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EFFECTIVENESS OF SEMI-ANALYTICAL FINITE ELEMENT METHOD IN THE NUMERIC ANALYSIS OF DEFORMATION OF NON-HOMOGENEOUS 3D CONSTRUCTIONS SUBJECT TO INITIAL DEVIATION OF FORM

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Static and dynamic numeric models to analyze stress-strain state of 3D uncanonical form bodies are considered in the range of semi-analytical finite element method. Effectiveness and simulation veracity analysis are provided on the base of specific test cases.
Key words: semi-analytical finite element method, body of revolution, geometric imperfection, deviation of form, curvature, stress-strain state.

Introduction

It is known that even a small change in curvature of the body surface has significant influence on distribution of stress-strain state parameters and considerably complicates the analysis of their behavior. Initial geometric imperfections, deflection of shapes, curves, which inevitably appear in the process of manufacturing and installation or designed-in directly, are the determining factors, which require careful reflection in finite element models to get correct solutions.

Successful solving of the problem by finite element method depends on many factors. Principal of them are efficient methods of construction of governing equation, the rational choice of integration algorithms, solution of large equation systems, eigenvalues problems etc. In order to solve these tasks, in many cases, additional hypotheses are introduced to downrange of the class of the objects and processes, but increase efficiency and reduce the computational burden.

Semi-analytical finite element method (SAFEM) is one of such approaches. SAFEM has been widely used to solve problems for the body of revolution and prismatic bodies, which are created by some movement of the generating surface along a closed or non-closed guide without gaps. It combines all the advantages of finite element method by the describing of complicated cross-section configuration of object and analytical approach by the approximation of known and unknown function (load, displacement, velocity, acceleration, strain, stress) along the generating direction. This approach allows to reduce three-dimensional linear problem to the bunch of two-dimensional and build a highly effective procedure of solving in case of non-linear problem.

Semi-analytical finite element method was proposed in [7, 13] as an effective method to research 3D axially symmetrically loaded bodies of revolution in the range of elasticity theory. The method is also known as discrete-continuous finite element method (DCFEM). Similar approaches are developed within the boundary element method [9] and variationally-difference method [8, 10]. Nowadays, SAFEM is used extensively to solve problems of static and dynamic analysis, continual fracture mechanics in case of creep, nonlinear deformation processes of reinforced concrete structures, thermal conductivity [1-4].

Modern engineering practice requires high precision of strength calculation for such kind of structures, which conduce to the necessity of essential clarification of design models, which simultaneously integrate three-dimensional, plate or shell elements. Usage of 3D finite elements (FE) with stiffness matrix, which have been obtained based on the general elasticity theory postulates without additional hypotheses and assumptions, that reflect the geometrical specific of objects, is not productive. Using of different dimension types of finite element approximations in the range of one model requires special rules of joining FE, numerical integration procedures and analysis of the results in the mentioned junction areas. Recent years are marked by extensive use of moment scheme of finite element (MSFE), which is the basis for building of rigidity matrix. General postulates of MSFE are formulated in [3, 11]. The paper [3] has proved effectiveness of prismatic and circular quadrangular finite elements with bilinear law for known and unknown functions for approximation of heterogeneous 3D bodies of revolution and prismatic bodies. Variability accounting of the metric tensor components in the plane of the element cross-section provides a fast convergence of results [5]. Averaging of finite element physical characteristics in its center allows to provide enough easy procedure of integration and decrease the numerical calculations [3].

This paper presents natural extension of semi-analytical finite element method for the behavioral analysis of inhomogeneous circular or prismatic bodies with imperfections under the external loads of different physical types. The objects of study are static and dynamic deformation processes of inhomogeneous circular and prismatic solids of non-canonical form. Purpose of the research are displacements, stresses, deformations and finite element mathematical models, which describe the strained state of three dimensional structures. All presented below solutions are based on generalized SAFEM finite-element models [12] for non-canonical prismatic bodies and bodies of revolution with complex structure of the cross section.

Research of the efficiency and reliability of SAFEM solutions in the analysis of stress-strain state of non-canonical heterogeneous bodies

It is considered the mode of deformation of a thin endless geometrically imperfect cylinder under the uniform external pressure $q=1$ (Fig. 1).

Equations, which describe actual geometry variability of the cylinder cross section:

$$y^{1''}=z^{1'},\ y^{2''}=Z_0^{2'}+c_1\cos 2Z^{3'},\ y^{3''}=Z^{3'}. \qquad (1)$$

Mechanical and geometrical characteristics: $h=1$ (thickness of the cylinder), $Z_0^{2'}=60$ (the radius of the circle, which defines the middle surface of the cylinder), $c_1=0.1$, $E=0.8\times10^6$, $\nu=0.22$.

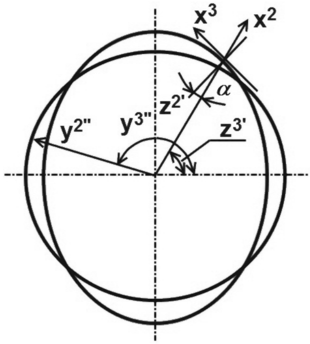

Fig. 1. Endless cylinder with a slight bend

Initially, in order to get stable results, the influence of solution accuracy of equations system ε was researched. Dependence of the hoop stress σ_θ on the value ε for sections I-I and II-II, corresponding to angle $Z^{3'}$, which is equal to 0° and 90°, is shown in the table 1. For the purpose of comparison the values from monograph [6] are represented, which were obtained by the formula:

$$\sigma_\theta = qZ_o^{2'}\left(1\pm\frac{6C_l}{h}\right)\frac{1}{h}. \qquad (2)$$

Table 1

The dependence of hoop stress σ_θ on the ε value

σ_θ	[6]	$\varepsilon=10^{-3}$		$\varepsilon=10^{-4}$		$\varepsilon=10^{-5}$	
		I-I	II-II	I-I	II-II	I-I	II-II
within	96	60.91	61.52	96.63	95.16	97.23	95.75
externally	24	60.11	59.53	24.61	25.75	23.97	25.05

We can see that value $\varepsilon=10^{-3}$ is not sufficient and provides axisymmetrical stress-strain state, which does not correspond to the real actual state. However, in the process of reduction of ε result is quickly stabilizing. In the case of $\varepsilon=10^{-5}$ the difference between examined values of stresses is no more than 1.3%.

Table 2 shows the results of calculations based on the number of Fourier series members.

Table 2

Dependence of stress σ_θ on membership Fourier series number

σ_θ	[6]	$L=2$		$L=3$		$L=4$	
		I-I	II-II	I-I	II-II	I-I	II-II
Within	96	95.49	94.21	96.63	95.16	96.5	95.03
externally	24	23.36	24.27	24.61	25.75	24.77	25.88

Saving even a minimum number of members, that equal to two, provides difference 1.86% between maximum values of stresses ($\varepsilon = 10^{-5}$).

The stress-strain state of thin built-in end cylinder under external pressure is analyzed. Form is described by equation (1). The length of the cylinder is $L = 8Z_o^{2'}$. Other parameters are the same as in the previous example.

Stable result has been got with 20 FE along the meridional direction and two members of Fourier series. Fig. 2 and 3 shows the results of solving by the proposed method and by the method described in [14], where curve 1 is stress distribution on the inner surface side, and curve 2 - on its outer surface. The solid and dotted lines are the result of semi-analytical solution, dashed - as described in [14]. The presence of a minor form deviation from the axially symmetric essentially affects the picture of stress-strain state. Especially substantially axial stresses σ_z are redistributed along circular coordinate, not only in value but also in sign. Moreover, in the nearest built-in end area on zero meridian, component of moment is missed for axial and the hoop stresses. At the same time, on meridian, which corresponds to $Z^{3'} = 90°$, moments are quite considerable.

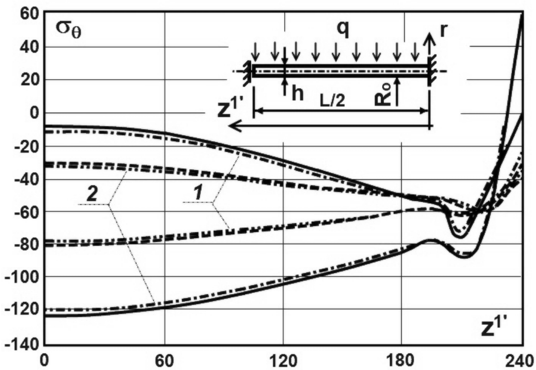

Fig. 2. Hoop stresses σ_θ on the inside and outer surface

The effect of bending size on the stress distribution in the massive endless cylinder, which is loaded by uniformly distributed external pressure $q = 1$ is researched. The body shape is defined as (1) in the previous examples. The following mechanical and geometrical characteristics are accepted: $Z_o^{2'} = 10$, $h = 1$, $\nu = 0.22$, $E = 0.8 \times 10^6$ (Fig. 4).

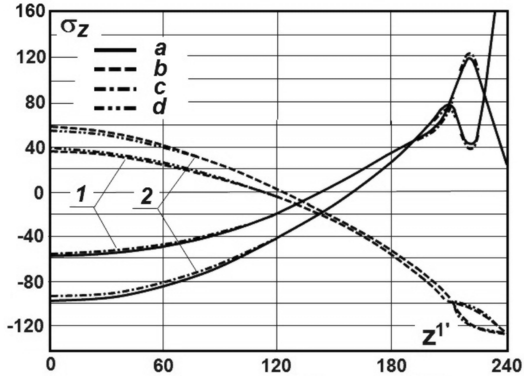

Fig. 3. Axial stresses σ_z . *a* - solution for meridian 90°, *b* - meridian 0°, *c* - method [14] for meridian 90°, *d* - method [14] for meridian 0°

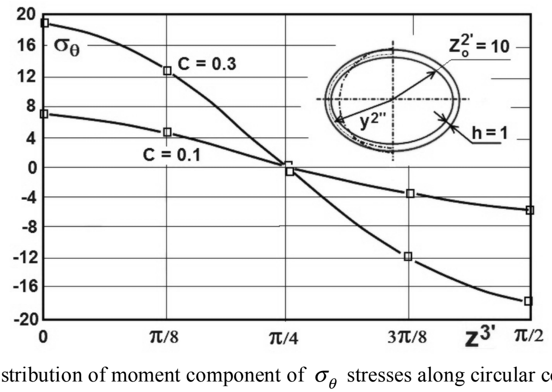

Fig. 4. Distribution of moment component of σ_θ stresses along circular coordinates

The graph shows the distribution of bending hoop component of stresses σ_θ depending on the angular coordinate $Z^{3'} (0 \le Z^{3'} \le \pi/2)$. The solid line describes results of a reference solution, which has been got with the help of traditional 3D finite element method. Squares - values, which calculated on the base of semi-analytical method.

The results are coincide with great precision. Three-dimensional finite element approximation needs 9 FE along circular direction, that, by the same other conditions, gives increasing in numerical operations for problem solution than semi-analytical based approach.

Conclusion

The efficient approach to numerical modeling of deformation of 3D bodies with complex configuration and structure subject to initial deviation of form is created on the base of new modifications of algorithms for solving equilibrium equations of semi-analytical finite element method.

Basic circle and prismatic finite elements with variable geometry and physical-mechanical parameters are developed. The usage of average values of known and unknown functions in the plane of finite element cross section combined with moment scheme and based on the tensor components of physical deformation allows to decrease the volume of mathematical operations associated with the numerical integration.

Effectiveness and simulation veracity analysis are provided on the base of specific test cases.

The developed methods and algorithms are implemented as a package of applications and can be used in the different directions of engineering analysis to determine the safety factors of constructions or their parts, which are three-dimensional bodies of revolution or prismatic bodies with complex structure.

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ЕФЕКТИВНІСТЬ НАПІВАНАЛІТИЧНОГО МЕТОДА СКІНЧЕННИХ ЕЛЕМЕНТІВ В ЗАДАЧАХ ДЕФОРМУВАННЯ НЕОДНОРІДНИХ ПРОСТОРОВИХ КОНСТРУКЦІЙ З УРАХУВАННЯМ ПОЧАТКОВИХ НЕДОСКОНАЛОСТЕЙ ФОРМИ

В рамках напіваналітичного методу скінченних елементів розглянуті чисельні моделі для аналізу напружено-деформованого стану кругових та призматичних неоднорідних тіл неканонічної форми, що знаходяться під дією довільного стаціонарного або нестаціонарного навантаження. Проведено апробацію підходу та дослідження ефективності і достовірності рішень на основі контрольних прикладів.

Ключові слова: напіваналітичний метод скінченних елементів, тіло обертання, геометричні недосконалості, відхилення форми, вигини, напружено-деформований стан.

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ЭФФЕКТИВНОСТЬ ПОЛУАНАЛИТИЧЕСКОГО МЕТОДА КОНЕЧНЫХ ЭЛЕМЕНТОВ В ЗАДАЧАХ ДЕФОРМИРОВАНИЯ НЕОДНОРОДНЫХ ПРОСТРАНСТВЕННЫХ КОНСТРУКЦИЙ С УЧЕТОМ НАЧАЛЬНЫХ НЕСОВЕРШЕНСТВ ФОРМЫ

В рамках полуаналитического метода конечных элементов рассмотрены численные модели для анализа напряженно-деформированного состояния круговых и призматических неоднородных тел неканонической формы, находящиеся под действием произвольного стационарного или нестационарного воздействия. Проведена апробация подхода, исследование эффективности и достоверности решений на основе контрольных примеров.

Ключевые слова: полуаналитический метод конечных элементов, тело вращения, геометрические несовершенства, отклонения формы, изгибы, напряженно-деформированное состояние.