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## CHOICE OF THE SHAPE IMPERFECTIONS MODEL IN DYNAMICS PROBLEMS OF A LONG FLEXIBLE CYLINDRICAL SHELL SUBJECTED TO FORCE COUPLES

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The issue of modeling geometrical imperfections in the dynamics problems of thin-walled shells was little researched. In cases when the natural modes of shell coincided with its buckling modes, the issue of choosing a dangerous imperfection model did not arise. When these shell modes did not coincide, it was important to investigate and compare the effect of different imperfections models on the static and dynamic characteristics of such shells. The choosing the shape imperfections model of a long flexible cylindrical shell subjected to force couples, the natural and buckling modes of which did not coincide, was studied using procedures of the finite element analysis software NASTRAN. The shell wall as a set of plat rectangular elements with six degrees of freedom at the node in the cylindrical coordinate system was modeled. The action of force couples as the concentrated forces were distributed at the nodes of the shell edges in accordance to the presentation of A.S. Volmir. The linear buckling problem and the geometrical nonlinear static analysis of the perfect shell by the Lanzosh method and the Newton-Raphson one were performed, respectively. The long half-waves buckling mode was taken as the first shell imperfections model. The modeling of the second shape imperfections as the first natural mode of the perfect shell using the natural vibration analysis by the Lanzosh method was performed. The different amplitudes of geometrical imperfections in proportion to the shell thickness using a program adapted to this software were set. The results of the geometrical nonlinear static analysis of the imperfect shell by the Newton-Raphson method showed that the shape imperfection model in the form of long half-waves more reduced the values of critical buckling loads. Investigations of natural shell vibrations by the Lanzosh method revealed the same influence of different imperfections models on the natural frequencies and natural forms. We think that the shape imperfections model in the form of long half-waves in studies of forced vibrations and dynamic stability of a long flexible cylindrical shell subjected to force couples will be more effective.

**Key words:** long flexible cylindrical shell, shape imperfections model, force couple, finite element method, stability, buckling, natural vibrations.

**Introduction.** Long flexible cylindrical shells are elements of pipelines, aircraft and other structures. The question of their stability under pure bending in two directions was studied [1-4]. For the first time, L. Brazier (1927) considered the geometrically nonlinear dependence of the shell deformation on the moment of the force couples under the assumption that all shell cross-sections are deformed in the same way during bending. Taking into account the formation of local dents, V. Flugge first theoretically investigated the stability of such a shell. In the future, this model of buckling analysis of the cylindrical shell during bending became the prevailing one. It was developed by many researchers, the results of whose works in the well-known monographs of S.P. Tymoshenko (1961), A.S. Volmir and others are detailed [2, 4].

The first researchers of the dynamic stability of elastic systems were V.M. Belyaev (1924), N.M. Krylov, N.N. Bogolyubov (1935), V.A. Bodner (1938), V.N. Chalomey (1939) and others. The dynamic stability of cylindrical shells was first investigated by A.N. Markov (1949) and O.D. Oniashvili (1950). But the problem of dynamic stability of long cylindrical shells

under pure bending remains insufficiently investigated. The problem lies in its complexity and the lack of the required number of experimental data.

It is known that the presence of small shape imperfections of thin-walled shells, which arise in the process of their manufacture, transportation and operation, can significantly reduce the critical value of static or dynamic buckling load and lead to emergency situations [1-14]. In the articles [7-10, 11-13], the authors presented a numerical technique that made it possible to estimate the effect of geometric imperfections of cylindrical shells on their bearing capacity under static loads. The first bifurcation mode as a model of shell imperfections under the action of one type of load (surface pressure, axial compression) was taken. When the shell was subjected to a combined load, two cases were considered: when two loads were orthogonal, the imperfection model was formed as the combination of buckling modes of the perfect shell subjected to individual load with the corresponding combination coefficients; when two loads were non-orthogonal – in the deformation form of the shell under operational loads or in the limit state, which by geometrical nonlinear static analysis was obtained.

The issue of modeling the shape imperfections of thin-walled shells in dynamics problems was little studied. In cases when the natural modes of the perfect shells coincided with the bifurcation modes, the question of choosing a of shape imperfection model did not arise. When these shapes do not coincide, it was important to investigate their effect on the dynamic characteristics and the critical dynamic load values. For example, in the article [12], the authors performed a modal and nonlinear dynamic analysis of the stability of the tank shell with variable thickness under surface pressure. The shape imperfections model in the lower bifurcation buckling mode was presented. The study of natural vibrations of the tank shell showed that an increase in the imperfection amplitude led to a slight decrease in the natural frequencies and amplitudes of the natural forms, the number of circumferential full waves in the corresponding modes did not change. Such an imperfections model in studies of the dynamic stability of the tank shell was effective. A significant influence of the shape imperfection amplitude on the critical values of the dynamic load and the corresponding stress-strain state of the shell was observed. In the article [13], the dynamic stability of the hemispherical shell under external pressure was investigated. The first bifurcation buckling form of static stability was taken as the imperfection model. A significant influence of imperfection on the critical values of the dynamic load and the deformation shape of the hemispherical shell had been also shown.

The issue of effective modeling of shape imperfections in problems of statics and dynamics of long flexible cylindrical shells during pure bending remains open. In the article, a comparative analysis of two models of shape imperfections of a long flexible cylindrical shell under force couples in the buckling form of long half-waves and the first natural mode was performed.

#### **Finite-element modeling of a long flexible cylindrical shell with shape imperfections.**

Considered a long flexible thin-walled cylindrical shell with a radius  $R = 1$  m, length  $L = 8$  m and thickness  $h = 0,002$  m, made of steel with mechanical characteristics:  $E = 2,06 \cdot 10^{11}$  Pa,  $G = 0,792 \cdot 10^{11}$  Pa,  $\mu = 0.3$ . The finite element model of the perfect shell using the software NASTRAN [15] was constructed. The shell wall was modeled by a set of flat rectangular finite elements with six degrees of freedom at the node in the cylindrical coordinate system. The nodes of the two shell ends were subject to restrictions on movement along the radius and tangent and on rotations around the origin. The action of force couples characterizing by the moments of couples were modeled in the form of concentrated forces, which were distributed in the nodes of the shell ends according to the cosine law with constant value  $F_0$  (N) similarly to the presentation of A.S. Volmir [2].

To determine the effective model of geometric imperfections of a long flexible cylindrical shell during pure bending, the problems of static stability and natural vibrations of a perfect shell were solved. First, the problem of stability of the shell under force couples in a linear formulation was solved by the Lanzosh method and the geometrical nonlinear static analysis using the Newton-Raphson method was solved [15]. The first bifurcation buckling mode (Fig. 3 (a)) and the long

half-waves buckling mode (Fig. 3 (b)) were obtained. The authors adopted the long half-waves buckling mode as the first shape imperfection model of the shell. Fig. 3 (c) showed the dependence of the maximum nodal total displacement of the shell on the load step change. This dependence was non-linear and after the loss of shell stability, the unloading curve coincided with the loading curve. The loss of shell stability occurred under the load, which corresponded to the critical normal stress  $2,4201 \cdot 10^8$  Pa in compressed zone of the shell.

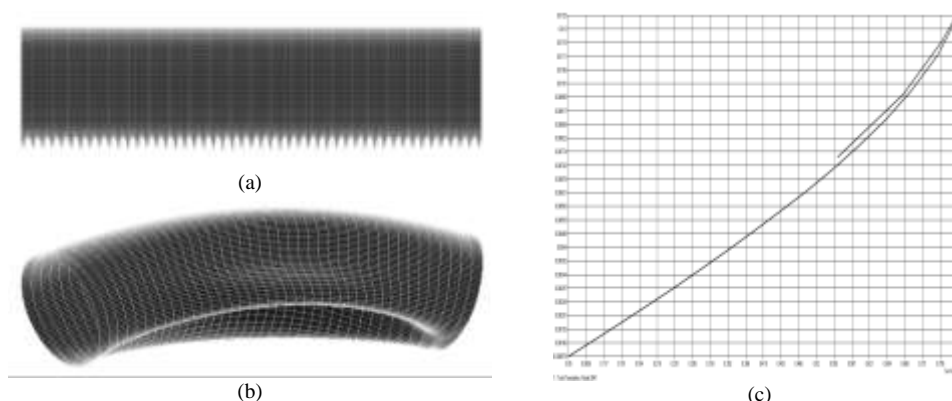


Fig. 1. The first bifurcation buckling mode (a), the long half-waves buckling mode (b), load curve (c) of the perfect shell

The construction of the shape imperfections model as the first natural mode of the shell was considered. For this purpose, the natural vibrations of a perfect shell were calculated using the Lanzosh method [15]. Fig. 2 presented the first five natural modes of the perfect shell and their corresponding natural frequencies. Natural modes had a different number of waves in the circular direction and one half-wave in the longitudinal one.

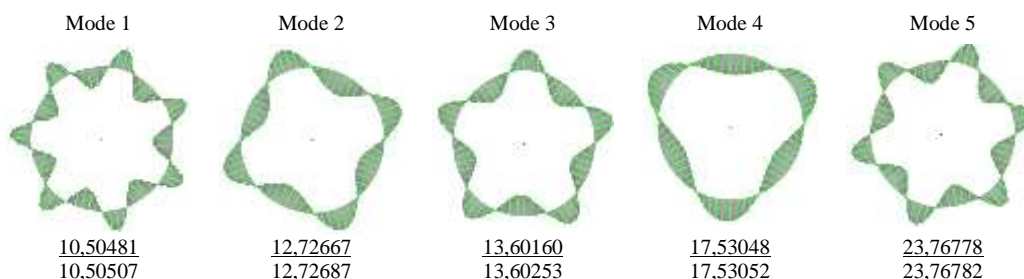


Fig. 2. The first five natural modes and natural frequencies (Hz) of the long perfect cylindrical shell

Thus, the first natural mode of the shell, which has seven waves in the radial direction and one half-wave in the longitudinal one, was taken as the second shape imperfections model of the shell.

**The influence of the shape imperfections models on the shell static stability.** The geometrical nonlinear analysis by Newton-Raphson method [15] was performed for half of the cylindrical shell. The half-shell wall using planar rectangular elements with six degrees of freedom at the node was modeled. The nodes of one shell end were subject to restrictions on movement along the radius and tangent and on rotations around the origin. At the nodes, which lay on the symmetry plane of the shell, restrictions were imposed on movements along the generator and on turns around the radius and tangent. The amplitude of the shell imperfections was equal to  $\delta = [0,5; 1,0; 1,5; 2,0]h$ ,  $h = 0,002$  m – shell thickness. The action of force couples was modeled for all settings in the form of concentrated forces, which were distributed in the shell end nodes according to the cosine law with  $F_0 = 25300$  N. In the tab. 1 the critical

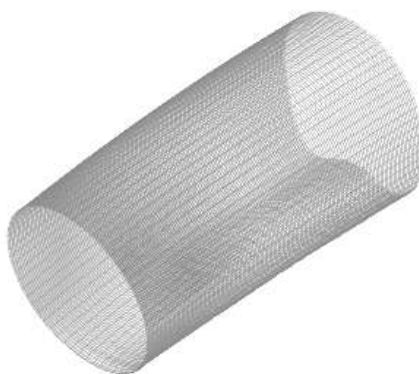
load/stress values (N/Pa) for the shell with different models and amplitudes of shape imperfections were showed. The long half-waves buckling mode of the shell influenced on the critical load/stress greater than the second imperfections model.

Table 1

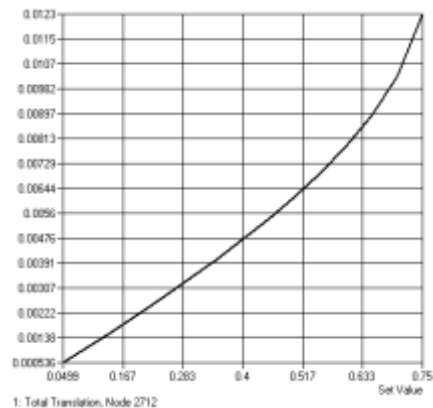
Critical values of load/stress on the shell with different models and amplitudes of shape imperfections (N/10<sup>8</sup> Pa)

Shape imperfections model	Amplitudes of shape imperfections ( $h=0,002$ m)			
	$\delta=0,5h$	$\delta=h$	$\delta=1,5h$	$\delta=2h$
The long half-waves buckling mode	<u>20240</u> 2,063	<u>18976</u> 1,934	<u>16445</u> 1,676	<u>15180</u> 1,547
The first natural mode	<u>23193</u> 2,364	<u>22939</u> 2,338	<u>22086</u> 2,251	<u>21244</u> 2,165

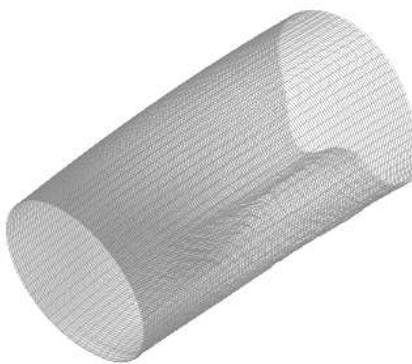
As an example, fig. 3 showed the results of the geometrical nonlinear static analysis of the shell with an amplitude  $\delta=h$  and  $\delta=2h$  of the imperfections, which were modeled in the form of the long half-waves buckling mode (Fig. 1 (a)). The pre-critical behavior of the shell in both cases was similar and nonlinear. Minor nodal deformations in the compressed zone of the shell near its attachment were observed. The maximum deformations had the form of densely located shallow dents in the compression zone of the shell middle. The maximum displacements were 12,5 mm and 10,8 mm for the shell with imperfections amplitude  $\delta=h$  and  $\delta=2h$ , respectively.



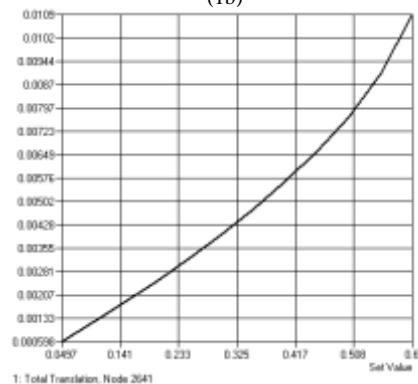
(1a)



(1b)



(2a)



(2b)

Fig. 3. The shell with imperfections in the form of the long half-waves buckling mode with  $\delta=h$  (1) and  $\delta=2h$  (2): buckling mode (a), load curve (b)

The results showed that with an increase in imperfections amplitude the critical load decreased maximum on 36,2% compared to the critical load for a perfect shell, the maximum displacement values also decreased.

Fig. 4 showed the results of the geometrical nonlinear analysis of the shell with an imperfections in the form of the first natural mode (Fig. 2), the amplitude of which was equal to  $\delta=h$  and  $\delta=2h$ .

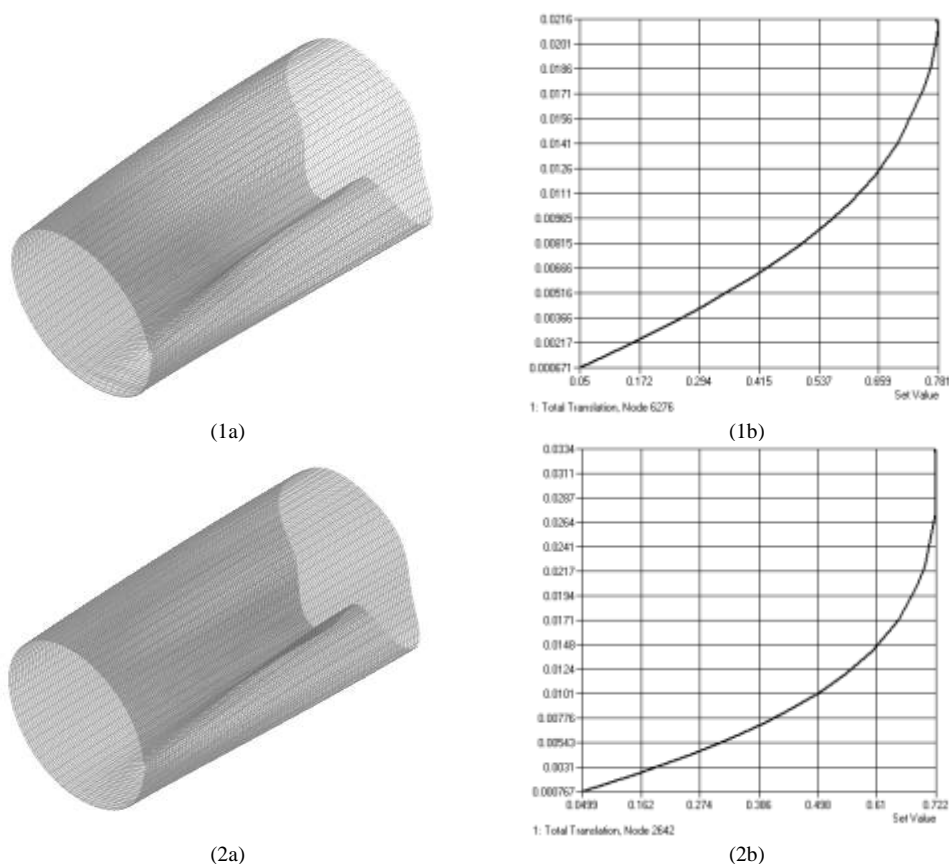


Fig. 4. The shell with imperfections in the form of the first natural mode with  $\delta=h$  (1) and  $\delta=2h$  (2): buckling mode (a), load curve (b)

It can be seen that the pre-critical behavior of the shell was much more nonlinear than in the case of the shell with the first imperfections model (Fig. 3). The increase in imperfections amplitude also affected the increase in nonlinear behavior of the shell (Fig. 4 (1b), (2b)). Minor deformations in the compressed zone of the shell near its attachment and maximum deformations in the compressed zone of the shell middle in the form of a long half-wave dent were observed. When the shell had lost of stability, the maximum displacements of the shell with imperfections amplitude  $\delta=h$  and  $\delta=2h$  were 21,6 mm and 33,4 mm, respectively. An increase in the imperfections amplitude reduced the critical load maximum on 10,5% compared to the critical load for a perfect shell, the maximum nodal displacement values increased.

**Modal analysis of a long flexible cylindrical shell with different shape imperfections models.** The natural vibrations of a flexible cylindrical shell without imperfections were studied by the Lanzosh method [15]. In the tab. 2 the first five natural frequencies of the shell with different models and amplitudes of the shape imperfections were presented.

Table 2

Natural frequencies (Hz) of the shell with different models and amplitudes of shape imperfections

Number frequency	First model – long half-waves buckling mode				Second model – the first natural mode			
	Imperfections amplitude				Imperfections amplitude			
	$\delta=0,5h$	$\delta = h$	$\delta =1,5 h$	$\delta =2 h$	$\delta=0,5h$	$\delta = h$	$\delta =1,5 h$	$\delta =2 h$
1	<u>10,50254</u>	<u>10,49721</u>	<u>10,48988</u>	<u>10,48171</u>	<u>10,50380</u>	<u>10,50077</u>	<u>10,49573</u>	<u>10,48867</u>
	10,50358	10,49941	10,49319	10,48576	10,50503	10,50490	10,50467	10,50436
2	<u>12,73322</u>	<u>12,75346</u>	<u>12,78790</u>	<u>12,83655</u>	<u>12,72606</u>	<u>12,72422</u>	<u>12,72113</u>	<u>12,71677</u>
	12,73436	12,75771	12,79716	12,85274	12,72614	12,72538	12,72122	12,71686
3	<u>13,59332</u>	<u>13,56788</u>	<u>13,52743</u>	<u>13,47379</u>	<u>13,60042</u>	<u>13,59689</u>	<u>13,59103</u>	<u>13,58288</u>
	13,59683	13,58134	13,55674	13,52393	13,60135	13,59782	13,59195	13,58380
4	<u>17,54469</u>	<u>17,58733</u>	<u>17,65830</u>	<u>17,75620</u>	<u>17,52856</u>	<u>17,52263</u>	<u>17,51252</u>	<u>17,49805</u>
	17,54516	17,58903	17,66178	17,76269	17,52864	17,52314	17,51423	17,50207
5	<u>23,78734</u>	<u>23,84533</u>	<u>23,94081</u>	<u>24,07174</u>	<u>23,76858</u>	<u>23,77089</u>	<u>23,77501</u>	<u>23,78061</u>
	23,79157	23,86247	23,97943	24,13963	23,76863	23,77103	23,77504	23,78064

We can see that in the case of modeling the shape imperfections of the shell in the form of a long half-waves buckling mode, with an increase in the imperfections amplitude, there was a decrease in the values of the first three natural frequencies and an increase in the fourth and fifth. In the case of modeling the shape imperfection of the shell in the form of the first natural form, with an increase in the imperfections amplitude, there was a decrease in the values of the first four natural frequencies and an increase in the fifth. The maximum decrease and increase in values of natural frequencies did not exceed 1%.

As an example, in fig. 5 presented the first five natural modes of the imperfect shell. The results showed that they had the same type and the same number of waves in the circular direction for different models and amplitudes of shape imperfections. In all productions one half-wave in the longitudinal direction of the shell was observed.

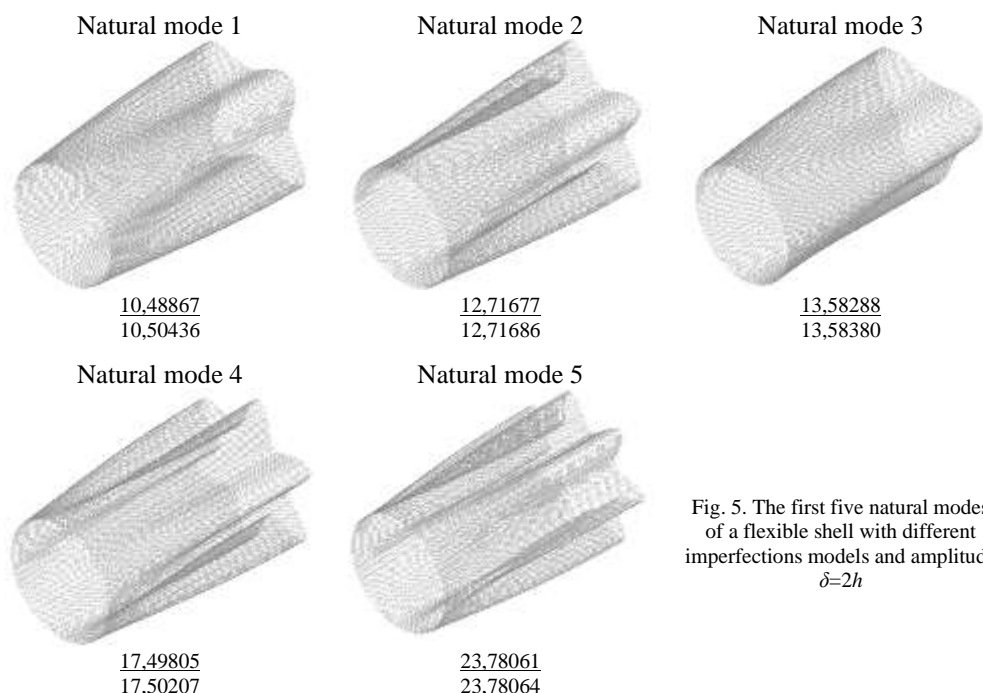


Fig. 5. The first five natural modes of a flexible shell with different imperfections models and amplitude  $\delta=2h$

Comparing the natural modes of the shell without (Fig. 2) and with shape imperfections (Fig. 5), we saw that they did not match. Thus, the first natural modes of the perfect shell, which had seven waves in the radial direction (Fig. 2), coincided with the fifth natural mode of the shell with imperfections (Fig. 5).

**Conclusion.** The choice of shape imperfections model in the problems of forced vibrations and dynamic stability of a long flexible cylindrical shell subjected to force couples is important and necessary. In this article the first step to solving these problems was a comparative assessment of the influence of the different models and the amplitude of the shape imperfections on the static stability and natural vibrations of such a shell. The long half-waves buckling mode and the first natural mode of the perfect shell were taken as the shell imperfections models. The results of the study of the shell with different models and amplitudes of shape imperfections showed that the imperfections model in the form of a long half-waves buckling mode was more effective. In studies of shell natural vibrations, two imperfections models equally affected the natural frequencies and natural modes. We think that the long half-waves buckling mode or a combination of two different imperfections models can be applied to the dynamics problems solving of such a shell.

#### REFERENCES

1. L.N.Donnel and C.C.Wan Effect of imperfections on the buckling of thin cylinders and coluns under axial compressions//J. Appl. Mech., 1950, 17, No.1, 73-83.
2. Volmir A.S. Ustojchivost deformirovanykh sistem [Stability of deformable systems], M.: Nauka, 1967, 984 s.
3. Ray W. Clough, C.Philip Johnson A finite element approximation for the analysis of thin shells//Int. J. Solids Struct., 1968, 4, No.1, 43-60.
4. Grigolyuk E.I., Kabanov V.V. Ustoichivost obolochek [Shell stability], M.: Nauka, 1978, 359 s.
5. Guliaev V.I., Bazhenov V.A., Gotsulyak E.A., Dekhtyaruk E.S., Lizunov P.P. Ustojchivost periodicheskikh processov v nelinejnykh mekhanicheskikh sistemah [Stability of periodic processes in the nonlinear mechanical systems]. Lviv, Vyschia shkola, 1983, 287s.(rus).
6. Grigorenko Ya.M., Guliaev V.I. Nelyneinye zadachy teoryi obolochek y metody ykh resheniya (obzor) [Nonlinear tasks of theory of shells and methods of their decision (review)] // Prykladnaia mekhanika, 1991, T. 27, No.10, 3-23 (rus).
7. Bazhenov V.A., Lukianchenko O.O., Kostina O.V., Gerashchenko O.V. Nonlinear Bending Stability of a Long Flexible Cylindrical Shell with Geometrical Imperfections //Strength of Materials, 2016, 48, No. 2, 308-314.
8. Lukianchenko O.O., Paliy O.M. Chyselne modeliuvannia stiikosti parametrychnykh kolyvan tonkostinnoi obolonky vidiemnoi hausovoi kryvyzny [Numeral design of vibrations stability of the thin-walled shell with negative raycovoï curvature] // Opir materialiv i teoriia sporud: nauk.-tekhn. zbirn., K.: KNUBA, 2018, 101, 45-59 (ukr).
9. Bazhenov V.A., Lukianchenko O.O., Vorona Yu.V., Vabyshchevych O.M. The influence of shape imperfections on the stability of thin spherical shells // Strength of Materials, 2021, 53, No.6, 842-851.
10. Bazhenov V.A., Krivenko O.P., Kozak A.A. Modal analysis of a complex shell structure under operational loads // Strength of Materials and Theory of Structures: Scientific and Technical collected articles, K.: KNUBA, 2021, 106, 3-13.
11. Lukianchenko O.O., Kostina O.V., Paliy O.M. Periodichni kolyvania obolonky rezervuaru z realnymi nedoskonalostiamy formy vid dii poverhnevogo tysku [Periodic vibrations of reservoir shell with the real shape imperfections under pressure] // Opir materialiv i teoriia sporud: nauk.-tekhn. zbirn., K.: KNUBA, 2022, 108, 255-266 (ukr).
12. Lukianchenko O.O., Gerashchenko O.V., Paliy O.M. Nonlinear dynamic analysis of reservoir shell with modeled shape imperfections // Strength of Materials and Theory of Structures: Scientific and Technical collected articles, K.: KNUBA, 2022, 109, 129-140.
13. Lizunov P.P., Lukianchenko O.O., Geraschenko O.V., Kostina O.V. Dynamic stability of a hemispherical shell with shape imperfections // Strength of Materials and Theory of Structures: Scientific and Technical collected articles, K.: KNUBA, 2023, 110, 97-107.
14. Krivenko O.P., Lizunov P.P., Vorona Yu.V., Kalashnikov O.B. A Method for Analysis of Nonlinear Deformation, Buckling, and Vibrations of Thin Elastic Shells of an Inhomogeneous Structure // Strength of Materials and Theory of Structures: Scientific and Technical collected articles, K.: KNUBA, 2023, 110, 131-149.
15. Rudakov K.N. FEMAP 10.2.0. Heometrycheskoe y konechno-elementnoe modelirovanye konstruktysyi [Geometrical and finite-element design of constructions] K: NTTU KPY, 2011, 317 s. (rus).

Лук'янченко О.О., Геращенко О.В., Костіна О.В., Палій О.М.

### ВИБІР МОДЕЛІ НЕДОСКОНАЛОСТЕЙ ФОРМИ В ЗАДАЧАХ ДИНАМІКИ НАВАНТАЖЕНОЇ ПАРАМИ СИЛ ДОВГОЇ ГНУЧКОЇ ЦИЛІНДРИЧНОЇ ОБОЛОНКИ

В задачах динаміки тонкостінних оболонок питання моделювання недосконалостей їх форм є мало дослідженим. У випадках, коли форми власних коливань оболонок збігаються з формами їх втрати стійкості, питання вибору небезпечної моделі недосконалості оболонок не виникає. Коли ці форми не збігаються, важливо дослідити і порівняти вплив недосконалостей на статичні і динамічні характеристики оболонок. В статті розглянуто питання вибору ефективної моделі недосконалостей форми довгої гнучкої циліндричної оболонки при дії пар сил, форми втрати стійкості і власних коливань якої не співпадають. Дослідження виконано із застосуванням обчислювальних процедур комплексу скінченноелементного аналізу NASTRAN. Стінка оболонки змодельована сукупністю плоских прямокутних скінченних елементів з шістьма степенями вільності у вузлі в циліндричній системі координат. Дія пар сил представлена у вигляді зосереджених сил, які розподілені у вузлах торців оболонки за законом косинуса згідно представленню А.С. Вольміра. Розв'язана задача стійкості оболонки в лінійній постановці методом Ланцоша і нелінійна задача статичної за допомогою методу Ньютона-Рафсона. Отримано перша біфуркаційна форма втрати стійкості оболонки і форма деформування оболонки в граничному стані у вигляді довгих півхвиль в стиснутій зоні стінки. За першу модель недосконалості прийнята форма втрати стійкості по довгим півхвилям. Моделювання недосконалостей у вигляді першої форми власних коливань оболонки виконано за допомогою розв'язання задачі на власні коливання методом Ланцоша. Амплітуда різних моделей недосконалостей задавалась пропорційно до товщини оболонки за допомогою адаптованої до комплексу програми. Результати дослідження статичної стійкості оболонки в нелінійній постановці методом Ньютона-Рафсона показали, що модель недосконалості у вигляді форми втрати стійкості по довгим півхвилям є більш ефективною. Дослідження власних коливань оболонки методом Ланцоша виявили однаковий вплив різних моделей недосконалостей на частоти і форми власних коливань. Вважаємо, що модель у вигляді згину оболонки по довгим півхвилям в дослідженнях вимушених коливань або динамічної стійкості довгої гнучкої циліндричної оболонки при дії пар сил є більш ефективною.

**Ключові слова:** довга гнучка циліндрична оболонка, недосконалисть форми, пара сил, метод скінченних елементів, стійкість, біфуркація, власні коливання.

УДК 539.3

Лук'янченко О.О., Геращенко О.В., Костіна О.В., Палій О.М. **Вибір моделі недосконалостей форми в задачах динаміки навантаженої пари сил довгої гнучкої циліндричної оболонки**// Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2023. – Вип. 111. – С. 65-73.

Досліджено вплив різних моделей і амплітуд недосконалостей форми на статичну стійкість і власні коливання довгої гнучкої циліндричної оболонки з урахуванням дії пар сил. За моделі недосконалостей прийнято форма втрати стійкості оболонки у вигляді довгої півхвилі та перша форма власних коливань досконалої оболонки. Задача власних коливань та геометрична нелінійна задача статичної оболонки досліджено методами Ланцоша та Ньютона-Рафсона відповідно. За модель недосконалостей в задачах вимушених коливань і динамічної стійкості даної оболонки може бути прийнята модель у вигляді довгої півхвилі або комбінація двох різних моделей недосконалостей.

Табл. 2. Іл. 5. Бібліогр. 15 назв.

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Lukianchenko O.O., Geraschenko O.V., Kostina O.V., Paliy O.M. **Choice of the shape imperfections model in dynamics problems of a long flexible cylindrical shell subjected to force couples** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – Kyiv: KNUBA, 2023. – Issue 111. – P. 65-73.

Influence of the different models and the amplitudes of the shape imperfections of a long flexible cylindrical shell subjected to force couples on the static stability and natural vibrations was investigated. The long half-waves buckling mode and the first natural mode of the perfect shell were taken as the shape imperfections models. The problem of natural vibrations and the geometrical nonlinear static analysis of the shell were performed by the Lanzosh method and the Newton-Raphson one, respectively. The long half-waves buckling mode or a combination of two different imperfections models can be applied to the solving of forced vibrations and dynamic stability of such a shell.

Tab. 2. Fig. 5. Ref. 15.

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