UDC 539.3

NUMERICAL SOLUTION OF THE PROBLEM OF POROUS SOLIDS VIBRATION

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In this paper vibration of fluid-saturated porous solids under the equal distribution load is studied using two different approaches. One of them is analytical way. Biot's equations in terms of displacement, pore pressure, porosity and effective densities are used for one-dimensional column. Using boundary conditions analytical expressions for parameters of stress-strain state: the solid and fluid displacements and stresses are obtained. Another way is Boundary Integral Equation Method. Equilibrium equations for 3-D linear dynamic poroelasticity are presented. Also required components of fundamental solution tensors as weighting displacement fields are obtained and analyzed with the help of the analogy between poroelastisity and thermoelastisity. The solution of the porous solid vibration problem for two types of boundary conditions is presented in the figures. Graphs present the comparison of the normalized solid displacement u_3 at the top and normalized pressure σ_{33} in elastic region and in porous solid of poroelastic region depending on frequency ω that are computed using Boundary Integral Equation and analytical methods. Figures show that graphs of the displacements and pressure in poroelastic and elastic region have the same character but different values. The numerical solution of this problem was calculated using material properties which are corresponding to the Barea Sandstone. It shows that massive porous bodies cannot be modeling as homogeneous elastic media but it is necessary to use two phase model and equations of poroelasticity. Since the agreement between the BEM results and the analytical solution is good so such an approach can be used for development and testing of numerical techniques for analyzing of 3-D porous solids vibration.

Key words: porous media, boundary integral equations, fundamental solution, forced vibrations of the layer.

Introduction

Porous materials are very spread in nature that's why they are widely used in technique and construction. Planning of underground structures, elaborating of fields by dynamic methods need knowledge of proceedings that are occurring in the time of wave propagation in porous saturated media. Laws of elastic theory can't be used for studying of wave propagations in saturated materials because of presence of filler changing the behavior of such materials. As a result three different compressional waves, the longitudinal fast wave, the second longitudinal slow wave, and the third transversal slow wave, are occurring during elastic vibration in saturated media. That's why we need another theory for calculating of porosity [1], [2], [3].

1. **Basic Relations**

Consider the two phase model of the poroelastic body. Following Biot [4] the system of differential equations for harmonic vibration is expressed as:

$$\mu u_{j,kk}+(\lambda+\mu)u_{k,kj}+\rho^*\omega^2 u_j-\gamma^*\tau_{,j}=0, \tag{1}$$

$$\tau_{,kk}+\frac{i\omega}{\kappa^*}\tau+i\omega\eta^* u_{k,k}=0, \tag{2}$$

where u_j – the jth component of the displacement vector complex amplitude of the porous body skeleton; τ – stresses in the fluid related to the fluid pressure p according to: $\tau=-\beta p$, where β is a porosity; ω – frequency; ρ* - the parameter, that characterize inertial properties of media during vibration proceed and can be written as follows: $\rho^*=\dfrac{\omega^2(\rho_{11}\rho_{22}-\rho_{12}^2)+i\omega b(\rho_{11}+\rho_{22}+2\rho_{12})}{i\omega b+\omega^2\rho_{22}}$, ρ_{11}, ρ_{22}, ρ_{12} – mass densities, that are relating to each other as: $\rho_{11}+\rho_{22}=(1-\beta)\rho_s$, $\rho_{12}+\rho_{22}=\beta\rho_f$, $\rho_{12}=-\rho_a$, where: ρ_s – the solid density, ρ_f – the fluid density, $\rho_a=c\cdot\beta\cdot\rho_f$ – the density of connecting mass, c – the coefficient that depend on the pores geometry and frequency of excitation.

Terms γ^*, κ^*, η^* can be written as follows:

$$\gamma^*=-\frac{Q}{R}-\frac{i\omega b+\omega^2\rho_{12}}{i\omega b-\omega^2\rho_{22}},\ \kappa^*=\frac{R}{b+i\omega\rho_{22}},\ \eta^*=-b\left(1+\frac{Q}{R}\right)+i\omega\left(\rho_{12}-\rho_{22}\frac{Q}{R}\right), \tag{3}$$

where b – a dissipation constant, Q, R – elastic modulus's of media that depend on a porosity β and drained and undrained bulk modulus's K, K_u.

Physical relations between stresses and strains in poroelasticity are expressed as:

$$\sigma_{ij}=\left(\lambda+\frac{Q^2}{R}\right)\delta_{ij}e+2\mu e_{ij}+Q\delta_{ij}\varepsilon,\ \ \tau=Qe+R\varepsilon, \tag{4}$$

where $e=u_{j,j}$ and $\varepsilon=w_{j,j}$ are the solid and fluid dilatation, respectively, $e_{ij}=0.5(u_{k,j}+u_{k,j})$ – the deformation in the solid, w_j – the component of the fluid displacement vector:

$$w_i=\frac{\tau_{,i}+(i\omega b+\omega^2\rho_{12})u_i}{i\omega b-\omega^2\rho_{22}}. \tag{5}$$

2. Integral Formulation and Boundary Integral Equation

The Boundary Integral Equation Method is used for solving of this problem. According to J.Dominguez [5] equilibrium equations for 3-D linear dynamic poroelasticity are written as:

$$c_{ij}u_i+\int_\Gamma T_{ij}^*u_i\,d\Gamma+\int_\Gamma U_{4j}^*v_n\,d\Gamma=\int_\Gamma U_{ij}^*t_i\,d\Gamma+\int_\Gamma T_{4j}^*\tau\,d\Gamma,$$

$$\frac{c\tau}{-i\omega b+\omega^2\rho_{22}}+\int_\Gamma\left(T_{i4}^*u_i+U_{44}^*v_n\right)d\Gamma=\int_\Gamma\left[t_iU_{i4}^*+\tau T_{44}^*\right]d\Gamma,$$

where j=1, 2, 3 subscript indicates the jth set of the fundamental solution; $c_{ij}=\delta_{ij}$ for points inside the body Ω, $c_{ij}=0$ for points outside Ω, $c_{ij}=0,5\delta_{ij}$ for points on Γ where the boundary is smooth, Γ – the boundary of the body Ω; $t_i=\tau_{ij}\cdot n_j$, $U_n=U_i\cdot n_i$, n_i – the ith component of the unit normal $\overline{n}$ to the boundary Γ. The variables denotes by superscript (*) are those associated with the weighting displacement fields (fundamental solutions).

The fundamental solutions matrix in equations (1), (2) using analogy with thermoelasticity can be expressed as totality of four components:

– the solid displacement in k direction derived from the unit point force applied to the solid following j direction:

$$U^*_{kj}(r,\omega)=\frac{\delta_{kj}}{4\pi\mu}\frac{e^{i\lambda_3(\omega)r}}{r}-\sum_{m=1}^{3}a_m(\omega)\left[\delta_{kj}U_0(r,\omega,m)+r,_j\, r,_k\, U_2(r,\omega,m)\right],$$

$$U_0(r,\omega,m)=\frac{e^{i\lambda_m(\omega)r}}{r^3}\left[i\lambda_m(\omega)r-1\right],$$

$$U_2(r,\omega,m)=\frac{e^{i\lambda_m(\omega)r}}{r^3}\left[3-3i\lambda_m(\omega)r-\lambda_m^2(\omega)r^2\right],$$

where r – the distance between point at which displacement calculated and force applied point,

$$\alpha_1(\omega)=\frac{\lambda_2^2-k_1^2}{4\pi\rho^*\omega^2(\lambda_2^2-\lambda_1^2)},\ \alpha_2(\omega)=\frac{k_1^2-\lambda_1^2}{4\pi\rho^*\omega^2(\lambda_2^2-\lambda_1^2)},\ \alpha_3(\omega)=-\frac{1}{4\pi\rho^*\omega^2},$$

$$k_1^2(\omega)=-\frac{\rho^*\omega^2}{\lambda+2\mu},\ \lambda_3^2(\omega)=-\frac{\rho^*\omega^2}{\mu},\ \lambda_1^2+\lambda_2^2=\frac{\rho^*\omega^2}{\lambda+2\mu}+\frac{i\omega}{\kappa^*}+\frac{i\omega\eta^*\gamma^*}{\lambda+2\mu},$$

$$\lambda_1^2\cdot\lambda_2^2=\frac{\rho^*\omega^2}{\lambda+2\mu}\cdot\frac{i\omega}{\kappa^*};$$

– the fluid stress derived from the unit point force applied to the solid following k direction:

$$U_{4k}(r,\omega)=\frac{i\omega\eta^*}{4\pi(\lambda_2^2-\lambda_1^2)(\lambda+2\mu)}r,_k\, r\left[U_0(r,\omega,2)-U_0(r,\omega,1)\right];$$

– the solid displacement in j direction derived from the following body forces $X_i'^*$ and X_i^* applied to the fluid and solid, respectively

$$X_i'^*=\left[\frac{-1}{4\pi r}\right],_i,\quad X_i^*=\frac{i\omega b+\omega^2\rho_{12}}{-i\omega b+\omega^2\rho_{22}}X_i'^*,$$

$$U_{j4}(r,\omega)=\frac{\gamma^*}{4\pi(\lambda_2^2-\lambda_1^2)(\lambda+2\mu)}r,_j\, r\left[U_0(r,\omega,1)-U_0(r,\omega,2)\right];$$

– the fluid stress derived from the body forces $X_i'^*$ and X_i^* applied to the fluid and solid, respectively:

$$U_{44}(r,\omega)=\frac{1}{4\pi(\lambda_2^2-\lambda_1^2)}\left[(\lambda_2^2-k_1^2)e^{i\lambda_2(\omega)r}-(\lambda_1^2-k_1^2)e^{i\lambda_1(\omega)r}\right].$$

Using equation for stresses on the plane with unit normal n_j :

$$t_{ij}=\sigma_{ij}n_j=n_i\left(\lambda u_{k,k}+\frac{Q}{R}\tau\right)+n_j\,(u_{i,j}+u_{j,i}),$$

and equation (5) for the fluid displacement one can obtain:

– the solid stresses on the plane with unit normal n_j derived from the unit point force applied to the solid following k direction:

$$T_{jk}\left(r,\omega\right)=n_j\left[\lambda U_{lk,l}+\frac{Q}{R}U_{4k}\right]+\mu n_l\left[U_{jk,l}+U_{lk,j}\right],\quad k,j,l=1,2,3;$$

– the fluid normal displacement derived from the unit point force applied to the solid following k direction:

$$T_{4k}\left(r,\omega\right)=\frac{(i\omega b-\omega^2\rho_{12})U_{jk}n_j-\dfrac{\partial U_{4k}}{\partial n}}{(i\omega b+\omega^2\rho_{22})};$$

– the solid stresses on the plane with unit normal n_j derived from the body forces $X_i'^*$ and X_i^* applied to the fluid and solid, respectively:

$$T_{j4}\left(r,\omega\right)=n_j\left[\lambda U_{lk,l}+\frac{Q}{R}U_{44}\right]+\mu n_l\left[U_{j4,l}+U_{l4,j}\right];$$

– the fluid normal displacement derived from the body forces $X_i'^*$ and X_i^* applied to the fluid and solid, respectively:

$$T_{44}\left(r,\omega\right)=\frac{(i\omega b-\omega^2\rho_{12})U_{j4}n_j-\dfrac{\partial U_{44}}{\partial n}-\dfrac{1}{4\pi r^2}\dfrac{\partial r}{\partial n}}{(i\omega b+\omega^2\rho_{22})}.$$

3. Solving of the test problem

Consider the problem of harmonic vibration of the porous elastic layer under the load equally distributed on its surface. In this case displacements u_1, u_2, v_1, v_2 not occur but displacements u_3 and v_3 are the functions of the coordinate x_3 [5]. Components of the stress state σ_{ij} and τ depend only on the coordinate x_3 as well. As a result the system of differential equations in partial derivatives can be rewritten as the system of two usual differential equations:

$$(\lambda+2\mu)\frac{d^2u_3}{dx_3^2}+\rho^*\omega^2u_3-\gamma^*\frac{d\tau}{dx_3}=0\,, \tag{6}$$

$$\frac{d^2\tau}{dx_3^2}+\frac{i\omega}{\kappa^*}\tau+i\omega\eta^*\frac{du_3}{dx_3}=0\,. \tag{7}$$

The solution of the system (6)-(7) is given as:

$$u_3(x_3)=ue^{ikx_3}\,,\quad \tau(x_3)=\tau e^{ikx_3}\,. \tag{8}$$

Substituting the solution (8) in the system (6)-(7) and reducing to e^{ikx_3} it's possible to obtain:

$$\begin{bmatrix}(\omega^2/c_1^2-k^2) & -ikm\\ -k\omega m & (i\omega/\kappa^*-k^2)\end{bmatrix}\cdot\begin{pmatrix}u\\ \tau\end{pmatrix}=0\,, \tag{9}$$

where $m\equiv\gamma^*/(\lambda+2\mu)$, $c_1^2\equiv(\lambda+2\mu)/\rho^*$, $c_2^2\equiv\mu/\rho^*$.

The nontrivial solution can be obtained only in following case:

$$Det\begin{bmatrix}(\omega^2/c_1^2-k^2) & -ikm\\ -k\omega m & (i\omega/\kappa^*-k^2)\end{bmatrix}=0$$

or

$$k^4-zk^2+q=0\,, \tag{10}$$

where $z=\frac{i\omega}{\kappa^*}+\frac{\omega^2}{c_1^2}+im\omega\eta$, $q=\frac{i\omega}{\kappa^*}\frac{\omega^2}{c_1^2}$.

Using roots of the biquadrate equation (10):

$$k_{1,2}^2=0,5\left\{z\pm\sqrt{z^2-4q}\right\}, \tag{11}$$

the solution can be expressed as:

$$u_3(x_3)=u_1^{(+)}e^{ik_1x_3}+u_1^{(-)}e^{-ik_1x_3}+u_2^{(+)}e^{ik_2x_3}+u_2^{(-)}e^{-ik_2x_3}\,. \tag{12}$$

Since the system of algebraic equations (6)-(7) is homogeneous then the fluid stress τ using expressions (8) may be written as:

$$\tau(x_3)=X_1\left[u_1^{(+)}e^{ik_1x_3}-u_1^{(-)}e^{-ik_1x_3}\right]+X_2\left[u_2^{(+)}e^{ik_2x_3}-u_2^{(-)}e^{-ik_2x_3}\right], \tag{13}$$

where $X_i=\frac{1}{ik_1m}\left(\frac{\omega^2}{c_1^2}-k_i^2\right)$, i=1,2.

Equations for other parameters of stress-strain state of the porous elastic layer can be written as:

$$\varepsilon_{33}(x_3)=\frac{du_3(x_3)}{dx_3}=ik_1\left[u_1^{(+)}e^{ik_1x_3}-u_1^{(-)}e^{-ik_1x_3}\right]+$$

$$+ik_2\left[u_2^{(+)}e^{ik_2x_3}-u_2^{(-)}e^{-ik_2x_3}\right]; \quad (14)$$

$$\sigma_{33}(x_3)=(\lambda+2\mu)\frac{du_3(x_3)}{dx_3}+\frac{Q}{R}\tau=Z_1\left[u_1^{(+)}e^{ik_1x_3}-u_1^{(-)}e^{-ik_1x_3}\right]+$$

$$+Z_2\left[u_2^{(+)}e^{ik_2x_3}-u_2^{(-)}e^{-ik_2x_3}\right], \quad (15)$$

where $Z_i=(\lambda+2\mu)\left(ik_i+\frac{Q}{R}\frac{1}{ik_i\gamma}\left(\frac{\omega^2}{c_1^2}-k_i^2\right)\right)$, i=1,2;

$$\frac{d\tau(x_3)}{dx_3}=X_1ik_1\left[u_1^{(+)}e^{ik_1x_3}+u_1^{(-)}e^{-ik_1x_3}\right]+X_2ik_2\left[u_2^{(+)}e^{ik_2x_3}+u_2^{(-)}e^{-ik_2x_3}\right]; \quad (16)$$

$$U_3(x_3)=\frac{1}{i\omega b+\omega^2\rho_{22}}\left[(i\omega b-\omega^2\rho_{12})u_3-\frac{d\tau(x_3)}{dx_3}\right]=$$

$$=Y_1\left[u_1^{(+)}e^{ik_1x_3}+u_1^{(-)}e^{-ik_1x_3}\right]+Y_2\left[u_1^{(+)}e^{ik_2x_3}+u_1^{(-)}e^{-ik_2x_3}\right], \quad (17)$$

where $Y_i=\frac{1}{i\omega b+\omega^2\rho_{22}}\left((i\omega b-\omega^2\rho_{12})-\frac{1}{m}\left(\frac{\omega^2}{c_1^2}-k_i^2\right)\right)$, i=1,2.

To obtain additional relations for $u_1^{(+)}$, $u_1^{(-)}$, $u_2^{(+)}$, $u_2^{(-)}$ following boundary conditions are used:

1) $u_3(0)=0$,
2) $U_3(0)=0$, using equation (17), can obtain: $\frac{d\tau(0)}{dx_3}=0$,
3) $\sigma_{33}(l)=-P_0$,
4) $\tau(l)=0$.

Taking into account equations (12), (13), (15), (17) after following transformations one can obtain the system of linear algebraic equations:

$$u_1^{(+)}+u_1^{(-)}+u_2^{(+)}+u_2^{(-)}=0,$$

$$X_1ik_1(u_1^{(+)}+u_1^{(-)})+X_2ik_2(u_2^{(+)}+u_2^{(-)})=0,$$

$$Z_1(u_1^{(+)}e^{ik_1l}-u_1^{(-)}e^{-ik_1l})+Z_2(u_2^{(+)}e^{ik_2l}-u_2^{(-)}e^{-ik_2l})=-P_0,$$

$$X_1(u_1^{(+)}e^{ik_1l}-u_1^{(-)}e^{-ik_1l})+X_2(u_2^{(+)}e^{ik_2l}-u_2^{(-)}e^{-ik_2l})=0\,.$$

Using the solution of the system it's possible to calculate values of other parameters of stress-strain state.

The numerical solution of this problem was calculated using material properties which are corresponding to the Barea Sandstone [5], [6]: $\lambda=4\cdot 10^9$ N/m^2, $\mu=6\cdot 10^9$ N/m^2, $\beta=0.19$, $Q=1.399$ N/m^2, $R=0.444$ N/m^2, $\rho_{11}=2418$ Kg/m^3, $\rho_{22}=340$ Kg/m^3, $\rho_{12}=-150$ Kg/m^3, $b=0.19\cdot 10^9$ Ns/m^4.

Figure 1 shows comparing of normalized displacement u_3 at the top in elastic region (the curve with designation 1) and in porous solid of poroelastic region (the curve with designation 2) versus frequency for a range going from 0 to $5\omega_1$ where: $\omega_1=\frac{\pi}{2l}\sqrt{E_u/\rho^{**}}$, $E_u=\frac{2\mu(1-\nu_u)}{(1-2\nu_u)}$, $\rho^{**}=\rho_{11}+\rho_{22}+2\rho_{12}$.

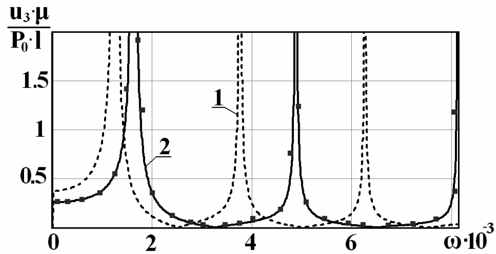

Fig. 1. Normalized displacement u_3 at the top versus frequency $\omega\cdot 10^{-3}$

Figure 2 presented comparing of normalized stresses σ_{33} in elastic region (the curve with designation 1) and in porous solid of poroelastic region (the curve with designation 2) versus frequency for the same range:

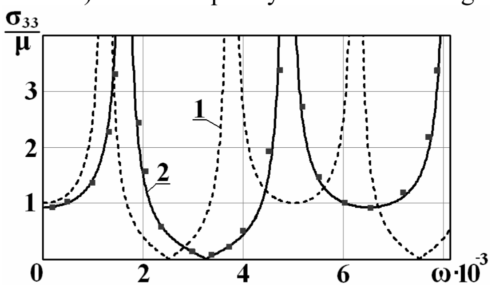

Fig. 2. Normalized stresses σ_{33} versus frequency $\omega\cdot 10^{-3}$

Consider this layer under other boundary conditions:

1) $u_3(0)=U_0$,

2) $U_3(0)=-m\omega^2 U_0(\rho_{12}+\rho_{22})$,

3) $\sigma_{33}(l)=0$,

4) $\tau(l)=0$.

The system of linear algebraic equations is following:

$$u_1^{(+)}+u_1^{(-)}+u_2^{(+)}+u_2^{(-)}=U_0\,,$$

$$X_1 ik_1(u_1^{(+)}+u_1^{(-)})+X_2 ik_2(u_2^{(+)}+u_2^{(-)})=m\omega^2 U_0(\rho_{12}+\rho_{22})\,,$$

$$Z_1(u_1^{(+)}e^{ik_1 l}-u_1^{(-)}e^{-ik_1 l})+Z_2(u_2^{(+)}e^{ik_2 l}-u_2^{(-)}e^{-ik_2 l})=0\,,$$

$$X_1(u_1^{(+)}e^{ik_1 l}-u_1^{(-)}e^{-ik_1 l})+X_2(u_2^{(+)}e^{ik_2 l}-u_2^{(-)}e^{-ik_2 l})=0\,.$$

Using the solution of the system it's possible to calculate values of other parameters of stress-strain state depending on frequency ω. Figures 3, 4 show graphs of normalized displacement u_3 at the top and normalized stresses σ_{33} in elastic region (the curve with designation 1) and in porous solid of poroelastic region (the curve with designation 2) versus frequency for a range going from 0 to $5\omega_1$ under other boundary conditions:

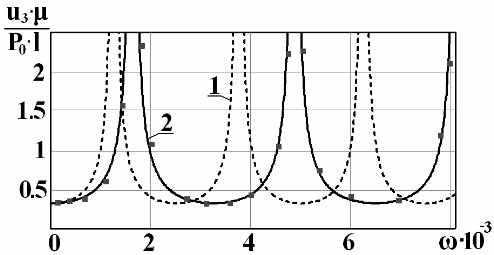

Fig. 3. Normalized displacement u_3 at the top versus frequency $\omega\cdot 10^{-3}$

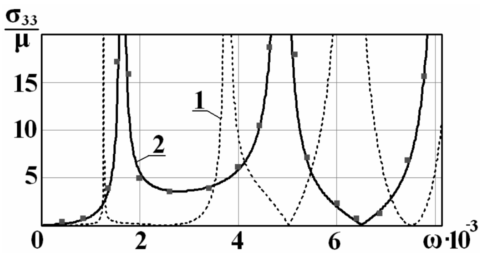

Fig. 4. Normalized stresses σ_{33} versus frequency $\omega\cdot 10^{-3}$

Points on figures present the BE results.

Conclusion

1. Figures show that graphs of the displacements and stresses in elastic region and in porous solid of poroelastic region have the same character but different values depending on frequency ω. It means that massive porous bodies can't be modeling as homogeneous elastic media but it is necessary to use the two phase model and equations of poroelasticity.

2. Since the agreement between the BE results and the analytical solution is good so such an approach can be used for development and testing of numerical techniques for analyzing of 3-D porous solids vibration.

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ЧИСЕЛЬНЕ РОЗВ'ЯЗАННЯ ЗАДАЧІ ПРО КОЛИВАННЯ ПОРОПРУЖНОГО МАСИВУ

Метод граничних інтегральних рівнянь застосовується для розв'язання задачі про тривимірні гармонічні коливання поропружного масивного тіла. Наведені основні розрахункові співвідношення та проаналізовано склад матриці фундаментальних розв'язків. Аналітично та чисельно розв'язана тестова задача про змушені коливання поропружного шару.

Ключові слова: пропружне середовище, граничні інтегральні рівняння, фундаментальний розв'язок, змушені коливання шару.

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ЧИСЛЕННОЕ РЕШЕНИЕ ЗАДАЧИ О КОЛЕБАНИЯХ ПОРОУПРУГОГО МАССИВА

Метод граничных интегральных уравнений применяется для решения задачи про трехмерные гармонические колебания пороупругого массивного тела. Приведены основные расчетные соотношения и проанализировано состав матрицы фундаментальных решений. Аналитически та численно решена тестовая задача про вынужденные колебания пороупругого слоя.

Ключевые слова: пороупругая среда, граничные интегральные уравнения, фундаментальное решение, вынужденные колебания слоя.

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Кара І.Д. **Чисельне розв'язання задачі про коливання поропружного масиву** // Опір матеріалів і теорія споруд. – К.: КНУБА, 2017. – Вип. 99. – С. 193 – 202.

Метод граничних інтегральних рівнянь застосовується для розв'язання задачі про тривимірні гармонічні коливання поропружного масивного тіла.

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Boundary Integral Equation Method is applied for numerical solution of the porous solids 3-D harmonic vibration problem.

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Кара И.Д. **Численное решение задачи о колебаниях пороупругого массива** // Опір матеріалів і теорія споруд. – К.: КНУБА, 2017. – Вып. 99. – С. 193 – 202.

Метод граничных итегральных уравнений применяется для решения задачи о трехмерных гармонических колебаниях пороупругого массивного тела.

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