure of the model generator, i.e. the scheme of sorting through a complete or incomplete set of complicating partial models in the selected basis.

During the development of the mathematical model, the block of formation of polynomial reference functions was implemented in the developed partial methodology.

For the measured values of operational factors $x_1...,x_m$ of the modeled process and the given degree of the polynomial q, the number of terms g in the complete polynomial of the degree q of m variables is determined by the expression: $g=\prod_{j=1}^{m}\frac{q+j}{j}$ Accordingly, the complete polynomial has the form:

$$y=\sum_{u=1}^{g}a_u\prod_{j=1}^{m}x_j^{q_{uj}} .$$

To form the degrees of arguments, a procedure for obtaining a sequence of m - digit numbers $q_u$= ($q_{u1}...,q_{um}$) with bases 1,2... and so on is organized.

Thus, for four arguments (m = 4) i q = 2 investigated in the course of the construction technique research, a sequence of g = 15 numbers $q_u$, u = 1,15 was obtained: 0000, 1000, 0100, 0010, 0001, 1100, 1010, 1001, 0110, 0101, 0011, 2000, 0200, 0020, 0002.

The partial model selection block includes the following operations: forming the structure of the next partial model; formation of the appropriate normal system of equations, estimation of model coefficients.

Formation of the structures of partial models in the combinatorial algorithm is carried out using a binary structural vector d = ($d_1,d_2,...,d_g$ ): if element d of this vector takes the value 1, then the corresponding u-th argument is included in the partial model, if 0 - it is not included (u = 1,g).

The change in the state of the vector d is organized by placing units in it: first one (total received $C_g^1$ = g variants), then two (total $C_g^2$ = 1/2 g(g+1) variants) and so on. The total number of different structures will be: $p_g=\sum_{u=1}^{g}C_g^u=2^g-1$ that is, a complete search.

The external criteria for the selection of mathematical process models are designed to solve problem (7) and are based on dividing the sample of initial data into two or more parts.

External criteria are called criteria that are calculated on the basis of new "external" information that was not used to build the model. Such criteria include: the criterion of regularity, the criterion of non-displacement (minimum shift), the balance of variables, predictability, absolutely obstacle-resistant, and others. All of them are based on the use of information that is new compared to that used to find the initial parameters.

The following groups of criteria [8] are distinguished among the external criteria of GMDH: accuracy criteria, which express the error of the model, which is calculated on different parts of the sample; consistency criteria, which are a measure of the closeness of estimates obtained on different parts of the sample; combined criteria is a convolution of several criteria, for example, in order to select a model that would be both the most accurate and the least biased.

A sequence of criteria is called several criteria that are applied one after the other.

At the same time, the data sample is divided, as a rule, into three non-intersecting subsamples: A, B, C, where A is an educational, B is a test, and C is an examination subsample; denote W - the working sample, and A U B = W. Let's consider in more detail the main criteria of the GMDH.

**Accuracy criteria.** The most widely used accuracy criterion is the regularity criterion, which is calculated for a given model complexity S:

$$AR_B(s) = AR_{B|A}(s) = \left\| y_B - \hat{y}_{B|A} \right\|^2 = \left\| y_B - X_B \hat{\theta}_A \right\|^2,$$

$$AR_B(s) = AR_{B|A}(s) = \Delta^2(B) = \left\| Y_B - X_B \theta_A \right\|^2,$$

where $AR_{B|A}(s)$ the entry means "error on B model of complexity S, the coefficients of which are obtained on A". In a similar way, the regularity criterion for the models obtained on B can be written:

$$AR_A(s) = AR_{A|B}(s) = \left\| y_A - \hat{y}_{A|B} \right\|^2 = \left\| y_A - X_A \hat{\theta}_B \right\|^2,$$

$$AR_A(s) = AR_{A|B}(s) = \Delta^2(A) = \left\| Y_A - X_A \theta_B \right\|^2.$$

In the future, we omit the dependence on $s$ in the formulas, remembering that the values of the criteria are considered for a specific complexity S.

The criterion of regularity, calculated on the same sample on which the model was obtained, is the residual sum of squares (RSS - Residual Sum of Squares). In addition, the stability criterion can be applied as an accuracy criterion [2]:

$$AS = AR_{W|A} + AR_{W|B} = \left\| y_W - X_W \hat{\theta}_A \right\|^2 + \left\| y_W - X_W \hat{\theta}_B \right\|^2.$$

Accuracy and unbiasedness are different indicators, one does not replace the other, especially with inaccurate or incomplete data. In the complete absence of noise, all criteria point to a physical model and any criterion (internal or external) can be used. The generally accepted approach, based on the calculation of the accuracy of the model, is effective only for overcomplicated models, in the presence of noise in the data. Accuracy on such a sample of data does not mean accuracy on the next sample at all, that is, it does not ensure consistency.

**Non-displacement criteria** (minimum shift, coherence). The criteria of this group reflect the requirement that the best models obtained on A and B differ minimally. The minimum of undisplaced coefficients has the form:

$$CB = CB_{W|A,B} = n_{\text{см}}^2 = \left\| X\theta_A - X\theta_B \right\|^2.$$

Consistency group criteria are the most important for choosing regularities, as they allow choosing the least contradictory models. In the presence of noise in the data, the computer finds increasingly simple structures of optimal models.

Purposeful search for an optimal structure is greatly facilitated if, firstly, based on physical considerations, the class of structures will be limited and, secondly, appropriate selection criteria will be applied. In [10] it is stated that the best results were shown by the criteria of the first two groups, and their application turned out to be sufficient for solving problems. Particularly effective use of these criteria, first of all, in connection with their high degree of objectivity, showed when searching for the correspondence of the physical content of the researched processes to its mathematical representation.

The regularity criterion makes it possible to obtain the most regular solution, i.e., a solution of minimum complexity in terms of content, which is insensitive to small changes in the initial data. The models found by the minimum of the regularity criterion are not accurate enough at a high level of noise, because when obstacles increase, the minimum of the regularity criterion shifts towards the selection of simpler models, and at high noise intensity, linear predictive models are the best.

Thus, the regularity criterion has a physical meaning, which is that it is oriented towards choosing a model that will be the most accurate for a set of points that will be obtained in the near future, it is used for short-term forecasting one or two steps ahead.

The criterion of non-displacement (minimum shift) compensates for the mentioned shortcomings of the criterion of regularity and allows to ensure a significant difference in the models obtained when using different parts of the table of initial data, which allows us to talk about the convergence of the mathematical representation of the physical content of the studied processes of changes in the technical state of the SCE at OATC. In the absence of a shift, the models obtained on the basis of different parts of the table should match. Thus, the criterion of unbiasedness allows solving the problem of changing the law, which is hidden in noisy experimental data, so it is recommended when solving the problem of identification. Criteria: regularity $\Delta^2(B)$ – and immutability $n^2_{см}$ – in the most general form [9] can be written:

$$\Delta^2(B) = \|Y_B - X_B\theta_A\|^2, \tag{10}$$

$$n^2_{см} = \|X\theta_A - X\theta_B\|^2, \tag{11}$$

where $X$, $Y$ are matrices containing all the initial information; $X_A$, $X_B$, $Y_A$, $Y_B$ – matrices, respectively $(n_A \times m)$, $(n_B \times m)$, $(n_A \times 1)$ and $(n_B \times 1)$; $n_A$, $n_B$ natural numbers whose sum is equal to $n$; $A$, $B$ are sets that are grouped on samples ($X_A$ $Y_A$) i ($X_B$ $Y_B$); $\theta_A, \theta_B$, are the coefficients found, respectively, on the sets $A$, $B$.

Let's determine the conditions for solving the problem for the system of equations of the form $Y = aX$, where $a$ is the vector of parameters corresponding to the selected structure, calculated by the method of least squares [9] and has the form: $a = (X^T X)^{-1} X^T Y$, $X^T$ where is the transposed matrix.

The regularity criterion for evaluating the structure of the model can be determined [9] by the formula:

$$\Delta^2(B) = (Y_B^* - Y_B)^T (Y_B^* - Y_B), \tag{12}$$

where $Y_B^*$ is the vector of estimates obtained by the method of least squares on $Y_A$ and $X_A$, that is, taking into account the previous expressions: $Y_B^* = X_B(X_A^T X_A)^{-1} X_A Y_A$.

The non-displacement criterion is, as a rule, given by the formula:

$$n_{\text{см}}^2 = (Y_A^* - Y_B^{**})^T (Y_A^{**} - Y_B^{**}), \tag{13}$$

where $Y_{A,B}^{**} = X(X_{A,B}^T X_{A,B})^{-1} X_{A,B}^T Y_{A,B}$.

This derivation of the unbiasedness criterion is equivalent [9] to the derivation of the unbiasedness criterion by the coefficients given by the formula: $n_{\text{см}}^2 = (a_A - a_B)^T X^T X (a_A - a_B)$.

Depending on the specifics of the task to be solved, taking into account the depth of forecasting and in case of insufficient accuracy of the obtained model, the question can be asked about extending the search to the area of higher-order structures. However, there is no reason to claim that a better solution will be found compared to the ones obtained, and that it will have a form that is acceptable for use, that is, recognized as rational. Therefore, on the basis of the above, it is possible to draw a conclusion about the expediency of focusing efforts not on finding solutions in the field of complex nonlinear structures, but on researching the solution of maximum accuracy from among the simplest optimal structures.

The outlined principles are used as starting points in the algorithm for finding optimal mathematical models when solving the task of assessing changes in the technical condition (reliability level) of equipment.

The solution to the problem of choosing the structure of the model is usually complicated by the small amount of information contained in the matrices $X, Y$. Therefore, in order to improve the quality of the forecast and obtain a reasoned decision regarding the appearance of the obtained model, there is a need to apply a modification of the criterion of regularity, or the criterion of predictable ability.

For this, the input and output samples are divided into three subsamples, respectively:

$$X = \begin{bmatrix} X_A \\ X_B \\ X_C \end{bmatrix} = \begin{bmatrix} X_W \\ X_C \end{bmatrix}, \quad Y = \begin{bmatrix} Y_A \\ Y_B \\ Y_C \end{bmatrix} = \begin{bmatrix} Y_W \\ Y_C \end{bmatrix}. \tag{13}$$

Accordingly, the predictability criterion uses the root mean square error calculated for the sample $Y$ with the parameters obtained on a separate examination sample C, which was not used to find the coefficients $\theta$ and during the selection of candidate models, and has the form:

$$\Delta^2(Y\backslash C)=\left\|Y\text{-}X\theta_C\right\|^2, \qquad (14)$$

where $X$, $Y$ are matrices containing all the initial information; $\theta_C$ is the coefficient found on the set C.

The predictability criterion for evaluating the model structure can be determined by [10] the formula:

$$\Delta^2(Y\backslash C)=\left(Y\text{-}X(X_C^TX_C)^{-1}X_CY_C\right)^T\left(Y\text{-}X(X_C^TX_C)^{-1}X_C^TY_C\right). \qquad (15)$$

When implementing forecasts, it is important to establish the compliance of the selected quality criterion of the obtained forecast results. A combined selection criterion is implemented in the algorithm for assessing the change in the level of reliability of the SCE. The general difficulty of direct application of combined criteria is that the maximum value of one of the criteria is often several orders of magnitude greater than the other.

In this regard, the GMDH algorithm implements not a two-criteria, but a sequential selection: first, $F_1$ models are selected according to the unbiasedness criterion, and then the best $F_2$ models are selected according to any accuracy criterion. Such a consistent application of criteria increases the effectiveness of modeling, in particular, immunity to interference. At the same time, the problem of rationing is avoided.

The most acceptable for solving the task of building models of the process of changing the level of reliability of SCE at OATC is the use of a combined criterion: the criterion of unbiasedness for the selection of $F_1$ models that are sufficiently tested in solving tasks of a similar class, and the combined criterion: the criterion of unbiasedness and predictive ability for the selection of $F_2$ models that allows you to successfully solve the tasks of medium-term and long-term forecasting under certain conditions [18, 19].

Solving problem (7), based on reliable reliability test data, a mathematical model of the form is searched for:

$$Y = f(X) \qquad (16)$$

and assessment of the technical durability of weapons samples after they have reached the minimum permissible level of failure: $Y \geq Y_{hm}$.

The process of building models stops when a model of the form is found:

$$\begin{cases} Y = f(X); \\ \sigma = \min; \\ \Delta^2(Y\backslash C) = \min; \\ Y \geq Y_{hm}, \end{cases} \qquad (17)$$

where $\sigma$ is the root mean square error; $\Delta^2(Y\backslash C)$ - the value of the predictability criterion.

The algorithm for modeling the process of changing the technical condition (reliability level) of anti-aircraft missile samples during operation according to the technical condition, which is based on the GMDH combinatorial algorithm, implemented by G.O. Ivakhnenko [19], and refined in terms of choosing the best predictive model according to the $\Delta^2(Y \backslash C) \to \min$ criterion, presented in Fig. 1.

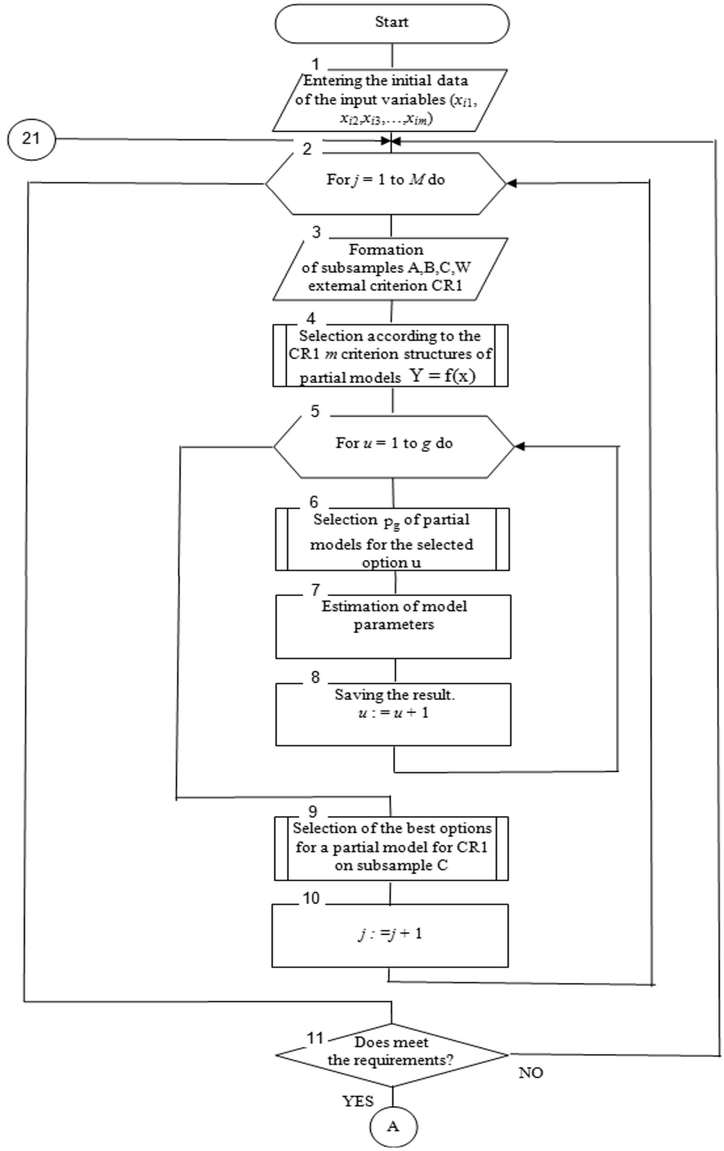


Fig. 1. Algorithm for modeling the process of changing the technical condition (reliability level) of samples of construction equipment during operation according to the technical condition (beginning)

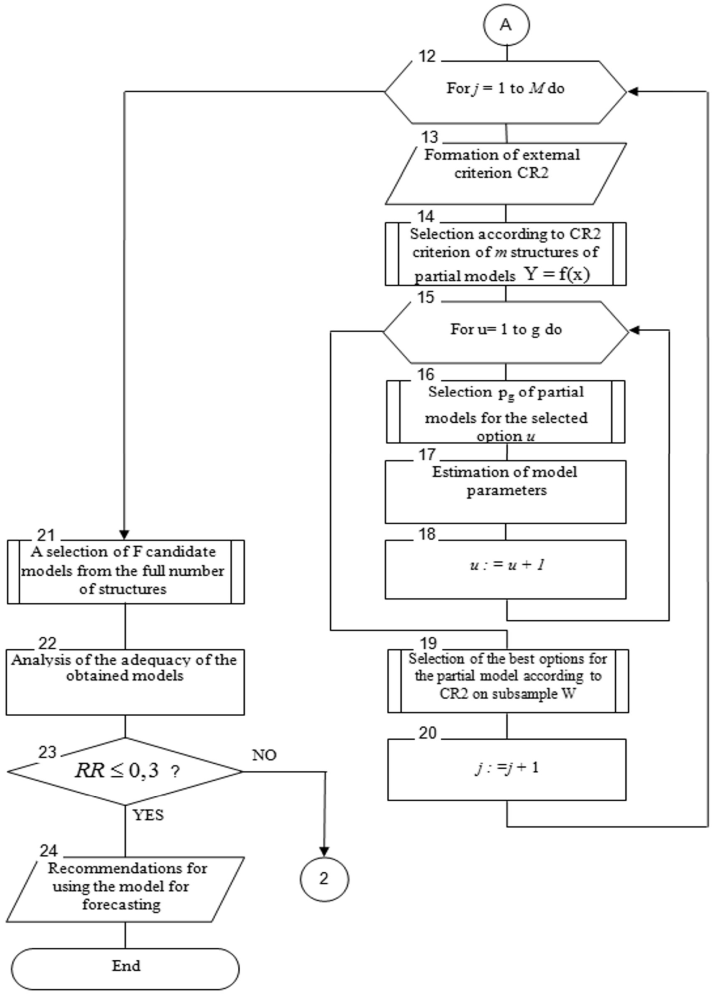


Fig. 1. Algorithm for modeling the process of changing the technical condition (reliability level) of samples of construction equipment during operation according to the technical condition (the end)

The issue of checking the adequacy of the models obtained as a result of solving problem (7) is considered differently under the conditions of application of one or another approach, and is mainly determined by the type of criterion used when comparing models. For example, within the framework of the classical approach, there are well-developed procedures for assessing the reliability of constructed models using the apparatus for testing hypotheses about the significance of the regression dependence as a whole (using F-statistics) and each regression coefficient separately (using t-statistics) [20,21]. Thus, in addition to specific means characteristic of certain methods, both classical and modern approaches actively use the technique of "crossvalidation", which boils down to splitting the sample, to check the predictive properties of models. It is on this technique that the GMDH methods are fundamentally based [22]. In particular, in the applied GMDH algorithms, the sample is divided into three parts: on the first (training) parameters of the generated models are calculated, on the second (testing) they are compared for accuracy with the selection of one or more of the best, on the third, independent (examination), quality is checked selected models in the forecasting mode. So, the obtained model is characterized by three error values (on each of the three parts of the sample), which can be used to draw a sufficiently justified conclusion about its quality: if they are all close to each other, then the model is not contradictory (adequate in structure)

$$RR = \frac{\sum_{i=NA+NB+1}^{N}\left(y_i - y_i^*\right)^2}{\sum_{i=NA+NB+1}^{N}\left(y_i - y_E\right)^2}, \tag{18}$$

where $y_i$ are the real values of the function on the examination sequence of length NC, i.e. from $N-(NA+NB)$ to $N$; $y_i^*$ - the function values predicted and determined using a mathematical model on the examination sequence; $y_E$ - the average value of the function on the examination sequence.

The degree of model adequacy is considered high if the RR value is less than 0.3 [9]. The difference between the presented quality indicator of the model (18) is the operation of external information in relation to the data used to obtain the mathematical model.

In addition, the assessment of the adequacy of the model to the investigated process is carried out using the criterion of predictive ability $\Delta^2(Y\backslash C)$. As a result, the one that has the best predictive properties is selected from among the many models.

**Conclusions.** Thus, the proposed mathematical model of the process of changing the technical condition of samples of anti-aircraft missile weapons during operation according to the technical condition is a logical continuation of the known similar procedures considered in [1,2,6,7], and unlike them, takes into account the use of raw data inconsistent results of evaluating the controlled reliability indicator based on the results of operational observations.

The use of the proposed mathematical model, which describes the dependence of the change of the reliability indicator on the set of parameters characterizing the operating modes of objects, instead of paired linear regression models of the change in the reliability indicator from the duration of operation (total earnings), makes it possible to ensure a more complete consideration of the specific features of SCE.

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**РОЗРОБКА МАТЕМАТИЧНИХ МОДЕЛЕЙ ПРОЦЕСУ ЗМІНИ ТЕХНІЧНОГО СТАНУ ЗРАЗКІВ БУДІВЕЛЬНОЇ ТЕХНІКИ ПРИ РЕАЛІЗАЦІЇ СТРАТЕГІЇ ТЕХНІЧНОЇ ЕКСПЛУАТАЦІЇ ЗА СТАНОМ**

У статті наведена процедура розробки математичної моделі процесу зміни технічного стану зразків будівельної техніки під час експлуатації за технічним станом. Наведена процедура ґрунтується на використанні методу групового урахування аргументів та враховує можливість використання в якості початкових даних оцінок показників безвідмовності, отриманих за нерівноточними результатами експлуатаційних спостережень. Наведена процедура дозволяє більш повно враховувати специфічні особливості використання зразків спеціальної техніки.

**Ключові слова**: метод групового урахування аргументів, зразок будівельної техніки, експлуатація за технічним станом.

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Fig. 1. Ref. 22.

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