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STABILITY OF CYLINDRICAL ANISOTROPIC SHELLS UNDER AXIAL PRESSURE IN THREE-DIMENSIONAL STATEMENT

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Approach to the solution of a problem of stability of cylindrical anisotropic shells under the influence of the axial compression, based on use of procedure of Bubnova-Galerkin taking into account boundary conditions on surfaces and end faces of a cylindrical shells and a numerical method of discrete orthogonalization is offered. The problem of stability of a cylindrical shells made of the material which characteristics are described by one plane of elastic symmetry is solved. Dependence of sizes of critical loadings on an angle of rotation of the main directions of elasticity of an initial material relatively curvature designs is investigated. Results are presented in the form of schedules and provided in the table, besides, the analysis is carried out them.

Introduction

Approach to the solution of a problem of stability of cylindrical anisotropic shells based on two dimensional theory is presented in detail in works [1,2]. Research on the stability of shell rotation from isotropic and orthotropic materials under axial compression in three-dimensional setting is described in works [3-9]. Solutions for three-dimensional stability of cylindrical orthotropic materials are presented in [5].

Absence of the research on the stability of anisotropic shells based on three-dimensional setting, made from the material that has elastic properties which are described by one plane of elastic symmetry, explains the complexity of the solution for such problems. As previously discovered, this is caused by the connectivity of the strains of tension and shift, bend and tension. Accounting for these in the calculation models results in more complex, comparing to orthotropic materials, equations of stability. Moreover, taking them into account in the models allows to develop and design shell systems from such materials that are capable of operation loading while remaining optimal, for example, based on weight, rigidity. In addition, the resulting three-dimensional solutions can serve as standards of measurement in calculating stability using numerical method of shell constructions that have more complicated geometry.

Problem Description

Under consideration are elastic cylindrical shells on a cylindrical coordinate system r, z, θ. Axis z and θ of which are matching with the lines of the main curves of the construction, r – normal coordinate or the radius of the cylinder that is not dependent on coordinates z and θ. Anisotropy of the material is described by the angle of rotation of the main directions of elasticity of the material relatively to z axis, of the adopted coordinate system, Fig. 1.

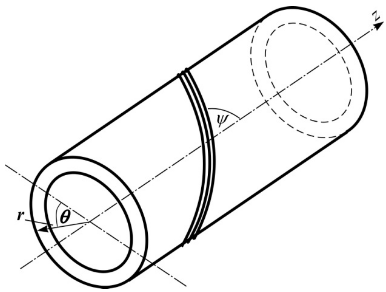

Fig. 1. Cylindrical thick-walled anisotropic shell

Equilibrium Equation written based on work [8]:

$$\frac{\partial \hat{\sigma}_{rr}}{\partial r} = -\frac{1}{r}\left[\hat{\sigma}_{rr} + r\frac{\partial}{\partial z}\left(\hat{\tau}_{zr}\right) + \frac{\partial}{\partial \theta}\left(\hat{\tau}_{\theta r}\right) - \hat{\sigma}_{\theta\theta} + rF_r\right];$$

$$\frac{\partial \hat{\tau}_{rz}}{\partial r} = -\frac{1}{r}\left[\hat{\tau}_{rz} + r\frac{\partial}{\partial z}\left(\hat{\sigma}_{zz}\right) + \frac{\partial}{\partial \theta}\left(\hat{\tau}_{\theta z}\right) + rF_z\right];$$

$$\frac{\partial \hat{\tau}_{r\theta}}{\partial r} = -\frac{1}{r}\left[\hat{\tau}_{r\theta} + \hat{\tau}_{\theta r} + r\frac{\partial}{\partial z}\left(\hat{\tau}_{z\theta}\right) + \frac{\partial}{\partial \theta}\left(\hat{\sigma}_{\theta\theta}\right) + rF_\theta\right], \tag{1}$$

Where F_r, F_z, F_θ - force projection of the vector volume in directions tangent to the coordinate lines r, z, θ; $\hat{\sigma}$, $\hat{\tau}$ – projection of stress on axis of the adopted system of coordinates of shell elasticity:

$$\hat{\sigma}_{rr} = \sigma_{rr} + \sigma_{rr}\frac{\partial u_r}{\partial r} + \tau_{rz}\frac{\partial u_r}{\partial z} + \tau_{r\theta}\frac{1}{r}\frac{\partial u_r}{\partial \theta} - \tau_{r\theta}\frac{1}{r}u_\theta;$$

$$\hat{\sigma}_{zz} = \sigma_{zz} + \sigma_{zz}\frac{\partial u_z}{\partial z} + \tau_{z\theta}\frac{1}{r}\frac{\partial u_z}{\partial \theta} + \tau_{rz}\frac{1}{r}\frac{\partial u_z}{\partial r};$$

$$\hat{\sigma}_{\theta\theta} = \sigma_{\theta\theta} + \sigma_{\theta\theta}\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \sigma_{\theta\theta}\frac{1}{r}u_r + \tau_{z\theta}\frac{\partial u_\theta}{\partial z} + \tau_{r\theta}\frac{\partial u_\theta}{\partial r};$$

$$\hat{\tau}_{rz} = \tau_{rz} + \tau_{rz}\frac{\partial u_z}{\partial z} + \sigma_{rr}\frac{\partial u_z}{\partial r} + \tau_{r\theta}\frac{1}{r}\frac{\partial u_z}{\partial \theta};$$

$$\hat{\tau}_{zr} = \tau_{rz} + \tau_{rz}\frac{\partial u_r}{\partial r} + \sigma_{zz}\frac{\partial u_r}{\partial z} + \tau_{z\theta}\frac{1}{r}\frac{\partial u_r}{\partial \theta} - \tau_{z\theta}\frac{1}{r}u_\theta;$$

$$\hat{\tau}_{r\theta} = \tau_{r\theta} + \tau_{r\theta}\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \tau_{r\theta}\frac{1}{r}u_r + \sigma_{rr}\frac{\partial u_\theta}{\partial r} + \tau_{rz}\frac{\partial u_\theta}{\partial z};$$

$$\hat{\tau}_{\theta r}=\tau_{r\theta}+\tau_{r\theta}\frac{\partial u_r}{\partial r}+\tau_{z\theta}\frac{\partial u_r}{\partial z}+\sigma_{\theta\theta}\frac{1}{r}\frac{\partial u_r}{\partial\theta}-\sigma_{\theta\theta}\frac{1}{r}u_\theta;$$
$$\hat{\tau}_{z\theta}=\tau_{z\theta}+\tau_{z\theta}\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\tau_{z\theta}\frac{1}{r}u_r+\sigma_{zz}\frac{\partial u_\theta}{\partial z}+\tau_{rz}\frac{\partial u_\theta}{\partial r};$$
$$\hat{\tau}_{\theta z}=\tau_{z\theta}+\tau_{z\theta}\frac{\partial u_z}{\partial z}+\tau_{r\theta}\frac{\partial u_z}{\partial r}+\sigma_{\theta\theta}\frac{1}{r}\frac{\partial u_z}{\partial\theta}. \quad (2)$$

Connection between the components of the elasticity and movements [8]:

$$e_{zz}=\frac{\partial u_z}{\partial z}; \qquad e_{\theta\theta}=\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{1}{r}u_r; \qquad e_{rr}=\frac{\partial u_r}{\partial r};$$
$$e_{z\theta}=\frac{\partial u_\theta}{\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial\theta}; \qquad e_{rz}=\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}; \qquad e_{r\theta}=\frac{\partial u_\theta}{\partial r}-\frac{1}{r}u_\theta+\frac{1}{r}\frac{\partial u_r}{\partial\theta}. \quad (3)$$

Here u_z, u_θ, u_r – shift of the cylinder points in direction of axis z, θ, r respectively.

Relationship of the generalized Hooke Law, that connects components of elasticity and tension in changing the angle of the axis of orthotropy relatively to axis z written below [7]:

$$e_{zz}=a_{11}\sigma_{zz}+a_{12}\sigma_{\theta\theta}+a_{13}\sigma_{rr}+a_{16}\tau_{z\theta};$$
$$e_{\theta\theta}=a_{12}\sigma_{zz}+a_{22}\sigma_{\theta\theta}+a_{23}\sigma_{rr}+a_{26}\tau_{z\theta};$$
$$e_{rr}=a_{13}\sigma_{zz}+a_{23}\sigma_{\theta\theta}+a_{33}\sigma_{rr}+a_{36}\tau_{z\theta};$$
$$e_{r\theta}=a_{44}\tau_{r\theta}+a_{45}\tau_{rz};$$
$$e_{rz}=a_{45}\tau_{r\theta}+a_{55}\tau_{rz};$$
$$e_{z\theta}=a_{16}\sigma_{zz}+a_{26}\sigma_{\theta\theta}+a_{36}\sigma_{rr}+a_{66}\tau_{z\theta}. \quad (4)$$

In (4) a_{ij} ($i,j=1,6$) – mechanical constants of the material that has one plane of elastic symmetry, connection of which with constants of the material axis of orthotropy matches with the coordinates $a_{ij}^{/}$, written in accordance with [7]:

$$a_{11}=a_{11}^{/}\cos^4\psi+\left(2a_{12}^{/}+a_{66}^{/}\right)\cos^2\psi\sin^2\psi+a_{22}^{/}\sin^4\psi;$$
$$a_{22}=a_{22}^{/}\cos^4\psi+\left(2a_{12}^{/}+a_{66}^{/}\right)\cos^2\psi\sin^2\psi+a_{11}^{/}\sin^4\psi;$$
$$a_{12}=a_{12}^{/}+\left(a_{11}^{/}+a_{22}^{/}-2a_{12}^{/}-a_{66}^{/}\right)\sin^2\psi\cos^2\psi;$$
$$a_{66}=a_{66}^{/}+4\left(a_{11}^{/}+a_{22}^{/}-2a_{12}^{/}-a_{66}^{/}\right)\cos^2\psi\sin^2\psi;$$
$$a_{16}=\left[2a_{22}^{/}\sin^2\psi-2a_{11}^{/}\cos^2\psi+\left(2a_{12}^{/}+a_{66}^{/}\right)\left(\cos^2\psi-\sin^2\psi\right)\right]\cos\psi\sin\psi;$$
$$a_{26}=\left[2a_{22}^{/}\cos^2\psi-2a_{11}^{/}\sin^2\psi-\left(2a_{12}^{/}+a_{66}^{/}\right)\left(\cos^2\psi-\sin^2\psi\right)\right]\cos\psi\sin\psi;$$
$$a_{13}=a_{13}^{/}\cos^2\psi+a_{23}^{/}\sin^2\psi;$$
$$a_{23}=a_{23}^{/}\cos^2\psi+a_{13}^{/}\sin^2\psi;$$

$$a_{36} = 2\left(a'_{23} - a'_{13}\right)\cos\psi\,\sin\psi\ ;$$
$$a_{33} = a'_{33}\ ;$$
$$a_{44} = a'_{44}\cos^2\psi + a'_{55}\sin^2\psi\ ;$$
$$a_{55} = a'_{55}\cos^2\psi + a'_{44}\sin^2\psi\ ;$$
$$a_{45} = \left(a'_{44} - a'_{55}\right)\cos\psi\,\sin\psi\ , \qquad (5)$$

here ψ – angle of rotation of the main direction of elasticity of fibrous orthotropic material relative to z axis of the adopted system of coordinates.

Methodology for solving the problem

Relationship of the generalized Hooke Law for materials with one plane of elastic symmetry (4), transformed to the following view and used in the system of equations (1)

$$\sigma_{zz} = b_{11}e_{zz} + b_{12}e_{\theta\theta} + b_{16}e_{z\theta} + c_1\sigma_{rr}\ ;$$
$$\sigma_{\theta\theta} = b_{12}e_{zz} + b_{22}e_{\theta\theta} + b_{26}e_{z\theta} + c_2\sigma_{rr}\ ;$$
$$\tau_{z\theta} = b_{16}e_{zz} + b_{26}e_{\theta\theta} + b_{66}e_{z\theta} + c_3\sigma_{rr}\ ;$$
$$e_{rr} = -c_1e_{zz} - c_2e_{\theta\theta} - c_3e_{z\theta} + c_4\sigma_{rr}\ ;$$
$$e_{rz} = a_{45}\tau_{r\theta} + a_{55}\tau_{rz}\ ;$$
$$e_{r\theta} = a_{44}\tau_{r\theta} + a_{45}\tau_{rz}\ , \qquad (6)$$

where b_{ij} ($i, j = 1,\ 2,\ 6$), c_i ($i = \overline{1,4}$) – characteristics, that are calculated using mechanical constants a_{ij} ($i, j = 1{,}3; 5; 6$) of the shell material [4].

Equations of stability based on static criteria of Euler, derived by using system (1), while taking into account the dependencies (2), (3):

$$\frac{\partial\sigma_{rr}}{\partial r} = -\frac{1}{r}\left[\sigma_{rr} + r\frac{\partial}{\partial z}\left(\tau_{zr} + \sigma^0_{zz}\left(\frac{\partial u_r}{\partial z}\right) + \tau^0_{z\theta}\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta} - \frac{1}{r}u_\theta\right)\right) + \frac{\partial}{\partial\theta}\left(\tau_{\theta r} + \tau^0_{z\theta}\left(\frac{\partial u_r}{\partial z}\right)\right) - \left(\sigma_{\theta\theta} + \tau^0_{z\theta}\left(\frac{\partial u_\theta}{\partial z}\right)\right)\right];$$

$$\frac{\partial\tau_{rz}}{\partial r} = -\frac{1}{r}\left[\tau_{rz} + r\frac{\partial}{\partial z}\left(\sigma_{zz} + \sigma^0_{zz}\frac{\partial u_z}{\partial z} + \tau^0_{z\theta}\left(\frac{1}{r}\frac{\partial u_z}{\partial\theta}\right)\right) + \frac{\partial}{\partial\theta}\left(\tau_{z\theta} + \tau^0_{z\theta}\frac{\partial u_z}{\partial z}\right)\right];$$

$$\frac{\partial\tau_{r\theta}}{\partial r} = -\frac{1}{r}\left[\tau_{r\theta} + \left(\tau_{r\theta} + \tau^0_{z\theta}\frac{\partial u_r}{\partial z}\right) + r\frac{\partial}{\partial z}\left(\tau_{z\theta} + \tau^0_{z\theta}\left(\frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{1}{r}u_r\right) + \sigma^0_{zz}\frac{\partial u_\theta}{\partial z}\right) + \frac{\partial}{\partial\theta}\left(\sigma_{\theta\theta} + \tau^0_{z\theta}\frac{\partial u_\theta}{\partial z}\right)\right], \qquad (7)$$

where σ_{zz}^{0} and $\tau_{z\theta}^{0}$ are known subcritical tension values. Since the problem of axial compression is considered here, prevailing tensions are σ_{zz}^{0} та $\tau_{z\theta}^{0}$, which are proved by the results of the research on the stress state of the cylindrical shells made from the orthotropic materials and materials with one plane of elastic symmetry [4, 7], that are represented in the system (7). This points to the heterogeneity of the critical loadings state of the cylindrical shell.

Deriving for such construction the connection between these loadings. Since until the moment when stability is lost the shell preserves non-deformed state, then in subcritical state of deformation $e_{z\theta}$ equals to zero and is described (4):

$$e_{z\theta} = a_{16}\sigma_{zz}^{0} + a_{66}\tau_{z\theta}^{0} = 0\,,$$

Where only prevailing subcritical tension in axial compression are accounted for.

Based on this a connection between the axial and tangential subcritical tensions can be identified:

$$\tau_{z\theta}^{0} = -\frac{a_{16}}{a_{66}}\sigma_{zz}^{0}\,. \tag{8}$$

By substituting in (6) elasticities e_{zz}, $e_{\theta\theta}$, $e_{z\theta}$, e_{rz}, $e_{r\theta}$, e_{rr} with their expressions from (3) and substituting resulting dependencies σ_{zz}, $\sigma_{\theta\theta}$, $\tau_{z\theta}$ in (7), system of equations of stability is obtained:

$$\frac{\partial\sigma_{rr}}{\partial r}=\frac{c_2-1}{r}\sigma_{rr}-\frac{\partial\tau_{rz}}{\partial z}-\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}+\frac{b_{22}}{r^2}u_r+\frac{b_{12}}{r}\frac{\partial u_z}{\partial z}+\frac{b_{26}}{r^2}\frac{\partial u_z}{\partial\theta}+$$

$$+\frac{b_{26}}{r}\frac{\partial u_\theta}{\partial z}+\frac{b_{22}}{r^2}\frac{\partial u_\theta}{\partial\theta}-\sigma_{zz}^{0}\frac{\partial^2 u_r}{\partial z^2}-\frac{2}{r}\tau_{z\theta}^{0}\frac{\partial^2 u_r}{\partial z\partial\theta}+\frac{2}{r}\tau_{z\theta}^{0}\frac{\partial u_\theta}{\partial z};$$

$$\frac{\partial\tau_{rz}}{\partial r}=-c_1\frac{\partial\sigma_{rr}}{\partial z}-\frac{1}{r}\tau_{rz}-\frac{b_{12}}{r}\frac{\partial u_r}{\partial z}-b_{11}\frac{\partial^2 u_z}{\partial z^2}-\frac{b_{66}}{r^2}\frac{\partial^2 u_z}{\partial\theta^2}-\frac{b_{12}+b_{66}}{r}\frac{\partial^2 u_\theta}{\partial z\partial\theta}-$$

$$-\frac{c_3}{r}\frac{\partial\sigma_{rr}}{\partial\theta}-\frac{b_{26}}{r^2}\frac{\partial u_r}{\partial\theta}-\frac{2b_{16}}{r}\frac{\partial^2 u_z}{\partial z\partial\theta}-b_{16}\frac{\partial^2 u_\theta}{\partial z^2}-\frac{b_{26}}{r^2}\frac{\partial^2 u_\theta}{\partial\theta^2}-\sigma_{zz}^{0}\frac{\partial^2 u_z}{\partial z^2}-\frac{2}{r}\tau_{z\theta}^{0}\frac{\partial^2 u_z}{\partial z\partial\theta};$$

$$\frac{\partial\tau_{r\theta}}{\partial r}=-\frac{c_2}{r}\frac{\partial\sigma_{rr}}{\partial\theta}-\frac{2}{r}\tau_{r\theta}-\frac{b_{22}}{r^2}\frac{\partial u_r}{\partial\theta}-\frac{b_{12}+b_{66}}{r}\frac{\partial^2 u_z}{\partial z\partial\theta}-b_{66}\frac{\partial^2 u_\theta}{\partial z^2}-\frac{b_{22}}{r^2}\frac{\partial^2 u_\theta}{\partial\theta^2}-c_3\frac{\partial\sigma_{rr}}{\partial z}$$

$$-\frac{b_{26}}{r}\frac{\partial u_r}{\partial z}-b_{16}\frac{\partial^2 u_z}{\partial z^2}-\frac{b_{26}}{r^2}\frac{\partial^2 u_z}{\partial\theta^2}-\frac{2b_{26}}{r}\frac{\partial^2 u_\theta}{\partial z\partial\theta}-\sigma_{zz}^{0}\frac{\partial^2 u_\theta}{\partial z^2}-\frac{2}{r}\tau_{z\theta}^{0}\frac{\partial u_r}{\partial z}-\frac{2}{r}\tau_{z\theta}^{0}\frac{\partial^2 u_\theta}{\partial z\partial\theta};$$

$$\frac{\partial u_r}{\partial r}=c_4\sigma_{rr}-\frac{c_2}{r}u_r-c_1\frac{\partial u_z}{\partial z}-\frac{c_3}{r}\frac{\partial u_z}{\partial\theta}-c_3\frac{\partial u_\theta}{\partial z}-\frac{c_2}{r}\frac{\partial u_\theta}{\partial\theta};$$

$$\frac{\partial u_z}{\partial r} = a_{55}\tau_{rz} + a_{45}\tau_{r\theta} - \frac{\partial u_r}{\partial z};$$

$$\frac{\partial u_\theta}{\partial r} = a_{45}\tau_{rz} + a_{44}\tau_{r\theta} - \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{1}{r}u_\theta . \quad (9)$$

To solve (9) free setting the value of σ_{zz}^0 and in accordance with (8) determining the value $\tau_{z\theta}^0$.

Solution of the system (9) derived by applying boundary conditions:

On inner $r = r_0$ and outer $r = r_N$ surfaces of the shell

$$\sigma_{rr} = \tau_{rz} = \tau_{r\theta} = 0; \quad (10)$$

On end faces

$$\sigma_{zz} = -\sigma_{zz}^0, \ u_r = u_\theta = 0, \quad (11)$$

That can point to the existence on them of the diaphragms of absolute rigidity in their planes and flexible among them.

In the subsequent transformation of the three-dimensional problem into one-dimensional, that will be solved using method of discrete orthogonalization, using the Bubnova-Galerkin procedure. In accordance with it, separate all functions into trigonometric rows based on coordinates along forming z so that all satisfy boundary conditions (11), and also take into account their frequency based on circular coordinate θ:

$$\sigma_{rr}(r,z,\theta) = \sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\left[y_{1,pk}(r)\cos k\theta + y_{1,mk}^{/}(r)\sin k\theta\right]\sin l_m z;$$

$$\tau_{rz}(r,z,\theta) = \sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\left[y_{2,pk}(r)\cos k\theta + y_{2,mk}^{/}(r)\sin k\theta\right]\cos l_m z;$$

$$\tau_{r\theta}(r,z,\theta) = \sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\left[y_{3,pk}(r)\sin k\theta + y_{3,mk}^{/}(r)\cos k\theta\right]\sin l_m z;$$

$$u_r(r,z,\theta) = \sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\left[y_{4,pk}(r)\cos k\theta + y_{4,mk}^{/}(r)\sin k\theta\right]\sin l_m z;$$

$$u_z(r,z,\theta) = \sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\left[y_{5,pk}(r)\cos k\theta + y_{5,mk}^{/}(r)\sin k\theta\right]\cos l_m z;$$

$$u_\theta(r,z,\theta) = \sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\left[y_{6,pk}(r)\sin k\theta + y_{6,mk}^{/}(r)\cos k\theta\right]\sin l_m z . \quad (12)$$

After performing mathematical transformations and separation of the variables in the equations (9) with the help of the relations (12), derived an infinite system of ordinary differential equations of stability in the normal form Cauchy:

$$\frac{d\bar{y}}{dr}=T(r)\bar{y}\ ,\quad T(r)=t_{i,j}(r)\ , \qquad (13)$$

$$\bar{y}=\left\{y_{1,pk};y_{2,pk};y_{3,pk};y_{4,pk};y_{5,pk};y_{6,pk};y'_{1,mk};y'_{2,mk};y'_{3,mk};y'_{4,mk};y'_{5,mk};y'_{6,mk}\right\}$$

– solving vector function. Non-zero elements of the matrix $T(r)$ shown below:

$$t_{1,1}=\frac{c_2-1}{r},\ t_{1,2}=l_p,\ t_{1,3}=-\frac{k}{r},\ t_{1,4}=\frac{b_{22}}{r^2}+\sigma^0_{zz}l_p^2,\ t_{1,5}=-l_p\frac{b_{12}}{r},$$

$$t_{1,6}=k\frac{b_{22}}{r^2},\ t_{1,10}=\sum_{m=1}^{\infty}\varphi(p,m)2\frac{k}{r}\tau^0_{z\theta}l_m,\ t_{1,11}=\sum_{m=1}^{\infty}\varphi(p,m)k\frac{b_{26}}{r^2},$$

$$t_{1,12}=\sum_{m=1}^{\infty}\varphi(p,m)\left(\frac{b_{26}}{r}l_m+\frac{2}{r}\tau^0_{z\theta}l_m\right),\ t_{2,1}=-c_1l_p,\ t_{2,2}=-\frac{1}{r},\ t_{2,4}=-\frac{b_{12}}{r}l_p,$$

$$t_{2,5}=b_{11}l_p^2+k^2\frac{b_{66}}{r^2}+\sigma^0_{zz}l_p^2,\ t_{2,6}=-k\frac{b_{12}+b_{66}}{r}l_p,\ t_{2,7}=-\sum_{m=0}^{\infty}\varphi(m,p)k\frac{c_3}{r},$$

$$t_{2,10}=-\sum_{m=0}^{\infty}\varphi(m,p)k\frac{b_{26}}{r^2},$$

$$t_{2,11}=\sum_{m=0}^{\infty}\varphi(m,p)\left(2\frac{kb_{16}}{r}l_m+2\frac{k}{r}\tau^0_{z\theta}l_m\right),\ t_{2,12}=\sum_{m=0}^{\infty}\varphi(m,p)\left(b_{16}l_m^2+k^2\frac{b_{26}}{r^2}\right),$$

$$t_{3,1}=k\frac{c_2}{r},\ t_{3,3}=-\frac{2}{r},\ t_{3,4}=k\frac{b_{22}}{r^2},\ t_{3,5}=-k\frac{b_{12}+b_{66}}{r}l_p,$$

$$t_{3,6}=b_{66}l_p^2+k^2\frac{b_{22}}{r^2}+\sigma^0_{zz}l_p^2,\ t_{3,7}=-\sum_{m=1}^{\infty}\varphi(p,m)c_3l_m,$$

$$t_{3,10}=-\sum_{m=1}^{\infty}\varphi(p,m)\left(\frac{b_{26}}{r}l_m+\frac{2}{r}\tau^0_{z\theta}l_m\right),\ t_{3,11}=\sum_{m=1}^{\infty}\varphi(p,m)\left(b_{16}l_m^2+k^2\frac{b_{26}}{r^2}\right),$$

$$t_{3,12}=\sum_{m=1}^{\infty}\varphi(p,m)\left(2\frac{kb_{26}}{r}l_m+2\frac{k}{r}\tau^0_{z\theta}l_m\right),\ t_{4,1}=c_4,\ t_{4,4}=-\frac{c_2}{r},\ t_{4,5}=c_1l_p,$$

$$t_{4,6}=-k\frac{c_2}{r},\ t_{4,11}=-\sum_{m=1}^{\infty}\varphi(p,m)k\frac{c_3}{r},\ t_{4,12}=-\sum_{m=1}^{\infty}\varphi(p,m)c_3l_m,\ t_{5,2}=a_{55},$$

$$t_{5,4}=-l_p,\ t_{5,9}=\sum_{m=0}^{\infty}\varphi(m,p)a_{45},\ t_{6,3}=a_{44},\ t_{6,4}=\frac{k}{r},\ t_{6,6}=\frac{1}{r},$$

$$t_{6,8}=\sum_{m=1}^{\infty}\varphi(p,m)a_{45}\,,\;\; t_{7,4}=-\sum_{m=1}^{\infty}\varphi(p,m)2\frac{k}{r}\tau_{z\theta}^{0}l_m\,,\;\; t_{7,5}=\sum_{m=1}^{\infty}\varphi(p,m)k\frac{b_{26}}{r^2}\,,$$

$$t_{7,6}=\sum_{m=1}^{\infty}\varphi(p,m)\left(\frac{b_{26}}{r}l_m+\frac{2}{r}\tau_{z\theta}^{0}l_m\right),\;\; t_{7,7}=\frac{c_2-1}{r}\,,\;\; t_{7,8}=l_p\,,\;\; t_{7,9}=-k\frac{1}{r}\,,$$

$$t_{7,10}=\frac{b_{22}}{r^2}+\sigma_{zz}^{0}l_p^2\,,\;\; t_{7,11}=-l_p\frac{b_{12}}{r}\,,\;\; t_{7,12}=k\frac{b_{22}}{r^2}\,,\;\; t_{8,1}=-\sum_{m=0}^{\infty}\varphi(m,p)k\frac{c_3}{r}\,,$$

$$t_{8,4}=-\sum_{m=0}^{\infty}\varphi(m,p)k\frac{b_{26}}{r^2}\,,\;\; t_{8,5}=\sum_{m=0}^{\infty}\varphi(m,p)\left(2\frac{kb_{16}}{r}l_m+2\frac{k}{r}\tau_{z\theta}^{0}l_m\right),$$

$$t_{8,6}=\sum_{m=0}^{\infty}\varphi(m,p)\left(b_{16}l_m^2+k^2\frac{b_{26}}{r^2}\right),\;\; t_{8,7}=-c_1l_p\,,\;\; t_{8,8}=-\frac{1}{r}\,,\;\; t_{8,10}=-\frac{b_{12}}{r}l_p\,,$$

$$t_{8,11}=b_{11}l_p^2+k^2\frac{b_{66}}{r^2}+\sigma_{zz}^{0}l_p^2\,,\;\; t_{8,12}=-k\frac{b_{12}+b_{66}}{r}l_p\,,\;\; t_{9,1}=-\sum_{m=1}^{\infty}\varphi(p,m)c_3l_m\,,$$

$$t_{9,4}=-\sum_{m=1}^{\infty}\varphi(p,m)\left(\frac{b_{26}}{r}l_m+\frac{2}{r}\tau_{z\theta}^{0}l_m\right),\;\; t_{9,5}=\sum_{m=1}^{\infty}\varphi(p,m)\left(b_{16}l_m^2+k^2\frac{b_{26}}{r^2}\right),$$

$$t_{9,6}=\sum_{m=1}^{\infty}\varphi(p,m)\left(2\frac{kb_{26}}{r}l_m+2\frac{k}{r}\tau_{z\theta}^{0}l_m\right),\;\; t_{9,7}=-k\frac{c_2}{r}\,,\;\; t_{9,9}=-\frac{2}{r}\,,$$

$$t_{9,10}=k\frac{b_{22}}{r^2}\,,\;\; t_{9,11}=-k\frac{b_{12}+b_{66}}{r}l_p\,,\;\; t_{9,12}=b_{66}l_p^2+k^2\frac{b_{22}}{r^2}\,,$$

$$t_{10,5}=-\sum_{m=1}^{\infty}\varphi(p,m)k\frac{c_3}{r}\,,\;\; t_{10,6}=-\sum_{m=1}^{\infty}\varphi(p,m)c_3l_m\,,\;\; t_{10,7}=c_4\,,\;\; t_{10,10}=-\frac{c_2}{r}\,,$$

$$t_{10,11}=c_1l_p\,,\;\; t_{10,12}=-k\frac{c_2}{r}\,,\;\; t_{11,3}=\sum_{m=0}^{\infty}\varphi(m,p)a_{45}\,,\;\; t_{11,8}=a_{55}\,,\;\; t_{11,10}=-l_p\,,$$

$$t_{12,2}=\sum_{m=1}^{\infty}\varphi(p,m)a_{45}\,,\;\; t_{12,9}=a_{44}\,,\;\; t_{12,10}=\frac{k}{r}\,,\;\; t_{12,12}=\frac{1}{r}\,. \qquad (14)$$

Here $l_m=\frac{m\pi}{L}$, $l_p=\frac{p\pi}{L}$, L – length of the cylinder forming, p, m – wave numbers in Fourier series (12).

Functions $\varphi(p,m)$ and $\varphi(m,p)$ dependent on the whole numerical parameters p and m are determined by the formulas:

$$\varphi(p,m)=\begin{cases}0, & \text{if } p+m \text{ – even number,}\\ \dfrac{2}{\pi}\left(\dfrac{1}{p-m}+\dfrac{1}{p+m}\right) & \text{if } p+m \text{ – odd.}\end{cases}$$

$$\varphi(m,p)=\begin{cases}0, & \text{if } p+m \text{ – even number,}\\ \dfrac{2}{\pi}\left(\dfrac{1}{m-p}+\dfrac{1}{m+p}\right) & \text{if } p+m \text{ – odd.}\end{cases} \qquad (15)$$

Implementation of the resulting system of equations (13) given boundary conditions (11) is conducted by using numerical method of discrete orthogonalization [11]. Problem solution algorithm of stability of shell of rotation that is under axial compression was developed as software package.

Results of the numerical calculations and their analysis

Testing of the solution results based on the recommended approach of stability of cylindrical orthotropic shells under the influence of axial compression were conducted with the values of the boundary voltages derived in [5]. Evaluated shells: radius R=0.6m, length L=2.15m, physical and mechanical properties of the material E_{11}=10,0E_0, E_{22}=2.8E_0, E_{33}=E_0, G_{12}=1.075E_0, G_{13}=G_{23}=2E_0, ν_{21}=0.3, ν_{12}=0.084, ν_{32}=0.22, ν_{31}=0.35, E_0=$1,0\cdot10^6$ MH/m^2. Comparison results of the boundary voltage values are presented in Table 1.

Table 1

Wall Thickness *h, m*	Calculation Results Based on Methodology [5]		Calculation Results Based on Recommended Methodology	
	Number of Waves	Critical Tension σ_{cr}, MH/m^2	Number of Waves	Critical Tension σ_{cr}, MH/m^2
0.012	6	$4.0\cdot10^4$	6	$4.0\cdot10^4$
0.02	5	$6.5\cdot10^4$	5	$6.5\cdot10^4$
0.025	4	$8.0\cdot10^4$	4	$8.0\cdot10^4$

Analysis of the results from test calculations on stability shown in Table 1 points to full conformity of the solutions derived based on the recommended approach, in comparison with the results in work [5]. Unfortunately, the authors of the article were not able to find in the literature reliable data calculations on the stability of anisotropic cylindrical shells in a three-dimensional setting.

In evaluating the recommended methodology, consider cylindrical shells of stable thickness that are created by the cross-laying layers of previously orthotropic material under angles $\pm\psi_i$ to the axis z. Parameters of the shell match with the one used in the previous problem, thickness h equal to 0.03m. Investigated three types of shells that are under axial pressure. First – single layer, second – double layer. Calculations of the third construction is conducted without accounting for anisotropic material constants a_{i6}, $i = 1 \div 3$, a_{45}, meaning – shell was orthotropic.

Since the accepted geometric characteristics of the shell match those of thin shell, the recommended approach to solving three-dimensional problem of stability can be compared to the solutions obtained using two-dimensional theory. Values of the critical loadings calculated using recommended in this work approach are compared with the results using the classical theory, obtained in the work [1].

Calculations of stability is shown in graphs in the Fig. 1 and detailed in Table 1, where number of waves in circular direction denote the moment of loss of stability.

On the graph all critical values F_{cr} are reduced to values, derived using Kirchhoff-Lyav hypothesis F_{cl}^0

$$F^* = \frac{F_{cr}}{F_{cl}^0}.$$

At the angle of laying of the composite $\psi = 0$, based on the working formula [10]

$$F_{cl}^0 = \frac{\sqrt{E_{11}E_{22}}}{\sqrt{(3(1-\nu_{12}\nu_{21}))}}\left[\frac{h^2}{r}\right]\sqrt{\frac{\sqrt{E_{11}E_{22}} + \nu_{12}E_{11} + 2G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}} - \nu_{12}E_{11} + \dfrac{E_{11}E_{22}}{2G_{12}}}}. \quad (17)$$

Markings of the curves on the Fig. 2 are as following. Numbers 1, 2 and 3 denote the types of the shells. Letters (a) and (b) on the Fig. 1 mark the type of the used calculation approach – recommended and the methodology of work [1] respectively. Location of the curves on the Fig. 1 suggest that the value of the critical loadings of axial compression significantly depends on the angle ψ of the home laying of orthotropic material.

Values of the critical axial compression loadings that are shown in Table 2, need to be multiplied by 10^2 *MH/m*.

As shown, the minimal values of the axial critical loadings are applicable to single layer anisotropic shell (curvature 1). As the number of layers increases to two critical axial loadings significantly increases (curvature 2) and gains the largest values in calculations without accounting for anisotropic constants of the material (curvature 3).

Table 2

Angle of Winding ψ, deg	Single Layer Shell				Double Layer Shell (cross-wound shell) $\pm\psi$				Calculation without accounting for anisotropic material constants			
	Recommended methodology		Methodology [1]		Recommended methodology		Methodology [1]		Recommended methodology		Methodology [1]	
	Number of Waves	F_{cr}	Number of Waves	F_{cr}	Number of Waves	F_{cr}	Number of Waves	F_{cr}	Number of Waves	F_{cr}	Number of Waves	F_{cr}
0	4	28.2	4	31.2	4	28.2	4	31.2	4	28.2	4	31.2
10	3	20.0	4	24.4	4	27.4	4	29.7	4	31.0	4	34.5
20	3	16.3	3	20.9	3	28.2	3	30.3	3	32.8	0	42.0
30	3	15.9	3	20.2	2	28.5	2	30.6	3	33.5	0	38.3
40	3	18.6	3	22.9	1	28.8	1	30.8	0	32.7	0	35.0
50	2	24.8	3	30.4	0	30.7	0	32.3	0	32.6	0	35.2
60	2	27.6	1	31.6	3	30.0	1	36.1	2	31.6	0	37.9
70	4	27.4	4	31.6	4	27.5	4	31.4	2	29.9	4	41.1
80	4	26.6	4	29.9	4	26.3	4	29.4	2	28.2	4	34.0
90	4	28.0	4	30.9	4	28.0	4	30.9	4	28.0	4	30.9

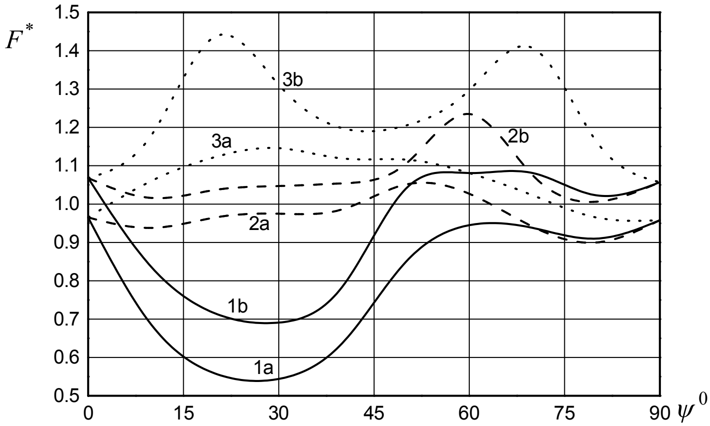

Fig.. 2. Dependencies of critical loadings on the angle of rotation of orthotropic axis

Moreover, in cases of single and double layer shells the results of the critical loadings received based on the recommended approach practically match values of the classical theory [1] (differences do not exceed 9%). Stability calculations without accounting for anisotropic constants of the material show significant differences between the results of critical loadings based on the recommended methodology and the classical theory [1], which, however, does not exceed 27%. It's important to point out that for all three types of cylinders critical values of the axial compression, calculated by using the recommended methodology, are smaller than the calculated values based on the classical approach.

Conclusion

As shown, the application of the recommended approach that is based on the usage of Bubnova-Galerkin procedure, that takes into account boundary conditions on surfaces and end faces of a cylindrical shells and a numerical method of discrete orthogonalization, allows to solve in three-dimensional settings the problems of stability of cylindrical shells that are under axial load, made from one plane of elastic symmetry materials in a wide range of geometric and mechanical characteristics.

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Семенюк М.П., Трач В.М., Подворний А.В.
СТІЙКІСТЬ ЦИЛІНДРИЧНИХ АНІЗОТРОПНИХ ОБОЛОНОК ПІД ОСЬОВИМ ТИСКОМ У ТРИВИМІРНІЙ ПОСТАНОВЦІ

Запропоновано підхід стосовно реалізації задачі стійкості циліндричних анізотропних оболонок під дією осьового стиску, що базується на застосуванні процедури Бубнова-Гальоркіна при урахуванні граничних умов на поверхнях і торцях циліндричної оболонки та чисельного методу дискретної ортогоналізації. Розв'язано задачу стійкості циліндричної оболонки, що виготовлена з матеріалу, характеристики якого описуються однією площиною пружної симетрії. Досліджено залежність величин критичних навантажень від кута повороту головних напрямів пружності вихідного матеріалу відносно кривин конструкції. Результати представлені у вигляді графіків і наведені у таблиці, окрім того, проведено їх аналіз.

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УСТОЙЧИВОСТЬ ЦИЛИНДРИЧЕСКИХ АНИЗОТРОПНЫХ ОБОЛОЧЕК ПОД ОСЕВЫМ ДАВЛЕНИЕМ В ТРЕХМЕРНОЙ ПОСТАНОВКЕ

Предложен подход к решению задачи устойчивости цилиндрических анизотропных оболочек под действием осевого сжатия, основанный на использовании процедуры Бубнова-Галеркина с учетом краевых условий на поверхностях и торцах цилиндрической оболочки и численного метода дискретной ортогонализации. Решена задача устойчивости цилиндрической оболочки изготовленной из материала, характеристики которого описываются одной плоскостью упругой симметрии. Исследована зависимость величин критических нагрузок от угла поворота главных направлений упругости исходного материала относительно кривизн конструкции. Результаты представлены в виде графиков и приведены в таблице, кроме того, проведен их анализ.