UDC 539.3

V.A. Bazhenov[1], Doctor of Sciences
O.S. Pogorelova[1], Candidate of Sciences
T.G. Postnikova[1], Candidate of Sciences

[1]*Kyiv National University of Construction and Architecture*
31 Povitroflotsky Avenue, Kyiv, Ukraine. 03680

APPLICATION OF PARAMETER CONTINUATION METHOD FOR INVESTIGATION OF VIBROIMPACT SYSTEMS DYNAMIC BEHAVIOUR. PROBLEM STATE. SHORT SURVEY OF WORLD SCIENTIFIC LITERATURE

Authors in their works study vibroimpact system dynamic behaviour by numerical parametric continuation technique combined with shooting and Newton-Raphson's methods. The technique is adapted to two-mass two-degree-of-freedom vibroimpact system under periodic excitation. Impact is simulated by nonlinear contact interaction force based on Hertz's contact theory. Stability or instability of obtained periodic solutions is determined by monodromy matrix eigenvalues (multipliers) based on Floquet's theory. In the present paper we describe the state of problem of parameter continuation method using for nonlinear tasks solution. Also we give the short survey of numerous contemporary literature in English and Russian about parameter continuation method application for nonlinear problems. This method is applied for vibroimpact problem solving more rarely because of the difficulties connected with repeated impacts.

Keywords: vibro-impact system, parametric continuation technique, shooting method, instability, monodromy matrix, multipliers, bifurcation points

Vibroimpact machines and equipment are often encountered in many engineering practice applications. In general, systems with impacts between matching elements play an important role in the theory of mechanical systems vibration. Therefore the study of their dynamic behaviour and vibroimpact motion characteristics in different function conditions is an object of interest for many scientists in the world. Such investigations were developed extensively during the last decades. Many monographs and papers are devoted to this topic, for example, such well-known monographs as [1-3]. But one can face the big difficulties while solving some real and several theoretical tasks. So impact and vibroimpact processes investigations are continued at present too. Among the contemporary publications we want to underline such encyclopedic works as [4-7].

A big attention is paid to stability studying in systems with impacts, periodic motions, bifurcations, grazing bifurcations, slipping bifurcations, singularities at vibroimpact dynamics and other specific problems (see for example [8-12]).

Sticking motion in impact oscillators particularly sticking periodic motions are examined in [13] by methodology of analysis, based on a Predictor-Corrector method. Authors apply analytic predictor-corrector method and continuation method in order to obtain bifurcation diagram. They have determined some specific features such as quasiperiodic sticking motions and so-called rising bifurcation.

In [14] authors study dynamic behaviour of two-degree-of-freedom (2-DOF) cantilever beam with impacts by two methods: the Peterka's method [15] and the method of numerical integration of motion equations. These methods show a good agreement of the results. In general, many authors have obtained bifurcation diagrams by time integration of the dynamics.

In [16] the control of vibroimpact dynamics of a single-sided Hertzian contact forced oscillator is investigated analytically and numerically. This control is focused on the response near the primary resonance.

We apply the parameter continuation method in order to study vibroimpact system dynamic behaviour and periodic motions stability, to find the bifurcation points. We use parametric continuation technique in conjunction with shooting and Newton-Raphson's methods. This technique is powerful numerical tool to solve nonlinear differential equations. It can shrink the time to solve motion equations in steady-state oscillatory regimes up to ten times. Technique also allows finding out solutions step by step for each value of continuation parameter passing by the transitional process. It also opens the possibility to easily distinguish the instability zones and determine the bifurcation points.

Parameter continuation method is known for a long time. The idea to use parameter continuation to investigate solutions of nonlinear equations is coming from H.Poincare (1881-1886) [17]. Apparently, it was M.E.Lahaye (1934) [18] who applied for the first time the continuation method for numerical solution of nonlinear equations. At present parameter continuation method obtains the powerful development and is used very widely for nonlinear problems solving. Very often this methodology is used jointly with shooting method.

There is the fundamental survey of numerical continuation methods [19]. Authors describe two different types of continuation methods – the Predictor-Corrector (P.C.) methods and Piecewise Linear (P.L.) methods. But authors believe "two numerical methods have many common features and are based on similar general principles". Therefore they are presenting both of these methods as continuation methods. There are the fundamental encyclopedic works where methods of nonlinear systems analysis are described [20, 21]. In particular, there is a detailed description of continuation technique combined with shooting method.

Among Russian language works we want to note such fundamental as [22,23,24,]. A parameter continuation method and it's applications to different nonlinear problems are described there. The problems of the best parameterization, particular points, periodic solution branches, periodic processes stability and so on are examined.

In [24] the method of periodic solutions construction for essentially nonlinear systems in conjunction with parametric continuation technique was proposed and developed. This methodology may be applied for the evolution analysis of steady-state oscillatory regimes and allows to solve many different problems for nonlinear systems.

The work [25] adapts the numerical continuation approach to find the periodic, forced steady-state response of a nonlinear system. The method uses an adaptive procedure with a predictor step and a model switching correction step based on Newton-Raphson method. Authors apply this approach to calculate nonlinear frequency response curves for a Duffing oscillator with nonlinear cubic spring and for a low order nonlinear cantilever beam.

In [26] authors use a shooting procedure combined with pseudo-arclength continuation method for the computation of nonlinear normal modes. The method relies on direct numerical time integration (e.g.Runge-Kutta or Newmark schemes) and on the Newton-Raphson algorithm. Authors demonstrate their algorithm for weakly and strongly nonlinear two-degree-of-freedom systems with one and two cubic stiffnesses.

A methodology based on shooting technique, Newmark time integration scheme and Newton-Raphson iteration method is used in [27] for predicting the periodic responses of nonlinear systems with large number of degrees of freedom.

Let us note by the way that one often considers parameter continuation method belonging to homotopy methods. The words "continuation" and "homotopy" are often used interchangeably as synonyms. However it is pointed out in [28] that between these methods there are both subtle and fundamental distinctions which have been discussed in the literature (see for example [29, 30]).

However, parameter continuation method is applied for vibroimpact problem solving more rarely because of the difficulties connected with repeated impacts.

In [31] the authors apply a parametric continuation scheme in conjunction with the shooting method, in order to study the dynamics of periodically forced piecewise non-linear systems. In particular they consider an impact pair; the continuation parameter is non-dimensional excitation frequency. In [32] authors also use parametric continuation scheme based on the shooting method with using of Newton-Raphson technique for obtain the steady-state response of two-degree-of-freedom piecewise non-linear system. They consider in particular three-degree-of-freedom torsional model of an autonomic transmission. In [33] author rearranges the numerical continuation method. He uses a shooting method for finding periodic solutions. It is based on direct numerical time integration and Newton-Raphson algorithm. Continuation is performed with these methods by incrementing the forcing frequency. Author recommends well developed continuation packages for nonlinear system calculations, AUTO and MATCONT which are suited towards calculating periodic solutions of autonomous systems. To validate the proposed scheme author re-investigated the specific impact pair problem analyzed in [31].

In [34, 35] authors investigate preloaded vibro-impacting Hertzian contact by experimental and numerical methods. They use a classical time integration method in conjunction with a shooting method and with continuation technique. Numerical simulations for single-degree-of-freedom impact oscillator show a very good agreement with experimental results.

In [36] authors consider the motion of the impact oscillator subjected to harmonic excitation. They show that an accurate description of the dynamics is possible by a continuation method and that the stability of multiple impact periodic responses can be studied analytically.

In [37] authors develop the shooting method for impact systems. They treat the impact phenomenon where the sign of the velocity changes instantaneously and it is expressed by using a coefficient of restitution. The authors propose a shooting method for impact vibration systems that contain not only discontinuous force but also the discontinuous changes in momentum. The method consisting of the Newton-Raphson method and analytical solutions of individual linear equations of motiom is formulized. The authors call this shooting method using analytical solutions as Exact+Shooting.

The differential equations of strongly nonlinear vibroimpact systems motion contain many different parameters. The question about solution changing under parameters changing is highly important. Therefore the parametric continuation method applying is sufficiently natural in this case. In our works [38-45] we have used the parameter of external loading amplitude λ and its frequency ω as continuation parameters.

We applied parametric continuation technique to study vibroimpact systems dynamics. Theory of parametric continuation use for investigation of multi-dimensional vibroimpact systems was worked out in [38]. In this work the impact was simulated by boundary conditions with restitution coefficient use based on classical stereomechanical shock theory. The theory and numerical analysis of dynamic states for two-degree-of-freedom systems with internal and external impact contact were performed when the impact was simulated in such way [39]. In particular, the frequency-response curves were constructed and oscillatory motion stability was analyzed [40]

In [41-45] the theory and technique of parametric continuation application to 2-DOF vibroimpact system analysis were elaborated when the impact was simulated by contact interaction force. The loading curves and the frequency-response curves were constructed in large range of parameters values. The stability of obtained oscillatory motions was analyzed.

The main idea of continuation by parameter λ consists in construction of the solution $x_k(\lambda_k)$ pushing from the certain solution $x_0(\lambda_0)$, moving along the solution curve K and using the information about previous solution on every step. Usually the numerical realization of solution continuation is fulfilled as some process stepped by parameter. The realization of such stepped processes may use different iterative techniques. Their large variety is known. Often one can understand and present them independently of common scheme parametric continuation method.

In our works [38-45] and in other ones we have used parametric continuation technique combined with shooting and Newton-Raphson's methods for analysis of dynamic behaviour of two-degree-of-freedom vibroimpact system. We have simulated the impact by nonlinear contact interaction force based on Hertz's contact theory. We have performed the detailed analysis of system dynamic states in large range of external loading amplitude and frequency values. We have found the stability and instability zones of oscillatory motion. We have determined other oscillatory regimes which also are realized in instability zones. We have marked the bifurcation points, in particular, the discontinuous bifurcation points. The Poincare sections have been constructad for examination of regular or chaotic system states.

So in our opinion such investigations are very actually and really have got a novelty.

REFRENCES

1. *Babickii V. I.* Theory of vibro-impact systems //Moskva, Nauka. – 1978.
2. *Kobrinsky A. E., Kobrinsky A. A.* Vibroimpact Systems. – 1973.
3. *Ragulskene V. L.* Vibro-impact Systems //Mintis, Vilna. – 1974.
4. *Ibrahim R. A.* Vibro-impact dynamics: modeling, mapping and applications. – Springer, 2009. – T. 43.
5. *Stronge W. J.* Impact mechanics. – Cambridge university press, 2004.
6. *Ivanov A. P.* Impact oscillations: linear theory of stability and bifurcations //Journal of Sound and Vibration. – 1994. – T. 178. – №. 3. – C. 361-378.
7. *Luo A. C. J., Guo Y.* Vibro-impact Dynamics. – John Wiley & Sons, 2012.
8. *Ma Y. et al.* The nature of the normal form map for soft impacting systems //International Journal of Non-Linear Mechanics. – 2008. – T. 43. – №. 6. – C. 504-513.

9. *Ajibose O. et al.* Influence of contact force models on the global and local dynamics of drifting impact oscillator //Proceedings of the 8th World Congress on Computational Mechanics (WCCM'08) and 5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS'08). – 2008.
10. *Bishop S. R.* Impact oscillators //Philosophical Transactions of the Royal Society of London. Series A: Physical and Engineering Sciences. – 1994. – Т. 347. – №. 1683. – С. 347-351.
11. *Mikhlin Y. V., Vakakis A. F., Salenger G.* Direct and inverse problems encountered in vibro-impact oscillations of a discrete system //Journal of sound and vibration. – 1998. – Т. 216. – №. 2. – С. 227-250.
12. *Foale S., Bishop S. R.* Bifurcations in impact oscillations //Nonlinear Dynamics. – 1994. – Т. 6. – №. 3. – С. 285-299.
13. *Toulemonde C., Gontier C.* Sticking motions of impact oscillators //European Journal of Mechanics-A/Solids. – 1998. – Т. 17. – №. 2. – С. 339-366.
14. *Blazejczyk-Okolewska B., Czolczynski K., Kapitaniak T.* Dynamics of a two-degree-of-freedom cantilever beam with impacts //Chaos, Solitons & Fractals. – 2009. – Т. 40. – №. 4. – С. 1991-2006..
15. *Peterka F.* An investigation of the motion of impact dampers, paper I, II, III //Strojnicku Casopis XXI, с. – 1971. – Т. 5.
16. *Bichri A., Belhaq M., Perret-Liaudet J.* Control of vibroimpact dynamics of a single-sided Hertzian contact forced oscillator //Nonlinear Dynamics. – 2011. – Т. 63. – №. 1-2. – С. 51-60.
17. *Poincaré H.* Sur l'équilibre d'une masse fluide animée d'un mouvement de rotation //Acta mathematica. – 1885. – Т. 7. – №. 1. – С. 259-380.
18. *Lahaye E.* Une méthode de résolution d'une catégorie d'équations transcendantes //CR Acad. Sci. Paris. – 1934. – Т. 198. – С. 1840-1842.
19. *Allgower E. L., Georg K.* Introduction to numerical continuation methods. – SIAM, 2003. – Т. 45..
20. *Nayfeh A. H., Balachandran B.* Applied nonlinear dynamics: analytical, computational and experimental methods. – John Wiley & Sons, 2008.
21. *Seydel R.* Practical bifurcation and stability analysis. – New York : Springer, 2010.
22. *Shalashilin V. I., Kuznetsov E. B. Parametric* Continuation Method and Optimal Parametrization. – 1999.
23. *Grigolyuk E. I., Shalashilin V. I.* Problems of Nonlinear Deformation: Parameter Continuation Method in Nonlinear Problems of Solid Mechanics. – 1988.
24. *Gouliaev V. I. et al.* Stability of Periodical Processes in Non-Linear Mechanical Systems //Vyshcha Shkola, Kiev. – 1983.
25. *.Sracic M. W., Allen M. S.* Numerical continuation of periodic orbits for harmonically forced nonlinear systems //Civil Engineering Topics, Volume 4. – Springer New York, 2011. – С. 51-69.
26. *Peeters M. et al.* Nonlinear normal modes, Part II: Toward a practical computation using numerical continuation techniques //Mechanical systems and signal processing. – 2009. – Т. 23. – №. 1. – С. 195-216.
27. *Patel B. P., Ibrahim S. M., Nath Y.* Periodic response of nonlinear dynamical system with large number of degrees of freedom //Sadhana. – 2009. – Т. 34. – №. 6. – С. 1033-1037.
28. *Watson L. T.* Theory of globally convergent probability-one homotopies for nonlinear programming //SIAM Journal on Optimization. – 2001. – Т. 11. – №. 3. – С. 761-780.
29. *Ge Y. et al.* Probability-one homotopy algorithms for full and reduced order H 2/H∞ controller synthesis //Decision and Control, 1994., Proceedings of the 33rd IEEE Conference on. – IEEE, 1994. – Т. 3. – С. 2672-2677.
30. *Watson L. T., Billups S. C., Morgan A. P.* Algorithm 652: HOMPACK: A suite of codes for globally convergent homotopy algorithms //ACM Transactions on Mathematical Software (TOMS). – 1987. – Т. 13. – №. 3. – С. 281-310.

31. *Padmanabhan C., Singh R.* Analysis of periodically excited non-linear systems by a parametric continuation technique //Journal of Sound and Vibration. – 1995. – Т. 184. – №. 1. – С. 35-58.
32. *Padmanabhan C., Singh R.* Dynamics of a piecewise non-linear system subject to dual harmonic excitation using parametric continuation //Journal of Sound and Vibration. – 1995. – Т. 184. – №. 5. – С. 767-799.
33. *Byrtus M. Dynamic* Analysis of Reduced Order Large Rotating Vibro-Impact Systems //World Academy of Science, Engineering and Technology, International Science Index 83, International Journal of Mechanical, Aerospace, Industrial and Mechatronics Engineering, 2003, 7(11), 1236 - 1243.
34. *Rigaud E., Perret-Liaudet J.* Experiments and numerical results on non-linear vibrations of an impacting Hertzian contact. Part 1: harmonic excitation //Journal of Sound and Vibration. – 2003. – Т. 265. – №. 2. – С. 289-307.
35. *Perret-Liaudet J., Rigaud E.* Response of an impacting Hertzian contact to an order-2 subharmonic excitation: Theory and experiments //Journal of Sound and Vibration. – 2006. – Т. 296. – №. 1. – С. 319-333.
36. *Gontier C., Toulemonde C.* Approach to the periodic and chaotic behaviour of the impact oscillator by a continuation method //European journal of mechanics. A. Solids. – 1997. – Т. 16. – №. 1. – С. 141-163.
37. *Yoshitake Y. et al.* Development of shooting method for impact systems //JSME International Journal Series C. – 2004. – Т. 47. – №. 3. – С. 834-844.
38. *Dekhtyaryuk E.S., Pogorelova O.S., Postnikova T.G., Goncharenko S.N.* Stability of Steedy-State Vibration Regimes// Strength of Materials and Theory of Structures 69 (2001): 10-18. (in Ukrainian)
39. *Dekhtyaryuk E.S., Pogorelova O.S., Postnikova T.G., Goncharenko S.N.* Analysis of Steady-State Vibroimpact Processes in Elastic systems at Internal Impact Contact// Strength of Materials and Theory of Structures 73 (2003): 31-44. (in Ukrainian)
40. *Dekhtyaryuk E.S., Pogorelova O.S., Postnikova T.G., Goncharenko S.N.* Чисельні методи побудови амплітудно-частотних характеристик періодичних режимів коливань віброударних систем // Strength of Materials and Theory of Structures 70 (2002): 69-81. (in Ukrainian)
41. *Bazhenov V.A., Pogorelova O.S., Postnikova T.G.* Dynamic behaviour analysis of different types vibroimpact systems. LAP LAMBERT Academic Publ. GmbH and Co. KG Dudweiler, Germany, 2013. (in Russian)
42. *Bazhenov V.A., Pogorelova O.S., Postnikova T.G.*The development of continuation after parameter method for vibroimpact systems provided the impact is simulated by contact interaction force// Strength of Materials and Theory of Structures 87 (2011): 63-73. (in Ukrainian)
43. *Bazhenov V.A., Pogorelova O.S., Postnikova T.G.*The realization of parameter continuation method for vibroimpact systems at loading curves construction// Strength of Materials and Theory of Structures 88 (2011): 56-64. (in Ukrainian)
44. *Bazhenov V.A., Pogorelova O.S.,* Postnikova T.G. Theoretical principles of dynamic behavior analysis for vibroimpact systems// Strength of Materials and Theory of Structures 89 (2012): 39-49. (in Ukrainian)
45. *Bazhenov V.A., Pogorelova O.S., Postnikova T.G.*principles Parameter continuation method using for analysis of vibroimpact system dinamic behaviour// Strength of Materials and Theory of Structures 90 (2012): 16-30. (in Ukrainian)

Стаття надійшла до редакції 13.12.2014 р.

Баженов В.А., Погорелова О.С., Постнікова Т.Г.

ВИКОРИСТАННЯ МЕТОДУ ПРОДОВЖЕННЯ РОЗВ'ЯЗКУ ЗА ПАРАМЕТРОМ ДЛЯ ДОСЛІДЖЕННЯ ДИНАМІЧНОЇ ПОВЕДІНКИ ВІБРОУДАРНИХ СИСТЕМ. СТАН ПРОБЛЕМИ. КОРОТКИЙ ОГЛЯД СВІТОВОЇ НАУКОВОЇ ЛІТЕРАТУРИ

Автори в своїх роботах вивчають динамічну поведінку віброударної системи за допомогою чисельної методики продовження розв'язку за параметром, комбінованим з методом стрільби та методом Ньютона-Рафсона. Методика адаптована до двомасової віброударної системи з двома ступнями вільності під періодичним навантаженням. Удар моделюється нелінійною силою контактної взаємодії на основі контактної теорії Герца. Стійкість чи нестійкість отриманих періодичних розв'зків визначається власними числами матриці монодромії (мультиплікаторами) на основі теорії Флоке. В цій статті описаний стан проблеми застосування методу продовження за параметром для розв'язку нелінійних задач. Також наведений короткий огляд численної сучасної світової літератури англійською та російською мовами про використання методики продовження до розв'язку нелінійних проблем. Для аналізу динаміки віброударних систем ця методика застосовується значно рідше через труднощі, які пов'язані з наявністю повторюваних ударів.

Ключові слова: віброударна система, методика продовження за параметром, метод стрільби, нестійкість, матриця монодромії, мультиплікатори, точки біфуркації.

Баженов В.А., Погорелова О.С., Постникова Т.Г.

ПРИМЕНЕНИЕ МЕТОДА ПРОДОЛЖЕНИЯ РЕШЕНИЯ ПО ПАРАМЕТРУ ДЛЯ ИССЛЕДОВАНИЯ ДИНАМИЧЕСКОГО ПОВЕДЕНИЯ ВИБРОУДАРНЫХ СИСТЕМ. СОСТОЯНИЕ ПРОБЛЕМЫ. КРАТКИЙ ОБЗОР МИРОВОЙ НАУЧНОЙ ЛИТЕРАТУРЫ

Авторы в своих работах изучают динамическое поведение виброударной системы с помощью численной методики продолжения решения по параметру, комбинированной с методом стрельбы и методом Ньютона-Рафсона. Методика адаптирована к двухмассовой виброударной системе с двумя степенями свободы при периодическом нагружении. Удар моделируется нелинейной силой контактного взаимодействия на основе контактной теории Герца. Устойчивость или неустойчивость полученных периодических решений определяется собственными числами матрицы монодромии (мультипликаторами) на основе теории Флоке. В настоящей статье описывается состояние проблемы использования метода продолжения по параметру для решения нелинейных задач. Также приведен краткий обзор многочисленной современной литературы на английском и русском языках о применении методики продолжения к решению нелинейных проблем. Для анализа динамики виброударных систем эта методика применяется значительно реже из-за трудностей, связанных с наличием повторяющихся ударов.

Ключевые слова: виброударная система, методика продолжения по параметру, метод стрельбы, неустойчивость, матрица монодромии, мультипликаторы, точки бифуркации.